DOCTORAL THESIS

Study and research paths for statistics teacher education in secondary school

Janielly Taila dos Santos Verbisck

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Study and research paths for statistics teacher education in secondary school

Els recorreguts d'estudi i investigació per a la formació del professorat d'estadística a l'educació secundària

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Study and research paths for statistics teacher education in secondary school

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BARCELONA, 2025

Dedico este trabalho ao meu anjo aqui encarnado:
José Gabriel dos Santos Verbisck

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Comunico ao leitor que, neste espaço dedicado aos agradecimentos, me expressarei na minha língua natal, de origem: o português brasileiro. Mas com algumas frases em catalão, castelhano e francês dirigidas especialmente a algumas pessoas. Também ocuparei um espaço um pouco maior do que normalmente se ocupa aqui. Por quê? Porque é um espaço mais livre e pessoal e posso me expressar não só como a doutoranda Janielly, mas simplesmente como Janielly. Um ser humano que ocupa muitas posições e se apresenta de muitas formas ao longo de sua existência. Um ser humano que é filha, irmã, amiga, companheira, estudante, professora, investigadora, mãe, e tantas outras... Um ser humano apaixonado por sua profissão que envolve estudar, estudar muito, aprender, planejar, ensinar, pesquisar, analisar, comunicar… um ser humano que se tornou professora de matemática após cursar a Licenciatura em Matemática no período de 2012-2016. Depois se aventurou em um mestrado em Educação Matemática no período de 2017-2019. E, por se inspirar em professores e colegas investigadores, também sentiu o desejo de seguir o trajeto, o percurso, de fazer um doutorado e obter o tão sonhado diploma de doutora em Educação Matemática.

Quanto custa realizar um doutorado?

Tanto financeiramente como em relação ao tempo dedicado em todo o processo, não há dúvidas, custa muito caro! Muitas vezes durante este doutoramento eu me perguntava “Mas, Janielly, por que escolher fazer uma coisa tão difícil, complicada e que toma tanta energia? Não era mais fácil seguir com a sua vidinha ‘tranquila’ de professora?”

Há diferentes fatores, situações, variáveis que movem cada ser humano ao largo de sua existência terrena. Eu comecei a entender que no meu caso, que não é único, o que me move é o desafio e o desejo de experenciar coisas novas. Quando alguém que eu conheço realiza ou realizou algo que me chama a atenção e que parece ressoar com meu processo de crescimento pessoal e professional, essa se torna uma inspiração, um exemplo.

Eu tive muitas inspirações para fazer o meu doutorado. A começar pela minha orientadora Marilena, que é uma mulher muito guerreira e que foi para fora do Brasil realizar um doutorado. A Danielly Kaspary também, que se aventurou neste desafio de realizar um doutorado entre duas universidades. Mas por que fazer um doutorado em outro país? O Brasil não é bom o suficiente para se obter um título de doutora? Na minha opinião, o Brasil possui instituições de investigação tão boas quanto qualquer outro país. Me arrisco a dizer que até superamos muitos por aí…

Realizar um doutorado nos proporciona um grande crescimento pessoal e profissional. Realizar um doutorado no exterior ou, no meu caso, entre uma instituição brasileira e uma estrangeira, me proporcionou e segue me proporcionando um crescimento que, custe o preço que custar, vale cada centavo e cada energia! Prefiro valorar este processo de crescimento em termos de tempo. No meu caso, foi um ciclo que custou cinco anos e dois meses (começando em março de 2020). Cinco anos é muito ou é pouco? Na linearidade em que vivemos é relativo. Pode ser “muito” para alguns ou “pouco” para outros. Eu passei por fases de “Meu Deus, isso não acaba nunca?” e outras de “Meu Deus, já está acabando? Preciso de mais tempo!”.

Como eu valoro estes cinco anos de minha vida?

Foram os melhores cinco anos da minha vida! Por um lado, foram os cinco anos de mais luta, choro, pranto, ansiedade, tristeza, saudade que vivi. Por outro lado, foram os cinco anos de maiores conquistas, resiliência, paciência, meditação, risadas, choros de alegria, conhecimento e autoconhecimento, força, persistência, amor, conexão.

Tentarei resumir os momentos que me marcaram mais nestes cinco anos para que fique registrado nesta memória. No meu primeiro ano, eu passei por uma pandemia em 2020, eu fui contaminada pelo COVID-19 e fiquei sem paladar e sem olfato por mais de quatro meses. Eu participei de um processo seletivo e fui comtemplada com uma bolsa de doutorado sanduíche em 11 de março de 2021. De março a agosto de 2021 foi todo um processo de idas ao Consulado, preparativos para vir a Barcelona, busca de moradia, seguro saúde, etecetera e tal. Eu cheguei em Barcelona em 2 de setembro de 2021. Os primeiros meses de adaptação me arrancaram muitas lágrimas, mas lá pelo quarto mês eu senti que deveria ficar mais porque o processo de crescimento estava apenas começando. A verdadeira Janielly estava apenas começando a dar os seus primeiros passos, estava começando a sair do casulo. Em 2022, conheci, encontrei e me reencontrei com muitos seres de luz. Visitei muitos países e cidades europeias a congressos e a passeio. Voltei ao Brasil em março de 2023 para matar as saudades e realizar mais estudos. Retornei a Barcelona em julho de 2023 com muitos dados para analisar e com as energias renovadas para seguir adiante. Qualifiquei em outubro de 2024 e dei aquele "gás" para finalizar o que precisava ser finalizado neste ciclo.

Há um aspecto que gostaria de ressaltar também: a Janielly de fevereiro de 2020 já não existe mais. Este doutoramento me impulsionou a um amadurecimento não esperado e o qual sou muito grata.

Sempre falta espaço para todas as entrelinhas e riqueza de detalhes de um pouco mais de cinco anos de estudos, vivências e experiências. Tenho a certeza de que estará registrado em um dos meus livros na biblioteca de meus *registros akáshicos*.

Agora passarei ao momento de agradecimentos mais formal.

O percurso para o desenvolvimento desta tese foi pavimentado e percorrido por meio do apoio de indivíduos e instituições que eu não poderia deixar de mencionar neste espaço de agradecimento.

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À minha ancestralidade e a minha família, agradeço por terem me proporcionado ser quem eu sou e chegar aonde cheguei.

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Aos meus amigos e amigas do Brasil, eu agradeço por existirem em minha vida e por todo amor e apoio que, ainda com uma distância de mais de nove mil quilômetros, fez toda a diferença para a conclusão desta etapa. Não mencionarei aqui um a um porque me faltaria espaço e minha memória poderia me fazer cometer o pecado de esquecer de alguém. Mas no meu coração não existe este tipo de limitação!

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À Capes, eu agradeço o apoio financeiro tanto para a realização do doutorado sanduíche (PDSE – Programada de Doutorado Sanduíche no Exterior) como durante toda esta jornada como bolsista do Programa de Demanda Social.

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Um ciclo precisa se encerrar para que novos possam começar. Eu escrevo essas últimas linhas de agradecimentos com lágrimas de muita alegria e gratidão. Que venham as novas aventuras. Que assim seja!

Partamos de uma análise da pergunta, da criatividade das respostas como ato de conhecimento, como processo de pergunta-resposta que deveria ser realizado por todos os que participam do processo educativo.

Paulo Freire e Antonio Faundez
(Por uma pedagogia da pergunta)

Caminante, son tus huellas
el camino y nada más;
caminante, no hay camino,
se hace camino al andar.
Al andar se hace el camino,
y al volver la vista atrás
se ve la senda que nunca
se ha de volver a pisar.
Caminante no hay camino
sino estelas en la mar.

Antonio Machado

i. Abstract

This dissertation contributes to the line of research developed within the Anthropological Theory of the Didactic (ATD) on the change of pedagogical paradigm that begins in our current educational systems, which intend to evolve from the *monumentalism* or *paradigm of visiting works* to the *paradigm of questioning the world*. The ATD proposes the design and implementation of new instructional formats called *study and research* paths (SRPs) and SRPs for teacher education (SRPs-TE) to progress in the study of the conditions that facilitate the change of paradigm in current education institutions, as well as the constraints that hinder its development. Within this context, our research focuses on secondary school mathematics teacher education in the domain of statistics. It is underpinned by the overarching methodology of didactic engineering—integrating preliminary analysis, *a priori* analysis, *in vivo* analysis, and *a posteriori* analysis—in the context of teacher education in Brazil.

The preliminary analysis examines dominant conceptions of statistics and teacher education concerning statistics in Brazilian universities, aligning them with two primary models: "conceptualist statistics" and "data science". It is followed by three studies based on the design and implementation of an SRP-TE following three rounds of didactic engineering. The first round consists of a pilot online SRP-TE with preservice mathematics teachers based on a generating question about water distribution in Brazil. The second round corresponds to an online SRP-TE based on the same generating question but that amounted for an entire course for in-service teachers. It's *in vivo* and *a posteriori* analysis highlighted the conditions and constraints associated with addressing data treatment at secondary level. The third and final round investigated a face-to-face SRP-TE with preservice teachers acting as pedagogical residents in secondary schools. This SRP-TE included various implementations of inquiry processes in secondary schools designed and managed by the preservice teachers.

The results highlight the role of SRP-TE in making visible and working with pre-service teachers two relevant didactic phenomena. The first and main one is the invisibility of data processing in the teaching of statistics at secondary and tertiary levels. The second corresponds to the predominance of a particular form of "applicationism" in statistics university education and in secondary school teacher education.

Keywords: Anthropological theory of the didactic, study and research path for teacher education, statistics education, secondary school, invisibility of data treatment.

ii. Resumo

Esta tese contribui para a linha de pesquisa desenvolvida no âmbito da teoria antropológica do didático (TAD) sobre a mudança de paradigma pedagógico que se inicia em nossos sistemas educacionais atuais, visando evoluir do *monumentalismo* ou *paradigma da visita às obras* para o *paradigma do questionamento do mundo*. A TAD propõe o desenho e a implementação de novos dispositivos didáticos denominados percursos de estudo e pesquisa (PEP) e PEP para a formação de professores (PEP-FP), com o objetivo de avançar no estudo das condições que facilitam a mudança de paradigma nas instituições educativas atuais, assim como das restrições que dificultam seu desenvolvimento. Nesse contexto, nossa pesquisa centra-se na formação de professores de matemática dos ensinos fundamental e médio no domínio da estatística. Apoia-se em uma metodologia global da engenharia didática — integrando a análise preliminar, a análise *a priori*, a análise *in vivo* e a análise *a posteriori* — no contexto da formação docente no Brasil.

A análise preliminar examina as concepções dominantes da estatística e da formação docente em estatística nas universidades brasileiras, alinhando-as com dois modelos principais: a "estatística conceitualista" e a "ciência de dados". A seguir, apresentam-se três estudos baseados no desenho e implementação de um PEP-FP, seguindo três rondas de um processo de engenharia didática. O primeiro consiste em um PEP-FP piloto baseado em uma questão geradora sobre a distribuição da água no Brasil, desenvolvido online com professores de matemática em formação inicial. O segundo corresponde a um PEP-FP com a mesma questão geradora, implementado online com professores em exercício. Sua análise *in vivo* e *a posteriori* permite destacar as condições e restrições associadas ao tratamento de dados estatísticos nos ensinos fundamental e médio. O terceiro e último estudo envolve um PEP-FP presencial com professores em formação inicial que atuavam como residentes pedagógicos em escolas de ensinos fundamental e médio. Esse PEP-FP incluiu diversas implementações de processos de investigação em escolas, desenhados e conduzidos pelos professores em formação.

Os resultados evidenciam o papel dos PEP-FP para tornar visíveis e trabalhar com os professores em formação dois fenômenos didáticos relevantes. O primeiro e principal é a invisibilidade do tratamento de dados no ensino de estatística nos ensinos fundamental e médio e na universidade. O segundo diz respeito ao predomínio de uma forma particular de "aplicacionismo" na formação estatística universitária e na formação de professores dos ensinos fundamental e médio.

Palavras-chave: Teoria antropológica do didático, percurso de estudo e pesquisa para a formação de professores, educação estatística, ensinos fundamental e médio, invisibilidade do tratamento de dados.

iii. Resum

Aquesta tesi contribueix a la línia de recerca desenvolupada dins de la Teoria Antropològica del Didàctic (TAD) sobre el canvi de paradigma pedagògic que s'inicia en els nostres sistemes educatius actuals, per evolucionar des del *monumentalisme* o *paradigma de la visita d'obres* al *paradigma del qüestionament del món*. La TAD proposa el disseny i la implementació de nous dispositius didàctics denominats *recorreguts d'estudi i investigació* (REI) i REI per a la formació del professorat (REI-FP) per avançar en l'estudi de les condicions que faciliten el canvi de paradigma a les institucions educatives actuals, així com les limitacions que en dificulten el desenvolupament. En aquest context, la nostra recerca se centra en la formació del professorat de matemàtiques de secundària en l'àmbit de l'estadística. Es recolza en una metodologia global de l'enginyeria didàctica —integrant l'anàlisi preliminar, l'anàlisi *a priori*, l'anàlisi *in vivo* i l'anàlisi *a posteriori*— en el context de la formació del professorat al Brasil.

L'anàlisi preliminar examina les concepcions dominants de l'estadística i la formació del professorat en estadística a les universitats brasileres, alineant-les amb dos models primaris: l'"estadística conceptualista" i la "ciència de dades". El segueixen tres estudis basats en el disseny i la implementació d'un REI-FP seguint tres rondes d'un procés d'enginyeria didàctica. La primera ronda consisteix en un REI-FP pilot basat en una qüestió generatriu sobre la distribució de l'aigua al Brasil, que es desenvolupa en línia amb professorat de matemàtiques en formació inicial. La segona ronda correspon a un REI-FP amb la mateixa qüestió generatriu que va ser implementat en línia amb professorat en actiu. La seva anàlisi *in vivo* i *a posteriori* permet destacar les condicions i restriccions associades al tractament de dades estadístiques a l'escola secundària. La tercera i última ronda implica un REI-FP presencial amb professorat en formació inicial que actuaven com a residents pedagògics en centres de secundària. Aquest REI-FP va incloure diverses implementacions de processos d'indagació en centres de secundària dissenyats i gestionats pel professorat en formació.

Els resultats posen en evidència el paper dels REI-FP per fer visibles i treballar amb el professorat en formació dos fenòmens didàctics rellevants. El primer i principal és la invisibilitat del tractament de dades en l'ensenyament de l'estadística a secundària i a la universitat. El segon correspon al predomini d'una forma particular d'"aplicacionisme" en la formació estadística universitària i en la formació del professorat de secundària.

Paraules-clau: Teoria antropològica del didàctic, recorregut d'estudi i investigació per a la formació del professorat, educació estadística, ensenyament secundari, invisibilitat del tractament de dades.

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CHAPTER I: RESEARCH CONTEXT

This first chapter explores the transition from the teaching profession to the formulation of a research problem in the domain of statistics education. It is divided into two main sections, each addressing different aspects of the relationship between mathematics teacher education and the teaching of statistics at the secondary school level.

The first section provides a historical overview of mathematics teacher education in Brazil, highlighting the evolution of teacher education models from the establishment of the first mathematics courses in the 1930s to the present. It examines the influence of the "3+1 model" and its impact on the preparation of mathematics teachers, particularly concerning statistical knowledge. The discussion then shifts to the author's personal journey through teacher education, illustrating how the dichotomy between mathematicians and didacticians affected her education and professional identity. The section concludes with an analysis of statistics in the Brazilian national curriculum, tracing its presence in key official documents and how it aligns (or fails to align) with contemporary statistical literacy demands. The final subsection formulates the research problem, identifying the double discontinuity between university-level statistics education and the practical statistical knowledge required for teaching in secondary schools.

The second section discusses the epistemological dimensions of statistical knowledge as both *subject to be taught* and as specialised knowledge for teaching. It reviews theoretical perspectives on statistical literacy, reasoning, and thinking, drawing from research in statistics education. Additionally, the chapter explores the professional knowledge required for effective statistics teaching, including pedagogical content knowledge (PCK) and statistical knowledge for teaching (SKT). The section underscores the necessity for teacher education programmes to integrate statistical investigation, data analysis, and technology use to equip teachers with the skills needed to foster students' statistical reasoning and decision-making abilities.

Through these discussions, the chapter lays the foundation for the research problem, addressing the gap between the statistics education provided in teacher education programmes and the practical needs of mathematics teachers in secondary education.

In all the dissertation, we will talk about "statistics" to refer to an ill-defined field of study that includes the work with data and the mathematisation of variability and randomness, including probability, descriptive statistics, informal inference, etc., depending on the institution we are considering (university statistics, school statistics, etc.).

1.1. FROM THE TEACHING PROFESSION TO THE FIRST FORMULATION OF THE RESEARCH PROBLEM

1.1.1. Mathematics teacher education in Brazil and the case of Statistics: a historical overview

In Brazil, secondary mathematics teacher education has been officially established since 1934, with the creation of the first mathematics course at the *Faculdade de Filosofia, Ciências e Letras* of the *Universidade de São Paulo*, followed by the implementation of this course also at the *Faculdade Nacional de Filosofia* at the *Universidade do Brasil* (in Rio de Janeiro), in 1939 (Silva, 2002). "These two institutions exercised leadership in the orientation of the courses that began to emerge in the other regions of the country, serving as models" (Silva, 2002, p. 104, our translation). In the case of the course at USP, the decision was to insert mathematics in the Science section with a curricular organisation based on:

> 1st year: Geometry (Analytical and Projective), Mathematical Analysis (1st part), General and Experimental Physics (1st part), Vector Calculus;
> 2nd year: Mathematical Analysis (2nd part), Rational Mechanics, General and Experimental Physics (2nd part);
> 3rd year: Mathematical Analysis (3rd part), Geometry, History of Mathematics.
> (Gomes, 2016, pp. 427-428, our translation).

These three years first led to the degree known as "*bacharelado em matemática*" (bachelor in mathematics). To obtain the aptitude as a mathematics teacher —the "*licenciatura em matemática*[1]"— a supplementary year of Didactics was required. This course was composed of disciplines with pedagogical and didactic training. This type of training was characterised, in the Brazilian context, as the "3 + 1 model", with three years of mathematics training and one year of didactic training. Silva (2002) stated that "the secondary school teacher appeared as a highly specialised by-product of that institution that sought, first and foremost, to promote research" (p.104, our translation). With this, it is evident that, in these early institutions, mathematics teacher education is a by-product of mathematics, "having given more emphasis to the constitution of scientists than to the training of teachers" (Gomes, 2016, p. 430, our translation). In this historical context neither the compulsory nature nor the importance of statistical knowledge was made explicit.

[1] It is difficult to translate into English. This term is closely related to the words "licence" and "licentiate", meaning that when one finishes the course, you have a "licence" to teach.

The 1950s witnessed the entrenchment of the "3 + 1" model in Brazilian mathematics higher education. During this period, the separation between rigorous disciplinary training and subsequent didactic instruction became deeply institutionalised. Although this model produced graduates with a strong foundation in mathematics, it often left them inadequately prepared for the practical challenges of teaching. The marginal attention to statistical content during this period reflects an educational culture that valued pure mathematical knowledge over applied one. Moreover, the expansion of mathematics courses beyond the initial centres in São Paulo and Rio de Janeiro further consolidated these early curricular models, reinforcing a legacy that would continue to influence teacher education for decades (Silva, 2002; Gomes, 2016).

The 1960s marked a critical turning point in Brazilian mathematics education, spurred by the international Modern Mathematics movement. This reform movement, which emphasised abstract structures, set theory, and investigative methodologies, had a transformative effect on both secondary and tertiary curricula. The Modern Mathematics movement emerged as part of a broader effort to reform mathematics education in response to the perceived inadequacies of traditional curricula. Influenced by the work of mathematicians such as Nicolas Bourbaki, the movement sought to emphasise the structural and axiomatic foundations of mathematics, moving away from rote memorisation and computational techniques. During this movement, statistics and probability underwent a significant reconfiguration within the mathematics curriculum.

The Modern Mathematics movement advocated for the inclusion of probability and statistics as fundamental topics within the mathematics curriculum. This shift was part of a broader effort to introduce students to the structures and principles underlying mathematical thought. In Brazil, this meant that statistics and probability began to appear more prominently in both secondary and tertiary education curricula, often framed within the context of set theory and abstract reasoning (Valente, 2007; Burigo, cited in Valente, 2008). The movement placed a strong emphasis on conceptual understanding over procedural skills. In the case of statistics, this meant moving beyond mere calculation and data manipulation to explore the theoretical foundations of probability, random variables, and statistical inference. For example, topics such as probability distributions and hypothesis testing were introduced in a way that highlighted their mathematical structure and logical coherence (Valente, 2008).

In the subsequent decades of the 1970s and 1980s, attempts were made to consolidate the reforms initiated during the Modern Mathematics movement. Nevertheless, this period was also marked by significant challenges. Although the importance of statistical education was increasingly acknowledged, it continued to receive only peripheral attention within teacher education programmes. The enduring dominance of the "3 + 1" model during this time further perpetuated the divide between advanced mathematical content and the pedagogical skills required for effective teaching (Gomes, 2016; Silva, 2002).

The 1990s brought growing recognition of the necessity for curricular reform in teacher education. Influenced by global educational trends and the emerging demands of a data-driven society, Brazilian educators began to advocate for a more integrated approach to mathematics education. This period also witnessed the emergence of new research on the teaching of probability and statistics, highlighting the need for a stronger focus on pedagogical application (Rodrigues & Silva, 2019).

Concerning statistics education, today, the official Brazilian document *Diretrices Curriculares Nacionais* (National Curriculum Guidelines) for courses in mathematics bachelor and teacher education (Brasil, 2001) did not present any explicit indication of addressing topics related to the study of statistics and probability in teacher education. It only mentions that, for the teacher education context, the contents of compulsory education can be included in the set of teacher professional contents, taking into account the national curricular guidelines for primary and secondary education. It therefore indirectly places them only in the field of the professional training.

Currently, in Brazil there is no longer the "3 + 1 model" for secondary school teacher education. The *bacharelado* and the *licenciatura* still coexist, but the last one includes a structure of disciplines with mathematical aspects and others with pedagogical aspects throughout the four years of training. However, according to the Brazilian Society of Mathematics Education (SBEM):

> Conceptually speaking, the current teacher education is still very similar to the first Mathematics course, created at the University of São Paulo (USP) in 1934. In most institutions, subjects are still grouped into specific contents and pedagogical contents, with a tendency to value the first group more than the second, even when it comes to the training of the mathematics teacher and not the mathematics bachelor (SBEM, 2013 pp. 3-4, our translation).

In each university, the secondary school teacher education programme must follow the curricular guidelines mentioned above (Brasil, 2001) and elaborate their specific pedagogical guidelines for both the course responsible for pure mathematics training and the course for mathematics teacher education. Regarding statistics, there is a predominance of the "conceptual approach as opposed to an approach more related to the pedagogical knowledge of the statistical contents that future teachers will work on in compulsory education" (Rodrigues & Silva, 2019, p.1, our translation). In general, there is a subject about “Probability and Statistics” and the same curricular structure and content is used for teacher education and other degree courses (bachelor) in Mathematics, Physics, Chemistry, and Engineering. The subject contains the following general topics: probability; random variables; distribution models; notions of sampling and estimation; descriptive statistics; confidence intervals; one- and two-sample hypothesis testing; analysis of variance; simple linear regression; and correlation.

1.1.2. From the teacher education programme to the teaching profession: my personal case

I received a mathematics teacher education degree (*licenciatura*) in 2012. At this moment, my university did not offer the bachelor course. During my studies, I thought was “just” following a course to train mathematicians, without paying much attention to the meaning of *licenciatura*. It took me a while to discover that I was trained to become a teacher. On many occasions, I had the feeling that the lecturers did not differentiate between training for future mathematics teachers and for future mathematicians. For example, in my first semester, the lecturers (mostly researchers in mathematics) gave their lessons about “Introduction to Calculus”, “Introduction to Logic”, and “Geometric Constructions”, and did not mention anything about school teaching or secondary education. I only got my bearings when I had the first lesson in the course “Mathematics teaching practices”, when the lecturer told everybody that, in the *licenciatura*, we were educated to become secondary teachers.

The mathematicians did not denote this distinction. However, over the four years of training, a distinct divide between mathematicians, who taught mathematical subjects, and *didacticians* who taught subjects related to pedagogical and didactical approaches, became evident. I also noticed how mathematicians underestimated didacticians by saying, for example, that they knew “less mathematics” than the former, or that they chose didactics as their research line because “it is easier than pure mathematics”. And then, at some point during the course, those of us who were receiving this training had to choose which “side” we were on, and we had to start defending our position.

Although I do not believe in what for me is clearly a false dichotomy, I began to assume the position of didactician or researcher in didactics of mathematics when I finished my degree and started a master’s degree in mathematics education in 2016. On this occasion, I was interested in studying something related to teaching and learning of probability. The domain of probability and statistics was the least addressed during my teaching training. It was present in only one subject called "Probability and Statistics" in the third year and without any approach or discussion about teaching and learning processes. My deficiencies in this domain became manifest when I started teaching mathematics and statistics in lower and upper secondary schools. I realised there was an important gap between the statistics I have been introduced to and the statistics I needed for teaching.

Therefore, in the research carried out during my master's degree, I addressed the problem of teaching probability and analysed a sample of Brazilian compulsory education textbooks. Through this study, it was possible to characterise what Gascón (2014) refers to as a *dominant epistemological model of school probability*. It was possible to see the little attention given to this domain in the textbooks, which are one of the main teaching tools used in Brazilian public schools (Bittar, 2021; Verbisck & Bittar, 2021). A more detailed discussion about the presence of statistics in the official curriculum of Brazilian compulsory education is presented in the following subsection.

1.1.3. Statistics in the official curriculum of Brazilian secondary education

1.1.3.1. National Curriculum Parameters

In this subsection, we present the main curricular guidelines presented in three Brazilian national curriculum documents that were published between 1998 and 2002: The "National Curriculum Parameters" (PCN) for lower secondary school, the "National Curriculum Parameters for upper secondary school" (PCNEM), and the "Complementary Educational Guidelines to the National Curriculum Parameters" (PNC+).

In Brazil, compulsory education is organised as: primary education (6-10 years old), lower secondary school (years 11-14 years old) and upper secondary school (15-17 years old). The "National Curriculum Parameters" (PCN) —in Portuguese: *Parâmetros Curriculares Nacionais*— were the first curriculum published and represented a significant step in the development of curriculum documents in Brazil containing objectives, competencies, skills, guidelines and suggestions for teaching and learning each subject at primary and secondary school levels.

The PCN for the lower secondary school level (11-14 years old) was published in 1998 and the discipline of mathematics was presented in four main domains: "Numbers and operations", "Space and shape", "Magnitude and measures", and "Information Treatment". It is widely agreed that primary school mathematics curricula should cover the study of numbers and operations (within arithmetic and algebra), space and shapes (in geometry), and quantities and measures (which facilitate connections among arithmetic, algebra, geometry, and other domains). However, a deeper examination of our society highlights the importance of incorporating additional content that equips individuals to process daily information, including understanding statistical data, interpreting tables and graphs, and reasoning with concepts related to probability and combinatorics. It is also stated that:

> [...] it is important to emphasise that understanding and making decisions in the face of political and social issues depends on critically reading and interpreting complex, often contradictory information, which includes statistical data and indices disseminated by the media. In other words, in order to exercise citizenship, it is necessary to know how to calculate, measure, reason, argue, treat information statistically, etc. (Brasil, 1998, p.27, our translation)

Thus, statistics is presented as part of the "Information Treatment" content block.

> In view of the link between the cross-cutting themes and mathematics, some considerations should be considered. The mathematical content set out in the Information Treatment block provides the tools needed to obtain and organise information, interpret it, make calculations and thus produce arguments to support conclusions about it. On the other hand, the questions and practical situations linked to the themes provide the contexts that make it possible to explore mathematical concepts and procedures in a meaningful way. (Ibid., p. 29, our translation).
>
> It is social demand that has led to this topic being highlighted as a content block, although it could be incorporated into the previous ones. The aim is to emphasise its importance, given its current use in society. This block will include studies on notions of statistics and probability, as well as counting problems involving the multiplicative principle. Of course, the aim is not to develop work based on defining terms or formulae involving these subjects. Concerning statistics, the aim is to get students to build procedures for collecting, organising and communicating data, using tables, graphs and representations that frequently appear in their day-to-day lives. In addition, to calculate some statistical measures such as mean, median and mode to provide new elements for interpreting statistical data. (Ibid., p. 52, our translation).

The PCN for the lower secondary school level highlighted the importance of integrating the Cross-Cutting Themes into mathematics teaching, emphasising that the "Information treatment" content block is not limited to the transmission of theoretical content, but offers essential tools for obtaining, organising and interpreting data.

Finally, the PCN for the lower secondary school level stressed that it is crucial that, when analysing graphs, students develop the ability to evaluate aspects that influence the reliability of the data presented, such as relative frequency and the appropriateness of scales. Graphs can often manipulate data in a way that leads to misinterpretation. To avoid these errors, it is important that students are trained to identify missing information and check the accuracy of the data collected during their research. The contents of the 'Information treatment' block can be used in interdisciplinary projects that connect Mathematics with other subjects, such as History and Geography, as well as relevant themes, such as Health and the Environment.

In 2000, the "National Curriculum Parameters for upper secondary school" (PCNEM) —in Portuguese: *Parâmetros Curriculares Nacionais para o ensino médio*— were published, specifying the competencies and skills for each subject area as well as the knowledge to be developed at this educational level. Mathematics is included in the area entitled *Natural Sciences, Mathematics and their Technologies*. Continuing with this final stage of compulsory education, the notions of statistics become even more useful for other scientific disciplines and fields of knowledge, as indicated by the following statements:

> The skills of describing and analysing large amounts of data, making inferences and predictions based on a population sample, and applying probabilistic and combinatorial ideas to natural and everyday phenomena are applications of Mathematics to real-world issues that have expanded significantly and become rather complex. Statistical and probabilistic techniques and reasoning are undoubtedly tools for both the Natural Sciences and the Humanities. This underscores the importance of a careful approach to counting, statistics, and probability content in upper secondary education, thus broadening the interface between learning Mathematics and other sciences and fields.
> Mathematical concepts pertaining to finite sets of data also take on special significance for the Humanities and for the ordinary citizen, who is immersed in vast amounts of statistical or probabilistic information. (Brasil, 2000, pp. 44–45, our translation)

In this statement it is highlighted the transversal nature of probability and statistics, which extend beyond Mathematics to a variety of knowledge areas. In today’s context, characterised by an information-driven era, the ability to interpret data and make inferences has become indispensable. Addressing these topics in upper secondary education, not only prepares students for academic challenges, but also equips them to deal with the complexity of the modern world, in which statistical and probabilistic information is present in everyday decisions, public policies, and scientific advances. Moreover, the integration of mathematics with other sciences reinforces the notion that knowledge is interdisciplinary, and that mastery of mathematical tools can enhance one’s understanding of natural and social phenomena.

Building on the curricular guidelines for upper secondary education, the “Complementary Educational Guidelines to the National Curriculum Parameters” (PCN+)—in Portuguese: *Orientações Educacionais Complementares aos Parâmetros Curriculares Nacionais*—, published in 2002, reaffirm that statistics and probability need to be addressed as a field of concepts and procedures that allow Mathematics to be applied to real-life situations, aiding in the quantification and interpretation of information not immediately calculable.

A key consideration in the study of statistics is that:

> Data analysis has proved essential in social and economic issues, such as statistics on health, population, transport, budgets, and market-related questions. It is proposed that this be the third pillar or structuring theme of teaching, focusing on finite data sets that may be numerical or qualitative, leading to procedures that differ substantially from other themes in how they are quantified, by means of combinatorial counting processes, frequencies, statistical measures, and probabilities. This theme can be divided into three thematic units: Statistics, Counting, and Probability.
> [...] Mathematics at the upper secondary level can be decisive in enabling students to read information conveyed in the media and in other areas of knowledge in the form of tables, graphs, and statistical data. Nonetheless, at this stage of schooling, students are expected to go beyond merely reading information and to reflect more critically on its meaning. Thus, the proposed theme should go beyond simple data description and representation, moving towards data investigation and decision-making.

> Statistics and Probability should therefore be viewed as a set of ideas and procedures that allow Mathematics to be applied to real-world issues, especially those arising from other fields. They should also be regarded as ways in which Mathematics can quantify and interpret data sets or information that cannot be quantified directly or precisely. Statistics, for instance, can be employed to analyse voting intentions in an election or the potential success of launching a product on the market prior to the election itself or the product's manufacture. This is done through statistical research, which involves sampling, data collection, and information analysis. (Brasil, 2002, p. 126, our translation)

It underscored the centrality of statistics and probability not merely as mathematical tools but as essential instruments for understanding and influencing social, economic, and scientific issues. Proposing teaching organisation into three thematic units—Statistics, Counting, and Probability—reflects the need for a structured and contextualised approach that enables students to comprehend the wide range of applications of these concepts. Furthermore, it highlights the importance of going beyond a superficial reading of data, encouraging a critical reflection on its meaning and implications. This perspective is crucial in an increasingly data-oriented world, where the ability to analyse information and make evidence-based decisions is an indispensable skill. The mention of practical examples, such as examining voting intentions or introducing new products, illustrates how statistics is present in both everyday and professional contexts, reinforcing its relevance for the comprehensive education of students.

The three curricular guidelines, PCN (for lower secondary school), PCNEM and PCN+ (both for upper secondary school), presented the competencies and skills to be developed at each level of schooling. Table 1.1.1 summarises the competencies and skills to be developed at each educational level.

Table 1.1.1. Competencies and skills for lower and upper secondary school

Curriculum Guideline	**Competencies and skills to be develop**
PCN Lower secondary school	**Grades 6 and 7**: • Collecting and organising data and using appropriate visual resources (flowcharts, tables and graphs) to summarise and communicate them and enable conclusions to be drawn. • Reading and interpreting data expressed in tables and graphs. • Understanding the meaning of the arithmetic mean as an indicator of the trend of a survey. (Brasil, 1998, p. 74, our translation)
	Grades 8 and 9: • Reading and interpreting data expressed in column graphs, sector graphs, histograms and frequency polygons. • Organising data and constructing suitable visual resources such as graphs (column graphs, sector graphs, histograms and frequency polygons) to present data globally, highlight relevant aspects, summarise information and enable inferences to be made. • Understanding terms such as frequency, relative frequency, sample of a population to interpret information from a survey. • Distributing the frequencies of a variable in a survey into classes in such a way as to summarise the data with a reasonable degree of precision. • Obtaining the measures of central tendency in a survey (mean, mode and median), understanding their meaning in order to make inferences. (Ibid., p. 90, our translation)

Curriculum Guideline	Competencies and skills to be develop
PCNEM Upper secondary school	• Identifying, analysing, and applying knowledge about variable values represented in graphs, diagrams, or algebraic expressions, carrying out trend forecasts, extrapolations, interpolations, and interpretations; • Qualitatively analysing quantitative data, represented graphically or algebraically, related to socio-economic, scientific, or everyday contexts. (Ibid, 2000, p. 96, our translation)
PCN+ Upper secondary school	• Identifying appropriate ways of describing and representing numerical data and information of a social, economic, political, scientific-technological, or abstract nature. • Reading and interpreting statistical data and information presented in different languages and representations, whether in the media or other texts and media outlets. • Calculating averages and evaluating the variability of data or information sets of various types. • Understanding and forming judgements about statistical information of a social, economic, political, or scientific nature presented in texts, news, advertisements, censuses, research, and other sources. (Ibid, 2002, pp. 127–128, our translation)

The competencies for Grades 6 and 7 focus on foundational skills in data collection and interpretation. Students should be introduced to basic visual resources, such as flowcharts and graphs, and learn to understand the arithmetic mean. This foundational stage emphasises the importance of collecting and organising data, which sets the groundwork for more complex analyses in later grades. For Grades 8 and 9, the competencies expand significantly. Students are expected to read and interpret more complex graphical representations, such as histograms and frequency polygons. Additionally, they should learn to apply terms like frequency and relative frequency, which enhances their analytical capabilities. The introduction of measures of central tendency (mean, mode, and median) allows students to make more informed inferences based on their analyses. This progression indicates a clear build-up of skills, where each competency builds upon the previous one, fostering a deeper understanding of data analysis.

The transition to upper secondary education shifts the focus towards more sophisticated analytical techniques. The competencies outlined in PCNEM require students to engage in qualitative analyses of quantitative data, implying a higher level of critical thinking and application of knowledge. Students are expected to apply their understanding of data in socio-economic and scientific contexts, thus bridging theoretical knowledge with practical application. In PCN+, the competencies further advance by requiring students to demonstrate a comprehensive understanding of statistical data across various contexts, including the one published through media and in research journals.. Skills such as calculating averages and evaluating variability showcase a more nuanced approach to data analysis, empowering students to form judgements based on diverse information sources.

While it seems to have a progression from basic data handling in lower secondary school, to complex data analysis in upper secondary school, the core focus on data interpretation and statistical understanding remains consistent. Each level introduces new dimensions of analysis, with lower secondary concentrating on foundational skills and upper secondary delving into application and judgement.

1.1.3.2. National Common Core Curriculum

The National Common Core Curriculum (BNCC)—in Portuguese: *Base Nacional Comum Curricular*—, for compulsory education, is the curriculum document currently in use in Brazil, having been implemented on 20 December 2017. It went through a process of revisions that generated two versions prior to the final version. With this final document, "education networks and public and private school institutions now have a mandatory national reference for the development or adaptation of their curricula and pedagogical proposals" (Brasil, 2018, p. 5, our translation). This shows how the BNCC establishes a unified guideline, forcing institutions to align their curricula to a national standard, which contributes to consistency and equity in Brazilian education. Moreover, the BNCC "is a normative document that defines the organic and progressive set of essential learning that all students should develop throughout the grades and modalities of compulsory education" (Ibid., p. 7, our translation). This underlines the structured and progressive nature of the document, highlighting that the BNCC not only guides but also organises learning in such a way as to ensure the continuous development of students throughout their school levels.

In this curriculum guideline, we find details for the primary and secondary school levels. We looked at the discipline of mathematics, with particular attention to the guidelines for the domain of statistics, especially in the secondary school level, the stage focused on in this research. Firstly, it should be said that, as far as mathematics is concerned in primary school and lower secondary school:

> […] through the articulation of its various domains – Arithmetic, Algebra, Geometry, Statistics and Probability – needs to ensure that students relate empirical observations from the real world to representations (tables, figures and diagrams) and associate these representations with a mathematical activity (concepts and properties), making inductions and conjectures. In this way, they are expected to develop the ability to identify opportunities to use mathematics to solve problems, applying concepts, procedures and results to obtain solutions and interpreting them according to the contexts of the situations. The deduction of some properties and the verification of conjectures based on others can be encouraged, especially towards the end of primary school. (Brasil, 2018, p. 265)

It stresses the importance of integrating theory with practice, emphasising that the articulation between the various fields of mathematics, fosters the development of critical thinking and the ability to solve problems through induction and the verification of hypotheses. The guidelines stated the importance of articulating the four domains, in which mathematics is divided into in these educational levels: Arithmetic, Algebra, Geometry, Statistics and Probability. According to the BNCC, this articulation will enable students to relate everyday empirical experiences to mathematical representations and thus correlate them with mathematical activities in which they can make inductions, conjectures, solve problems, apply concepts, interpret situations and contexts, among other mathematical activities. Focusing specifically on statistics, the BNCC also stated that the domain (or thematic unit, in the terms of BNCC) “Probability and Statistics” deals with uncertainty and data processing:

> [...] proposes the approach of concepts, facts and procedures present in many problem situations of everyday life, science and technology. Thus, all citizens need to develop the skills to collect, organise, represent, interpret and analyse data in a variety of contexts to make well-founded judgements and appropriate decisions. This includes reasoning and using statistical concepts, representations and indices to describe, explain and predict phenomena.
> It is worth highlighting the use of technology – such as calculators to evaluate and compare results, and spreadsheets to help construct graphs and calculate measures of central tendency. Consulting the websites of research institutes – such as the Brazilian Institute of Geography and Statistics (IBGE) – can offer potentially rich contexts not only for learning statistical concepts and procedures, but also for using them to understand reality. (Brasil, 2018, p. 274, our translation).

The relevance of statistics is emphasised not just as a theoretical field but as a practical tool for understanding reality. The use of technology and reference to official data sources such as the IBGE reinforce the importance of contextualising statistical teaching with real-life applications. News and research (of different natures) are published all the time, and analyses are carried out using statistical tools, such as data collection and sampling, representation and interpretation of data in tables and graphs, etc. That is why students need to develop skills in collecting, organising, representing, interpreting and analysing data in a variety of contexts, especially the real ones. These skills also enable judgement and decision-making, which are part of human activity. Many technological tools can also help develop these skills, as highlighted in this excerpt from the document.

Still concerning statistics, some of its primary concepts are already being worked on from the first year of primary school, e.g.:

> [...] the first steps involve working on collecting and organising data from a survey of interest to the students. Planning how to carry out the research helps students understand the role of statistics in their everyday lives. Thus, reading, interpreting and constructing tables and graphs play a fundamental role, as well as how to produce written text to communicate the data, since it is necessary to understand that the text must summarise or justify the conclusions. In lower secondary school level, students are expected to know how to plan and construct descriptive statistical research reports, including measures of central tendency and the construction of tables and various types of graphs. This planning includes defining the relevant questions and the population to be researched, deciding whether or not to use a sample and, where appropriate, selecting its elements using a suitable sampling technique. (Ibid, p. 275, emphasis added, our translation).

As the study of statistics begins in the early years of primary school, in the lower secondary school level students are expected to be able to develop the planning and construction of descriptive statistical research reports, calculations of mean, median and mode (and the reason for their use in statistical research), as well as the construction of tables, graphs and their various types. The choice (or not) of samples for the inquiry to be carried out is also considered relevant at this school level, and this is if the student is already able to use sampling techniques adequately.

After these discussions and guidelines on what is considered essential in the teaching of statistics, the BNCC presents the delimitation of the objects of knowledge and skills to be worked on in each grade of primary and lower secondary school levels. With this delimitation, it is considered that:

> [...] mathematical notions are taken up, broadened and deepened year by year. However, it is essential that these skills are not read in a fragmented way. Understanding the role that a particular skill plays in all the learning requires understanding how it connects with skills from previous years, which leads to identifying the learning that has already been consolidated, and to what extent the work on developing the skill in question serves as a basis for subsequent learning. (Brasil, 2018, p. 276, our translation).

This final quote reinforces the importance of an integrated and cumulative approach to teaching mathematics, where each new skill builds on the previous ones, guaranteeing the continuity and deepening of knowledge throughout the school career.

Given this orientation and in an attempt to understand which knowledge objects and skills are envisaged for the teaching of statistics in each grade of primary and lower secondary school levels, we looked at these knowledge objects and skills for the study of statistics. The BNCC explained this knowledge in terms of "objects of knowledge" and "skills" for each grade and we organised and presented they in Table 1.1.2.

Table 1.1.2. "Objects of knowledge" and "skills" proposed for the study of statistics in primary school and lower secondary school

Grade	Objects of knowledge	Skills
1st	Reading tables and simple column graphs.	Read data expressed in tables and simple column graphs.
	Collecting and organising information. Personal records for communicating information collected.	Carry out survey involving up to two categorical variables of interest and a universe of up to 30 elements and organise data using personal representations (Brasil, 2018, pp. 280-281, our translation).

Grade	Objects of knowledge	Skills
2nd	Collecting, classifying and representing data in simple and double-entry tables and column graphs.	Compare information from surveys presented in double entry tables and simple column or bar graphs in order to better understand aspects of the surrounding reality. Carry out survey in a universe of up to 30 elements, choosing up to three categorical variables of interest, organising the data collected in lists, tables and simple column graphs (Ibid, pp. 283-284, our translation).
3rd	Reading, interpreting and representing data in double-entry tables and bar charts.	Solve problems whose data are presented in double-entry tables, bar charts or column graphs. Read, interpret and compare data presented in double-entry tables, bar or column graphs, involving significant survey results, using terms such as higher and lower frequency, appropriating this type of language to understand significant aspects of socio-cultural reality.
	Collecting, classifying and representing data on categorical variables using tables and graphs.	Carry out survey involving categorical variables in a universe of up to 50 elements, organise the data collected using lists, simple or double-entry tables and represent them in simple column graphs, with and without the use of digital technologies (Ibid, pp. 288-289, our translation).
4th	Reading, interpreting and representing data in double entry tables, single and grouped column graphs, bar and column graphs and pictorial graphs.	Analyse data presented in simple or double-entry tables and column or pictorial graphs, based on information from different areas of knowledge, and produce a text summarising their analysis.
	Differentiating between categorical and numerical variables. Collecting, classifying and representing research data.	Carry out survey involving categorical and numerical variables and organise data collected using tables and simple or grouped column graphs, with and without the use of digital technologies (Ibid, pp. 292-293, our translation).
5th	Reading, collecting, classifying, interpreting and representing data in double entry tables, grouped column graphs, pictorial graphs and line graphs.	Interpreting statistical data presented in texts, tables and graphs (columns or lines), referring to other areas of knowledge or other contexts, such as health and traffic, and producing texts with the aim of summarising conclusions. Carry out survey involving categorical and numerical variables, organise data collected using tables, column, pictorial and line graphs, with and without the use of digital technologies, and present a written text on the purpose of the survey and a summary of the results (Ibid, pp. 296-297, our translation).
6th	Reading and interpreting tables and graphs (single or multiple columns or bars) relating to categorical and numerical variables.	Identify the variables and their frequencies and the constituent elements (title, axes, legends, fonts and dates) in different types of graphs. Interpreting and solving situations involving survey data on environmental contexts, sustainability, traffic, responsible consumption, among others, presented by the media in tables and different types of graphs, and writing texts with the aim of summarising conclusions.
	Data collection, organisation and tabulation. Constructing different types of graphs to represent them and interpreting the information.	Plan and collect survey data on social practices chosen by the students and use spreadsheets to table, represent and interpret the information in tables, various types of graphs and text.
	Different types of information representation: graphs and flowcharts.	Interpret and develop simple flowcharts, identifying the relationships between the objects represented (e.g. the position of towns considering the roads linking them, the hierarchy of employees in a company, etc.) (Ibid, pp. 304-305, our translation).

Grade	Objects of knowledge	Skills
7th	Statistics: mean and range of a data set.	Understand, in meaningful contexts, the meaning of the statistical mean as an indicator of the trend of a survey, calculate its value and relate it intuitively to the amplitude of the data set.
	Sample survey and census survey. Planning a survey, collecting and organising data, constructing tables and graphs and interpreting information.	Plan and carry out survey involving social issues, identifying the need for a census or sample, and interpreting the data to communicate it in written reports, tables and graphs, with the support of spreadsheets.
	Sector graphs: interpretation, relevance and construction to represent a set of data.	Interpret and analyse data presented in sector graphs published by the media and understand when it is possible or appropriate to use them (Ibid, pp. 310-311, our translation).
8th	Bar, column, line or sector graphs and their constituent elements and suitability for a given set of data.	Evaluate the suitability of different types of graphs to represent a set of survey data.
	Organising the data of a continuous variable into classes.	Classify the frequencies of a continuous variable in a survey into classes to summarise the data in a way that is suitable for decision-making.
	Measures of central tendency and dispersion.	Obtain the values of measures of central tendency in a statistical survey (mean, mode and median) with an understanding of their meanings and relate them to the dispersion of data, indicated by the range.
	Census or sample surveys. Planning and carrying out a sample survey.	Select reasons of different kinds (physical, ethical or economic) that justify carrying out sample and non-census surveys, and recognise that the sample can be selected in different ways (simple casual, systematic and stratified sampling). Plan and carry out a sample survey, selecting an appropriate sampling technique, and write a report containing the appropriate graphs to represent the data sets, highlighting aspects such as measures of central tendency, amplitude and conclusions (Ibid, pp. 314-315, our translation).
9th	Analysing graphs published by the media: elements that can lead to misreading or misinterpretation.	Analyse and identify the elements in graphs published by the media that can lead to reading errors, sometimes deliberately, such as inappropriate scales, legends that are not correctly explained, omission of important information (sources and dates), among others.
	Reading, interpreting and representing survey data expressed in double entry tables, single and grouped column graphs, bar and sector graphs and pictorial graphs.	Choosing and constructing the most appropriate graph (columns, sectors, lines), with or without the use of spreadsheets, to present a given set of data, highlighting aspects such as measures of central tendency.
	Planning and carrying out a sample survey and presenting a report.	Plan and carry out a sample survey involving a social issue and communicate the results in a report containing an assessment of measures of central tendency and amplitude, appropriate tables and graphs, constructed with the support of spreadsheets (Ibid, pp. 318-319, our translation).

From a critical perspective of the objects of knowledge and skills indicated for primary and lower secondary school levels, we can see advances and, at the same time, limitations in the curriculum proposal for teaching statistics. Although the guidelines present indications from the literature on statistics education, "it seems to us that the BNCC only draws up a list of knowledge objects and skills in probability and statistics, which makes it impossible to have an articulated movement of concepts and procedures that promotes working on probabilistic and statistical reasoning throughout schooling" (Lopes et al., 2024, p. 14, our translation). In this sense, although the BNCC includes these topics from the early years, as Almeida (2023, p. 91, our translation) recognises, "if on the one hand the BNCC advances in the sense of dealing with Probability and Statistics topics from the early years of schooling, on the other it goes backwards by treating these topics in a deterministic, fragmented and decontextualised way".

The organisation of the content, distributed progressively from 1st to 9th grade, demonstrates the concern to establish, in a sequential manner, skills related to reading, organising and interpreting data, as well as conducting surveys and solving problems involving statistical and probabilistic variables. However, the segmentation into isolated items and the order in which they are presented —in which, for example, skills related to carrying out research often appear at the end of each year's set of skills. Thus, although the Brazilian curriculum recommendations make an effort to include statistics and probability in the maths curriculum, they are insufficient to promote the progressive and integrated development of these competencies (Lopes et al., 2014; Righi & De Paula, 2021, our translation).

These curriculum guidelines also provide orientations on the continuity of mathematics teaching in the last level of compulsory education, the upper secondary school. The aim is to promote continuous, in-depth teaching of the thematic units worked on in lower secondary school, focusing on applying maths to reality in different contexts. As Giordano et al. (2021) point out, statistics education, in particular, is gaining relevance in the BNCC as a tool for developing students' ability to interpret data, make informed decisions and actively participate in society.

> [...] in upper secondary school, the focus is on building an integrated view of maths, applied to reality, in different contexts. Consequently, when reality is the reference, it is necessary to take into account the daily experiences of secondary school students – impacted in different ways by technological advances, the demands of the labour market, the projects for the good life of their people, the potential of social media, among others. In this context, the importance of using digital technologies and applications both for mathematical investigation and to continue the development of computational thinking, which began at the previous stage, is also highlighted.
>
> Given these considerations, the area of Maths and its Technologies has a responsibility to take advantage of all the potential already built up by these students in primary school, to promote actions that expand the mathematical literacy begun at the previous stage. This means that new specific knowledge must stimulate more elaborate processes of reflection and abstraction, which support ways of thinking that allow students to formulate and solve problems in various contexts with more autonomy and mathematical resources. (Brazil, 2018, pp. 528–529).

This statement reinforces the importance of contextualising maths teaching, connecting it to students' experiences and contemporary demands. Giordano et al. (2019) complement this idea by pointing out that statistics education, when worked on through projects, allows students to experience real situations of collecting, analysing and interpreting data, developing essential skills for exercising citizenship. However, it is necessary to question whether schools are prepared to implement this approach, especially in regions that lack technological resources and adequate teacher education. It reinforces the need to consolidate and deepen mathematical literacy in secondary education. Giordano et al. (2019) point out that statistics education when worked on in a contextualised way and through projects, can be an effective way of achieving this goal. However, it is essential that teachers have adequate training and access to pedagogical resources that allow them to implement these practices in the classroom.

In this last stage of compulsory education, the skills to be developed by students have an even more reality-orientated focus. In problem-solving processes, skills related to reasoning, representation, communication and argumentation are fundamental to the development of this citizen. The BNCC presented five specific competencies for maths and its technologies in upper secondary school. Each competence is associated with the skills expected to be developed by students. We emphasise that these specific competencies and skills are not presented by grade, as we saw in the primary and lower secondary school levels. They are proposed to be developed over the three years of the upper secondary school. So, among the skills mentioned for each specific competence, we have highlighted those relating to the teaching of statistics and presented them in Table 1.1.3.

Table 1.1.3. Specific competencies and related skills proposed for the study of statistics in upper secondary school

Specific competencies	Related skills
(1) Use mathematical strategies, concepts and procedures to interpret situations in various contexts, be they everyday activities, facts from the natural and human sciences, socio-economic or technological issues, disseminated by different media, in order to contribute to a general education (Brasil, 2018, p. 532, our translation).	Analyse tables, graphs and samples of statistical survey presented in reports published by different media, identifying, where appropriate, inadequacies that could lead to misinterpretation, such as inappropriate scales and samples (Ibid, p. 533, our translation).
(2) Propose or participate in actions to investigate the challenges of the contemporary world and make ethical and socially responsible decisions, based on the analysis of social problems, such as those related to health situations, sustainability, the implications of technology in the world of work, among others, mobilising and articulating concepts, procedures and languages specific to mathematics (Ibid, p. 534, our translation).	Plan and carry out sample survey on relevant issues, using data collected directly or from different sources, and communicate the results in a report containing graphs and interpretation of measures of central tendency and measures of dispersion (amplitude and standard deviation), whether or not using technological resources (Ibid, p. 535, our translation)i

Specific competencies	Related skills
(3) Use mathematical strategies, concepts, definitions and procedures to interpret, build models and solve problems in various contexts, analysing the plausibility of the results and the suitability of the proposed solutions, in order to build consistent arguments (Ibid, p. 537, our translation).	Solve and prepare problems in different contexts involving the calculation and interpretation of measures of central tendency (mean, mode, median) and measures of dispersion (amplitude, variance and standard deviation) (Ibid, p. 538, our translation).
(4) Understand and use, with flexibility and precision, different mathematical representation formats (algebraic, geometric, statistical, computational, etc.) in the search for solutions and communication of problem results (Ibid, p. 539, our translation).	Constructing and interpreting frequency tables and graphs based on data obtained from statistical sample surveys, whether or not using software that interrelates statistics, geometry and algebra. Interpreting and comparing sets of statistical data using different diagrams and graphs (histograms, box plots, branches and leaves, among others), recognising the most efficient for analysis (Ibid, p. 540, our translation).

The competencies and skills approach, rather than a division by grades, allows for greater flexibility in curriculum planning. However, as Giordano et al. (2019) pointed out, this flexibility requires careful planning on the part of teachers, especially concerning integrating statistics education with other areas of knowledge and interdisciplinary projects. These competencies and skills prescribed by the BNCC for upper secondary school (Table 1.1.3) are comprehensive and in line with contemporary demands, promoting the integration of statistics with other areas of knowledge and with students' everyday lives. As Giordano et al. (2019) pointed out, statistics education, when worked on through projects and in a contextualised way, can be a powerful tool for developing students' critical thinking and autonomy.

1.1.3.3. International influences in the Brazilian curriculum

The changes operated in the new curriculum seem clearly aligned with the international renewal of statistics education promoted by the American Statistical Association through the GAISE reports. The significance of statistical literacy in contemporary society was underlined in the *2007 Guidelines for Assessment and Instruction in Statistics Education Report* (GAISE I). It makes the case that statistics are essential to decision-making for professionals, individuals, and legislators. According to the document, statistical literacy is crucial in several important areas:

- *Citizenship* – Understanding statistical concepts helps citizens interpret opinion polls, government data, and public policy decisions. For example, knowledge of sampling error and data reliability is necessary when evaluating poll results or government statistics.
- *Personal Choices* – Individuals encounter statistics daily in areas such as health, finance, and consumer choices. Whether assessing the effectiveness of a medical

treatment, evaluating product reliability, or interpreting financial investments, statistical reasoning is critical.

- *Workplace and Professions* – Many careers, including medicine, law enforcement, manufacturing, and business, rely on statistical thinking for quality control, forecasting, and decision-making. A statistically competent workforce is essential for economic competitiveness.
- *Science* – Advances in medicine, technology, and social sciences depend on statistical methods to validate findings. Statistical literacy enables individuals to critically evaluate scientific studies and claims, ensuring informed decision-making.
- *Education* – The report calls for an improved and structured approach to statistics education in K–12 curricula. It supports early exposure to statistical concepts and continued reinforcement through high school to develop essential data analysis skills.

The GAISE I report differentiated statistics from mathematics by emphasising that statistics deals with data variability and context-dependent reasoning. Unlike pure mathematics, statistics interprets real-world data rather than abstract numbers. It also aligns with national mathematics education standards, advocating for data analysis and probability as integral parts of the curriculum. It stresses the need for teachers to develop a deeper understanding of statistics to effectively teach these concepts.

In this report, a structured approach to statistical problem-solving is presented, emphasising the role of variability in the investigative process. It outlines four key components:

1. *Formulate Questions* – Identify and clarify the problem, distinguishing between deterministic questions (single-answer) and statistical questions (answers based on variable data).
2. *Collect Data* – Design and implement data collection methods that acknowledge and manage variability, such as random sampling and controlled experiments.
3. *Analyse Data* – Apply appropriate graphical and numerical techniques to account for variability, using statistical distributions to describe patterns and uncertainties.
4. *Interpret Results* – Draw conclusions while allowing for data variability, recognising that statistical results are estimates influenced by random and systematic differences.

Variability is central to statistical reasoning. The framework emphasises anticipating variability when formulating questions, acknowledging variability when designing data collection strategies, accounting for variability in analysis through probability models and distributions, and allowing for variability when making inferences and generalisations beyond collected data. Statistical literacy develops over time, with students progressing through three levels—A, B, and C—based on understanding rather than grade level. In Level A (basic understanding), teacher-driven learning where students begin recognising variability in data. In Level B (intermediate understanding), students start applying statistical concepts more independently. And in Level C (advanced understanding), Student-driven exploration, where learners make deeper connections between data collection, analysis, and interpretation. In Table 1.1.4 we can see the framework in detail.

Table 1.1.4. The framework in GAISE I report (ASA, 2007, pp. 14–15)

Process Component	Level A	Level B	Level
I. Formulate Question	**Beginning awareness of the *statistics question distinction*** Teachers pose questions of interest Questions restricted to the classroom	**Increased awareness of the *statistics question distinction*** Students begin to pose their own questions of interest Questions not restricted to the classroom	**Students can make the *statistics question distinction*** Students pose their own questions of interest Questions seek generalization
II. Collect Data	Do not yet *design for differences* Census of classroom Simple experiment	Beginning awareness of *design for differences* Sample surveys; begin to use random selection Comparative experiment; begin to use random allocation	Students make design for *differences* Sampling designs with random selection Experimental designs with randomization
III. Analyze data	Use particular properties of *distributions* in the context of a specific example Display variability within a group Compare individual to individual Compare individual to group Beginning awareness of group to group Observe association between two variables	Learn to use particular properties of *distributions* as tools of analysis Quantify variability within a group Compare group to group in displays Acknowledge sampling error Some quantification of association; simple models for association	Understand and use *distributions* in analysis as a global concept Measure variability within a group; measure variability between groups Compare group to group using displays and measures of variability Describe and quantify sampling error Quantification of association; fitting of models for association

Process Component	Level A	Level B	Level
IV. Interpret Results	Students do not look *beyond the data* No generalization beyond the classroom Note difference between two individuals with different conditions Observe association in displays	Students acknowledge that *looking beyond the data* is feasible Acknowledge that a sample may or may not be representative of the larger population Note the difference between two groups with different conditions Aware of distinction between observational study and experiment Note differences in strength of association Basic interpretation of models for association Aware of the distinction between association and cause and effect	Students are able to look beyond the data in some contexts Generalize from sample to population Aware of the effect of randomization on the results of experiments Understand the difference between observational studies and experiments Interpret measures of strength of association Interpret models of association Distinguish between conclusions from association studies and experiments
Nature of Variability	Measurement variability Natural variability Induced variability	Sampling variability	Chance variability
Focus on Variability	Variability within a group	Variability within a group and variability between groups Covariability	Variability in model fitting

With this framework, the GAISE I report provided a roadmap for developing statistical literacy through structured learning. It stresses the importance of variability in all aspects of statistical problem-solving and promotes a progressive, developmental approach to mastering statistical concepts.

In 2020, the Pre-K-12 Guidelines for Assessment and Instruction in Statistics Education II (GAISE II) was published as an update of GAISE I to reflect the evolving nature of data, advancements in technology, and the growing role of statistics across disciplines. If GAISE I report presented a four-step framework of the statistical problem-solving process, GAISE II report expanded on: different types of data (e.g., non-traditional data like images, text, and sound), multivariable thinking (analysing multiple factors simultaneously), probabilistic thinking (understanding randomness and uncertainty), and the role of technology (use of software, simulations, and data visualisation).

GAISE II report emphasises questioning, multivariable analysis, probabilistic thinking, and technology as essential skills for statistical literacy. The report underscores that statistical education should begin early and continue throughout schooling to prepare students for a data-rich world. It provides a structured approach to teaching statistical literacy through a two-dimensional model that integrates the statistical problem-solving process, as we can see in Figure 1.1.1.

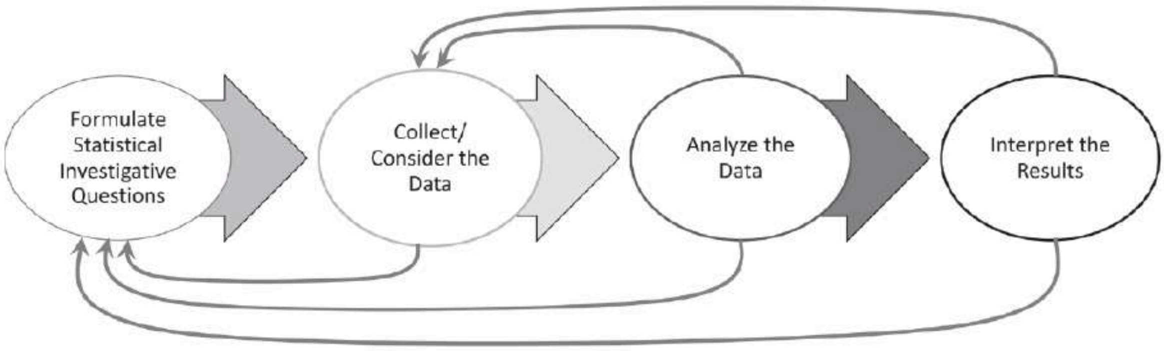


Figure 1.1.1. GAISE II framework: Statistical problem-solving process (ASA, 2020, p.13)

This framework seems particularly relevant in today's data-driven world, where individuals must critically evaluate data, question sources, and make informed decisions based on evidence. The emphasis on multivariable thinking, probability, and technology prepares students for the complexities of modern statistics and data science. Moreover, GAISE II report recognises that statistical learning is a continuous process, requiring years of experience and reinforcement. By structuring instruction across Levels A, B, and C (see Appendix A), it allows for flexibility based on students' exposure and understanding, rather than rigid grade-level expectations.

In the subsection 1.1.3.2 we saw that the BNCC included Probability and Statistics as a thematic unit in Mathematics domain, emphasising data processing and uncertainty. Although it does not explicitly discuss statistical literacy, its approach is aligned with the need to teach citizens who are able to interpret quantitative information critically. Furthermore, the GAISE reports propose a two-dimensional model for teaching statistics, structured into three progressive levels of statistical literacy development (beginner, intermediate and advanced), regardless of schooling.

The GAISE II guidelines are organised around the process of solving statistical research problems, which involves four stages: formulating an investigative question, collecting data, analysing data and interpreting results. The BNCC also emphasises the importance of contextualised statistics teaching, encouraging the use of surveys and information analysis in the classroom, but without explaining the structure of the process of solving statistical research problems.

As emphasised by Rangel et al. (2024), combined reading of the BNCC and GAISE reports reveals convergences and challenges. The BNCC favours the progression of statistical learning throughout the school grades, but the order of the skills can hinder the investigative approach, since carrying out research appears as one of the last skills in each school year. In addition, the lack of a systematic focus on summary measures in the early years can limit students' understanding of data distributions. Their analyses highlight the need for teacher education to ensure that statistics is taught effectively in compulsory education. This leads us to formulate the first research problem, which will be made explicit below.

1.1.4. First formulation of the research problem

Although the curriculum guidelines emphasise the importance of teaching statistics and probability in basic education, the implementation of this domain still faces numerous challenges. These difficulties are related to both teacher education and the structural conditions of schools, as well as pedagogical and cultural aspects that influence the teaching and learning of these subjects (Batanero et al., 2011b, Lopes, 2008; da Silva et al., 2015).

One of the main obstacles to effective teaching of statistics is teachers' initial and continuing education. During their academic training, many teachers were not adequately prepared to work with statistical and probabilistic concepts. Preservice teachers also relate that they do not feel prepared to teach statistics (Souza, 2023; Souza & Silva, 2024). As Lopes (2008) and da Silva et al. (2015) pointed out, mathematics teacher education often prioritises traditional content, such as algebra and geometry, to the detriment of more applied subjects, such as statistics and probability. This results in a gap in knowledge and confidence to approach these contents in the classroom. In addition, Lopes (2008) pointed out that many teachers are not familiar with teaching methodologies that promote the contextualisation and practical application of statistical concepts. Without training that enables them to relate statistics to real problems, teachers tend to adopt more traditional approaches centred on mechanical and repetitive calculations, which makes it difficult for students to engage with and understand the concepts.

Another significant challenge is the lack of resources and infrastructure in schools. Teaching statistics often requires the use of technology, such as data analysis software, and teaching materials that allow for the collection and interpretation of information. However, as da Silva et al. (2015) pointed out, many Brazilian public schools do not have adequate computer labs, internet access or even basic materials such as calculators and printed graphs. This lacks limits teachers' ability to develop practical and interactive activities that facilitate student learning. Furthermore, there is a school culture that often underestimates the importance of statistics, prioritising content considered more "essential" for exams and entrance exams. As Lopes (2008) pointed out, this view reduces statistics to a set of formulas and mechanical procedures, ignoring its potential for developing critical analysis and decision-making skills.

Along the same lines, the study presented by Giordano (2022) emphasises that the training of teachers to teach statistics faces significant challenges in Brazil. Primary teacher education and secondary mathematics teacher education programmes have gaps in teacher preparation, with few subjects focused on statistics and an excessively mechanistic approach that does not favour statistical and investigative thinking. In addition, the BNCC emphasises problem solving and data investigation, but many teachers are not prepared to apply active methodologies and the use of technologies. There is a clear misalignment between initial training and the practices required by the BNCC, which is exacerbated by the scarcity of effective continuing education programmes and the lack of adequate infrastructure in schools for the use of technological tools. Implementation of the BNCC also faces difficulties due to a lack of institutional support and resistance to methodological changes, making it difficult the effectiveness of the innovations proposed for teaching probability and statistics.

Hence, despite the relevance of the teaching of statistics in secondary education curriculum, this domain is relatively neglected, both in teacher education and at school in Brazil (Costa, 2007; Estevam & Cyrino, 2014, Ody & Viali, 2016). With this first research on probability teaching and the professional background of the author, the PhD research topic arose from my perception of a deficit in the pre-existing teacher education process, especially in the statistics domain, with a strong double discontinuity between the statistics taught at university and the needs one can experienced as a teacher at the beginning of the teaching profession (as it was my case). We can interpret this experience in terms of the double discontinuity described by Felix Klein (Eichler & Isaev, 2023). The first discontinuity occurs when students enter university teacher education courses, and few relationships are established between university mathematics and what they previously learned as secondary school students. The second discontinuity refers to the distance between the knowledge studied at the university by preservice teachers and the knowledge they need to teach at secondary school when they enter the teaching profession. In the case of statistics, it seems that the double discontinuity appears more acutely than in other topics or domains.

Based on the analysis of Brazilian curriculum documents for secondary school teacher education, Silva (2011) highlights a gap between the curriculum guidelines for teacher education and the teaching of statistics in secondary school. This gap can also be seen as a manifestation of the double discontinuity since the curricular guidelines for teacher education in statistics do not go in the same direction as the curricular guidelines proposed for statistics teaching at the secondary school level, showing little or no relationship between the statistical knowledge to teach (teacher education) and the statistical knowledge for teaching in secondary school. The subject Probability and Statistics in teacher education also highlights this double discontinuity and, more than that, does not provide preservice teachers with a practical vision of how to bring this subject into the classroom. It is common to hear accounts of novel mathematics teachers expressing their difficulty in working with this subject in secondary education in Brazil, as was my case. In fact, the double discontinuity problem is closely linked to the lack of consideration of mathematics teachers' needs in university teacher education, especially of the "mathematics needs", that it, needs in terms of mathematical knowledge.

The first question that follows is about the vision of the profile of the professional to be trained and the education process that is organised according to this profile's needs or concerns. In the terminology used in current educational institutions, we speak in terms of the knowledge and competencies needed for teachers to invest in the statistical learning of secondary school students in an effective and relevant way. So, an initial formulation of the problem could be expressed in the following terms:

> $P_{0\text{-}TE}$: *What knowledge and competencies are necessary for secondary school teachers to teach statistics?*

We will take the institutional context of Brazil as the main case of study and complement it with other European situations that have been or are currently under research.

It is important to emphasise that the initial focus of this research was to address the $P_{0\text{-}TE}$ problem within the context of the initial mathematics teacher education and the statistics teaching at the lower secondary school level (for students aged 11–14). However, as will become apparent throughout the studies presented in each chapter, this focus evolved during the research process. Additionally, our research expanded to include continuing mathematics teacher education and statistics teaching at the upper secondary school level (for students aged 15–17). This broadening of the unit of analysis was prompted by opportunities for collaboration among various institutions and researchers.

1.2. STATISTICS AS KNOWLEDGE TO BE TAUGHT AND KNOWLEDGE FOR TEACHING

Ruiz-Olarría (2015), based on studies of Cirade (2006) and Chevallard and Cirade (2009), presented an epistemological dimension of the teachers' need problem regarding mathematics: the knowledge to be taught, the knowledge for teaching, and the knowledge of the teaching profession. We are also interested in discussing this epistemological dimension regarding statistics by questioning, for example, *what is statistics as knowledge to be taught (school statistics), as knowledge for teaching (pedagogical content knowledge), and as knowledge of the teaching profession?* For this question, especially regarding statistics as knowledge to be taught and as knowledge for teaching, we present a first discussion below along with a literature review.

Over the past decades, statistics has strongly evolved in line with the development of technological resources for data handling and processing, giving rise to what is known today as "data science" (ASA, 2020; Cleveland, 2001; Holmes, 2017). Statistics education has always been aware of this change. A review of research literature shows the importance of taking a broad view of this field, including aspects like searching and collecting data in real-world contexts, selecting, organising, tabulating, and visualising them, using specific software, simulation, and reporting results, among others, all to answer open-ended questions and study the variability of concrete phenomena. To refer to this perspective on statistics, didactic research presents different terms or fundamental statistical ideas depending on what aspect it wants to emphasise (Burrill & Biehler, 2011; Shaughnessy, 2007).

"Statistical reasoning" (Ben-Zvi & Garfield, 2004) is the process by which statistical concepts are used to understand, interpret, and make sense of numerical and graphical information. Reasoning in statistics is defined as "understanding and being able to explain statistical processes and being able to fully interpret statistics results" (Op. Cit, p.7). Rather than being connected to deductive proofs, statistical reasoning is the process of analysing a statistical problem and applying how-and-why reasoning. In addition to examining and testing statistical models, statistical reasoning also investigates whether the models make sense in given situations.

"Statistical literacy" has been widely discussed in the literature as an essential competence for interpreting and critically evaluating data-based information, as well as for making informed decisions in a variety of contexts. This literature review looks at the contributions of Jane

Watson, Iddo Gal and Robert Gould, highlighting the evolution and broadening of the concept of statistical literacy over time. Watson (1997) was one of the pioneers in defining and operationalising the concept of *statistical literacy*. In her work, she identifies three essential stages or components for the development of this competence:

- *Basic understanding of statistical terminology*: This refers to mastering fundamental concepts such as mean, median, standard deviation and probability, which are essential for interpreting data.
- *Understanding statistical language in social contexts*: This involves the ability to understand how statistical concepts are applied in everyday situations, such as opinion polls, news and reports.
- *Questioning attitude*: This consists of the ability to apply more sophisticated statistical concepts to question claims that are not supported by adequate evidence.

Watson (1997) emphasises the importance of contextualising the teaching of statistics so that students can not only understand the concepts but also apply them critically in real situations.

Gal (2002 and 2004) refers to statistical literacy as the ability to interpret and critically evaluate statistical information, data-related arguments or stochastic phenomena that may be encountered in various contexts. It is broadly described as “being able to organise data, construct and display tables, and work with different representations of data. Statistical literacy also includes an understanding of concepts, vocabulary and symbols, and includes an understanding of probability as a measure of uncertainty” (Ben-Zvi & Garfield, 2004, p.7) and the “ability to access, understand, interpret, critically evaluate, and if relevant express opinions, regarding statistical messages, data-related arguments, or issues involving uncertainty and risk” (Gal, 2019, p. 2). Gal (2002 and 2004) also highlighted two skills interconnected:

- *Interpretation and critical evaluation of statistical information*: involves the ability to analyse data, identify trends, recognise biases and assess the validity of arguments based on statistical evidence.
- *Communicating reactions to and understanding of statistical information*: This refers to the ability to discuss, question, and communicate insights derived from data, as well as express concerns about the validity of the conclusions presented.

Robert Gould (2017) proposes an expanded view of statistical literacy, which he calls “data literacy”. He argues that in an increasingly data-driven world, it is essential to understand not only statistical concepts, but also the processes behind collecting, analysing and interpreting data. Gould argues that statistical literacy should include:

- *Understanding the origin and collection of data*: Knowing who collects the data, why and how it is collected.
- *Analysing and interpreting data*: Ability to work with random and non-random samples, as well as create basic descriptive representations to answer questions about real processes.
- *Ethical and technical aspects*: Understanding issues related to data privacy, ownership and storage, as well as understanding how computational representations can vary and why data is sometimes altered prior to analysis.
- *Predictive modelling*: Understanding the basic principles of statistical modelling and their application in practical contexts.

Gould (2017) emphasises that statistical literacy is not limited to interpreting graphs and tables but involves a deep understanding of the processes of producing and analysing data, as well as the ethical and social implications of these processes.

"Statistical thinking" (Wild & Pfannkuch, 1999) refers to a deep understanding of the nature and importance of variability in data and its impact on decision-making and understanding of the world around us. Wild and Pfannkuch (1999) provide a widely cited framework that focuses on the thinking processes required to solve statistical problems and considers four dimensions: investigative cycle, interrogative cycle, types of thinking and dispositions. It can be difficult to distinguish statistical thinking from statistical reasoning, nevertheless, statistical thinking also entails knowing how and why statistical investigations are carried out, even while statistical reasoning also requires describing statistical processes: "Statistical thinking involves an understanding of the nature of sampling, how we make inferences from samples to populations, and why designed experiments are needed to establish causation" (Ben-Zvi & Garfield, 2004, p.7).

"Informal statistical inference" (ISI) (Makar et al., 2011; de Vetten et al., 2019) and "Informal inferential reasoning" (IIR) (Pfannkuch, 2006) are related to the process by which conclusions are drawn from data primarily through observation, comparison and reasoning based on data distributions. IIR "is interconnected to reasoning from distributions, reasoning with measures of centre, and sampling reasoning within an empirical enquiry cycle. All these aspects are underpinned by a fundamental statistical thinking element, consideration of variation" (Pfannkuch, 2006, p. 1). ISI can be considered a statement and the IIR is the reasoning that underlies it (Makar et al., 2011).

"Statistical modelling" or "data modelling" involves deciding what attributes to collect, modifying or adding new attributes, adjusting the data structure, creating data visualisations, summarising observed patterns and variations, and is inseparable from the data handling process (Ben-Zvi, 2020; Dvir et al., 2024). "An additional important dimension of reasoning with data models and modeling relates to more general modeling skills such as fitting a model and evaluating the extent of the explanation, description, or prediction, that a specific data model" (Dvir et al., 2024, p. 8).

"Civic statistics" is the use and interpretation of statistical data to better understand and address issues of public interest and societal concern, focusing on collecting, analysing, and presenting data on a variety of civic topics, including but not limited to demographics, economics, health, education, crime, and environmental issues. Six key recommendations are provided by the ProCivicStat Report "Engaging civic statistics: a call for action and recommendations": promote engagement with social issues, use relevant real data and texts, embrace technological tools, develop critical interpretation skills, assess critical understanding, and foster systemic change and collaboration (ProCivicStat Partners, 2018).

"Statistical sense" or "stochastic sense" (Batanero et al., 2013; Burrill & Biehler, 2011) is the union of civic statistics and statistical reasoning and implies a critical understanding of the uncertainty and variability inherent in the cultural phenomena studied, as well as the ability to appropriately apply statistical and probabilistic concepts in different contexts and situations. As presented by Batanero (2013), statistical sense consists of three main components: understanding fundamental statistical ideas at an appropriate level, which are applicable in various contexts, that can be taught at any educational level, and essential for statistical modelling. "Secondly, a certain competence in data analysis is required, which is nowadays facilitated by the abundance of software, both for data storage and transmission, as well as for calculation and graphing. The third component is statistical reasoning, which is the most difficult to convey" (Op. Cit, p.2, our translation).

Summarising this overview of statistics as *knowledge to be taught*, we highlight the descriptions presented by Burrill and Pfannkuch (2024) about four major trends in statistics education, based on a narrative review of responses from members of the statistics education research community in describing observed trends in the field and identifying interesting and relevant articles related to those trends. The first is "data science", characterised by ongoing advances in software, integration with computational thinking and a renewed focus on predictive modelling. The second is the emphasis on "visualising statistical concepts", highlighted by a significant focus on young students, the potential of interactive visualisations and the continuing importance of inference. The third is the emphasis on "social statistics" which stresses the need to pay attention to risk, foster critical literacy and improve communication, whether in the media or for awareness raising and advocacy on important issues. Finally, the fourth is the emphasis on "new contexts for learning", which includes adopting new methods of data collection and analysis, different types of data, new forms of representation, and changes in curricula or curricular orientations.

Both statisticians and didacticians advocate the need for teaching to incorporate this vision of statistics, which is already beginning to be included in the curricular documents of many European and American countries (ASA, 2020; Brasil, 2018). Teacher education appears here as a determining variable to enable the change of perspective in teaching. Most researchers in statistics education (Batanero et al., 2011a; Ben-Zvi et al., 2018; Burrill & Ben-Zvi, 2019; Garfield & Everson, 2009; Lopes, 2013; Makar & Fielding-Wells, 2011; Zapata-Cardona, 2020) mention the need to update the knowledge of teachers who received limited training in the field and very much based on the mathematical foundations of statistics and probability. Under the influence of the models established previously by Shulman (1986) and Ball et al. (2008), which address "pedagogical content knowledge" (PCK) and "mathematical content knowledge for teaching" (MCKT), respectively, some authors propose more specific terms for the context of statistics teacher education. This is because, also a solid foundation in mathematics is required for teaching statistics, there are specific aspects of the statistical field that require particular attention.

Groth (2007) offers a conceptualisation of "statistical knowledge for teaching" that considers common knowledge, referring to skills learned in traditional mathematics courses, and specialised knowledge, which is developed by carefully addressing the specific mathematical challenges and problems that arise in educational settings. This conceptualisation is articulated to the four components of a statistical investigation based on a presented in the *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report* (Franklin et al. 2007): formulating questions, collecting data, analysing data, and interpreting results. Henriques and Ponte (2014) propose that the professional development of statistics teachers should not only focus on specialised knowledge of statistical content but also on the use of exploratory data analysis and digital technologies. They argue that teachers need to understand how students develop statistical ideas and how to use technological tools to support their learning process. For further research on the subject, Batanero and Díaz (2010) and Ponte and Noll (2018) present extensive literature reviews and discussions on the different approaches to the professional development of statistics teachers. Both reviews highlight that collaboration between mathematicians and statisticians is essential to address the challenges of teaching statistics and teacher education. Although statistics is different from other areas of mathematics, there are synergies between mathematics education and statistics education that can be leveraged in such training.

In summary, there is a line of research carrying out investigations that focuses on statistics as *knowledge to be taught* and *knowledge for teaching* either by emphasising "statistical reasoning", "statistical literacy", "statistical thinking", "informal statistical reasoning", etc. However, the changes necessary for progress in statistics teaching in this direction, and teacher education in coherence are far from assured. There is little research on the teacher profession problem from an institutional perspective that seeks to identify the conditions needed for change to take place and, above all, to study the constraints that hinder progress in this direction. This further underlines the justification of our research topic in this investigation and leads us to reformulate our research question by introducing our theoretical background.

CHAPTER II. THEORETICAL FRAMEWORK AND RESEARCH METHODS

Chapter II provides the theoretical and methodological foundation for this dissertation, outlining key concepts, research questions, and approaches to investigating the teaching profession. The chapter begins with the theoretical framework, where the anthropological theory of the didactic (ATD) is introduced as the lens to identify and analyse the conditions and constraints in the implementations of *study and research path for teacher education* (SRP-TE) regarding statistics teaching in secondary school level. This is followed by an exploration of the ecological dimension of the teaching profession, considering environmental, institutional, and social factors that impact educational practices.

The discussion then shifts to the need for a change in pedagogical paradigms, emphasising educational challenges. We also present the notions of reference epistemological model and that of SRP-TE, providing a methodological device for teacher education. Additionally, the main didactic tools for the analysis of the SRPs-TE are presented, culminating in a discussion on addressing ecological challenges through SRPs-TE.

Building up this theoretical framework, we reformulate the research question and articulate the specific hypothesis, derived research questions, and general and specific objectives. By grounding the research in a specific theoretical perspective, in the research methods section, we detail the methodological approach adopted, beginning with didactic engineering as a general research methodology, consisting of the preliminary, *a priori*, *in vivo*, and *a posteriori* analysis. We also detail the methods employed during the research process. It elucidates the rationale behind our chosen methods, explaining why they are well-suited to address our research questions. Additionally, it provides insights into the data-gathered procedures, including sampling techniques, data sources, and data analysis methods.

Finally, we offer an overview of the structure of the remaining chapters of the dissertation. This overview aims at ensuring clarity and coherence in the presentation of our research findings. It outlines the organisation of the subsequent chapters, highlighting the progression of hypothesis explored, and the results reached throughout the dissertation.

2.1. THEORETICAL FRAMEWORK

2.1.1. Anthropological theory of the didactic (ATD) and the teaching profession

The ATD has its origins in the contributions of the French researcher Yves Chevallard and, in particular, in the theory of didactic transposition (Bosch & Gascón, 2006; Chevallard, 1985; Chevallard & Bosch, 2020). It is part of the *epistemological research programme* initiated by the theory of didactic situations (TDS) proposed by Guy Brousseau (Brousseau, 1997).

From the point of view of the ATD, the aim of didactics is to find the mechanisms by which, in a given society, knowledge is spread between people and institutions and, like most approaches in didactics of mathematics, it has always been closed-related to the initial (preservice) and continuing (in-service) teacher education. Since the evidence of the phenomenon related to the didactic transposition processes (Chevallard, 1985), ATD has been one of the first approaches to consider not only the teaching and learning activities in the classroom but also the entire process from creation and use of mathematical knowledge to its incorporation into school as taught knowledge, as an object of study and research. This object of study also includes all the institutions involved in this process, including the teachers themselves, which we consider as an institution, and those involved in their preservice and in-service teacher education.

Many ATD researchers have been involved in teacher education at various educational levels, hence, the development of this approach has always been enhanced by the problems that arise in such training process and the effort to provide elements of response. In the present conception of the teaching profession, the teacher is viewed and identifies him/herself as a small independent producer who must source his/her own resources. As a result, they are obliged to accept that the challenges and difficulties they encounter in developing their professional stem are due to personal limitations. If a teacher saw him/herself as a member of a profession (which would necessitate the profession's ongoing development), his/her profession would change profoundly since he/she could interpret many of the teaching problems as professional problems. The responsibility for seeking answers to these problems would not fall on the individual teacher but on the institution of professional development.

As emphasised above (section 1.2), focusing on the statistics teaching profession problem, many approaches, such as the current educational institutions, formulate this problem in terms of knowledge or competencies necessary for teachers to be able to invest in an effective and relevant way in teaching statistics in secondary school level. Based in our theoretical background, we presented the first formulation of our research problem in terms of praxeologies ($P_{0\text{-}TE}$). One of the main postulates of the ATD is that any kind of knowledge can be modelled in terms of *praxeologies*: a combination of *praxis* (ways of doing) and *logos* (the discourse justifying praxis). Within *praxis* we can find the *types of tasks* (T_i) and the *techniques* (τ_i), and within *logos* we can find the *technology* (θ_i) (understood as the discourse on technique) and the *theory* (Θ_i) (the justification of technology). We can get a better idea by following Chevallard's text:

> [...] *a* praxeology is, in some way, the basic unit into which one can analyse human action at large. (The concept of *a* praxeology is, therefore, basic to praxeology as a science – in the dictionaries' definition of the word.) What exactly is *a* praxeology? We can rely on etymology to guide us here – one can analyse any human doing into two main, interrelated components: *praxis*, i.e. the practical part, on the one hand, and *logos*, on the other hand. "*Logos*" is a Greek word which, from pre-Socratic times, has been used steadily to refer to human thinking and reasoning – particularly about the cosmos. Let me represent the "praxis" or practical part by *P*, and the "logos" or noetic or intellectual part by *L*, so that a praxeology can be represented by [*P/L*]. How are P and L interrelated within the praxeology [P/L], and how do they affect one another? The answer draws on one fundamental principle of ATD – the anthropological theory of the didactic –, according to which no human action can exist without being, at least partially, "explained", made "intelligible", "justified", "accounted for", in whatever style of "reasoning" such an explanation or justification may be cast. *Praxis* thus entails *logos* which in turn backs up *praxis*. For *praxis* needs support – just because, in the long run, no human doing goes unquestioned. Of course, a praxeology may be a *bad* one, with its "praxis" part being made of an inefficient technique – "technique" is here the official word for a "way of doing" –, and its "logos" component consisting almost entirely of sheer nonsense – at least from the praxeologist's point of view! (Chevallard, 2006, p. 23).

An advantage of the notion of praxeology is that it combines under the same concept the "knowledge" or organised pieces of knowledge and the "practice" or practical activity at stake. A domain in mathematics discipline (such as statistics, theory of big numbers, or the didactics of mathematics) refers to a set of praxeologies that we designate metonymically by alluding to one of its components, in this case, its technological-theoretical component $[\theta, \Theta]$. Speaking of "teaching practice", the teacher's "expertise" or "know-how" is to emphasise the practical-technical block $[T, \tau]$ of praxeology, omitting or leaving implicit its technological-theoretical component (often very naturalised and difficult to describe). ATD postulates that all "knowledge" (all reasoned or explanatory discourse) has a potential praxis associated with it, even if it is implicit, and that there is a descriptive and justifying discourse for all "practice", however rudimentary it may be.

There is a wide range of praxeologies of varying proportions in the activities that mathematics teachers engage in. On the one hand, there are the "mathematical (or statistical) praxeologies" that the teacher must teach. These are *punctual* praxeologies when they focus on a single type of task, which is typically associated with a set of techniques, such as calculating the chances or total number of possible events, calculating the probability of a favourable event, constructing a table with absolute and relative frequencies of a set of data, calculating the arithmetic mean of a set of data. When the practical blocks are articulated around a common technological discourse, we develop *local* praxeologies, such as the “themes” into which we structure the teaching: descriptive statistics, inferential statistics, proportionality, etc. and finally, if local praxeologies are constructed around a theory, they form *regional* praxeologies, which are commonly referred to as “thematic blocks” or “sectors” in school mathematics, such as statistics, geometry, arithmetic etc. However, we should keep in mind that the punctual, local, or regional character of a praxeology is relative to the institution considered: a regional praxeology in one institution, such as classical (Laplacian) probability or statistics in secondary education, could be considered a local praxeology in another institution, such as the field of different “types of probabilities (classical, frequential, subjective, axiomatic, marginal, conditional, Bayesian, geometric)” or “descriptive statistics”.

In addition to these mathematical (or statistical) praxeologies *to be taught*, the teacher must activate other types of praxeologies useful for teaching. To designate those praxeologies specific to the teaching profession, Ruiz-Olarría (2015) refers to “teaching praxeologies” or “didactic praxeologies of teachers”. It is also possible to speak of *learner praxeologies* or *student didactic praxeologies* in the case of the students. In summary, didactic praxeologies are generally conceived as involving both the teacher and the students and are therefore cooperative praxeologies. Within the *teachers’ praxeological equipment*, i.e. his/her set of praxeologies, some are certainly mathematical (or statistical) but the *teachers’ mathematical (or statistical) praxeological equipment* cannot be reduced to what he or she has to teach. Cirade (2006) designated “mathematical praxeologies for teaching” which we will focus on here as “statistical praxeologies for teaching”, which contain the mathematical (or statistical) knowledge required to delimit, interpret, relate, and make apparent the *raison d’être* of the mathematics (or statistics) to be taught, as well as to conceptualise and build the didactic praxeologies linked with the praxeologies to be taught.

Both teaching praxeologies and mathematical (or statistical) praxeologies for teaching are part of a broader set of praxeologies that Ruiz-Olarría (2015) called praxeologies for the *teaching profession*, as they are focused on the social diffusion of mathematical praxeologies. These praxeologies include knowledge that, although not directly part of teaching praxeologies, is necessary to design and manage didactic praxeologies. The latter may be punctual, local, or regional, depending on the degree of cohesion of the technological-theoretical discourse that organises them. Unlike mathematics or other disciplines with a long history and tradition, it is more difficult to find a technological-theoretical discourse that systematically describes, structures and justifies specific didactic praxeologies. In fact, one of the main objectives of research in the didactics of mathematics is to advance the knowledge and development of these didactic praxeologies.

In summary, Ruiz-Olarría (2015), based on studies of Cirade (2006) and Chevallard and Cirade (2009), presented an epistemological dimension of the teachers' need problem regarding mathematics: the praxeologies *to be taught* (or knowledge to be taught), the praxeologies *for teaching* (or knowledge for teaching), and the praxeologies *for the teaching profession* (or knowledge of the teaching profession). We are also interested in discussing this epistemological dimension regarding statistics by investigating statistics as *knowledge to be taught (school statistics), as knowledge for teaching (pedagogical content knowledge), and as knowledge of the teaching profession*.

If we adopt the approach provided by the ATD, we can reformulate the first teaching profession problem $P_{0\text{-}TE}$ (as initially presented in section 1.1) in terms of "praxeologies" instead of "knowledge and competencies":

$P_{0\text{-}TE}$: What praxeological needs do secondary school teachers have regarding statistics?

2.1.2. Ecological dimension of the teaching profession problem

It is important to point out that the ATD defines the didactics of mathematics as *the science that studies the conditions and constraints of the social diffusion of mathematical praxeologies*. This diffusion encompasses both the processes of teaching and learning in educational and training institutions, as well as the processes of transposition between different types of institutions, be they for the teaching, production or use of mathematics. In this context, the problem of teacher education is a central part of his field of study. Since the school is the main institution for the dissemination of mathematics and the teacher is its main actor, the teaching profession and its problems are, as we mentioned at the beginning, one of the priority axes of research.

The *conditions* and *constraints* of the dissemination of mathematics are understood as everything that enables, facilitates or hinders such dissemination. Constraints are those conditions that, at a given moment and from a specific institutional position, are difficult to modify. When the teacher and student develop a study of a certain knowledge, it is necessary to realise that there are various factors involved in this knowledge, i.e. "what can happen is determined by conditions and constraints that cannot be reduced to those immediately identifiable in the classroom" (Wozniak, 2005, p. 187). That certain mathematical notions or tools are required before others can be constructed, that mathematical praxeologies can be organised into local, regional, etc. praxeologies based on certain criteria, etc. To better understand the study of these conditions and constraints, the *scale of levels of didactic codeterminacy* is used.

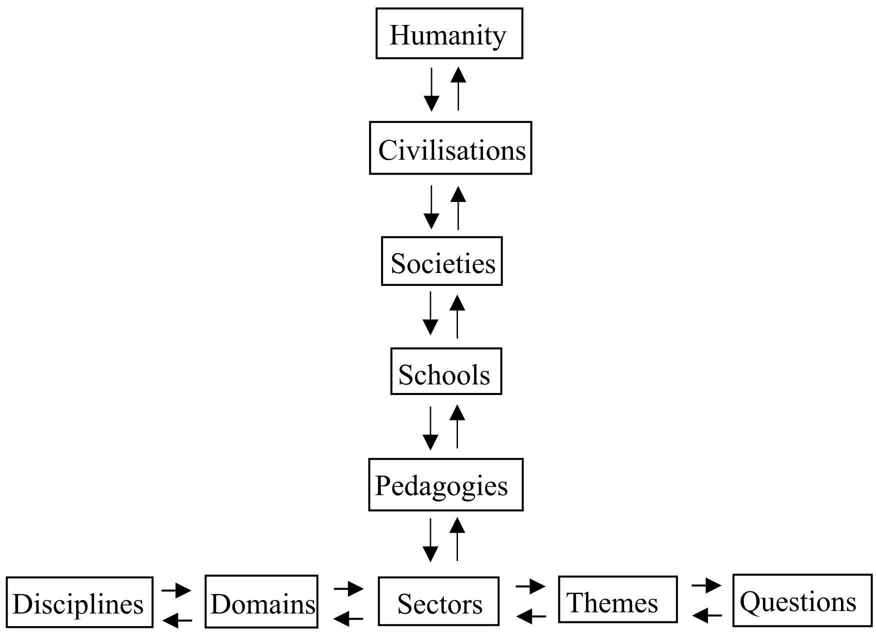


Figure 2.1. Scale of levels of didactic codeterminacy (adapted from Chevallard, 2002)

Regarding *humanity*, at the higher levels of the scale we have *civilisations* which "in a deliberately minimalist way, we will understand here, under this name, a set of complex praxeologies present and mobilised in a *set* of societies [...]" (Wozniak, 2005, p. 187). *Societies* are those belonging to civilisations, such as Brazilian society, German society, French society, among others. We also have *schools*, which include all "the infrastructures provided by educational institutions to organise didactic systems and help them run: organisations of groups of teachers and students, structures in courses and modules, physical and virtual spaces, time schedules, final exam obligations, access to knowledge resources and experts, accreditations, etc." (Bosch, 2018, p. 4003). *Pedagogies*, on the other hand, are understood as everything that is used by the teacher and student for learning, which does not necessarily depend on a specific object of study.

We can see that in the current educational context of many societies, there is a set of praxeologies that form a curriculum, leading to specific knowledge organisations. These structures are taken into consideration by the lower levels of the scale of levels of didactic codeterminacy (disciplines, domains, sectors, themes, and questions/tasks), which differentiate between various praxeological organisations of different "sizes".

Teachers can act on some of these factors (usually those connected to the level of the themes or local praxeology) but not on others (e.g., on the 'content blocks' or a specific way of structuring them), conditions that operate as discipline-level constraints. They also face constraints and conditions imposed by higher levels of the scale of levels of didactic codeterminacy, which affect the way they organise the study of a discipline at the centre (pedagogical level), or in a specific type of school (school level), or in one society or another, and according to the principles and values of a particular civilisation.

> The scale of didactic codeterminacy is first of all a tool for researchers in didactics to question the reality they aim at studying. Its main utility is to enlarge our vision towards certain empirical fields that are traditionally kept outside the didacticians' perspective and are thus taken for granted. In effect, a great amount of research in didactics that focuses on the levels of the questions or the themes that are taught and learnt, rarely questions the specific structuring of disciplines, sectors or domains these questions or themes belong to. Therefore, many of the conditions and constraints that come from the lower levels remain hidden, as if they were not part of the problems addressed. (Bosch, 2018, p. 4005).

By recognising the complexity of these influences and the limited control teachers have over higher-level constraints, the ATD provides a comprehensive framework for understanding and addressing the multifaceted challenges in mathematics education. This understanding is essential for developing teacher educational proposals for secondary school teachers that better support the study of data and variability in the transition between paradigms (from the visiting works to questioning the world), aligning with contemporary educational needs.

2.1.3. Towards a change in pedagogical paradigms

In the ATD, the *didactic system* is defined as the relationship $S\ (X;\ Y;\ Q)$ (Chevallard, 2004a), where the variables of the relationship are:

- Work (W): Praxeological organisation. Everything created by deliberate human action, to achieve a purpose.
- Student (x) or students (X): Person or persons who engage in the study and research of a question, task or work.
- Teacher (y) or team of teachers (Y): Director/s of the study.

The need to develop didactic tools and devices for the teaching profession, as well as to study their functionality, impact, and diffusion, is supported by recent research in the ATD in terms of a change of pedagogical paradigm from the *paradigm of visiting works* towards a new *paradigm of questioning the world* (Chevallard, 2006 and 2015). Within this framework, we find didactic systems immersed in the conditions and constraints of certain paradigms, among which Chevallard (2015) distinguishes two pedagogical paradigms:

- The paradigm of *visiting works*. In this paradigm, works of study W are proposed by Y to be studied by X, without the need to justify their reason for being. In other words, why we study that is not questioned, but rather that the work is assigned an intrinsic historical value or a technical importance for other works. Within this paradigm, there appears the phenomenon that (Chevallard, 2015) calls the *monumentalisation of the curriculum*, *i.e.*, that the curriculum that is taught is "approached as a monument that stands on its own, that students are expected to admire and enjoy, even when they know next to nothing about its *raisons d'être*, now or in the past" (Chevallard, 2015, p. 175).
- The paradigm of *questioning the world*. In this new paradigm, a model appears to describe study processes: the *Herbartian scheme* in its extended version (Bosch, 2018; Chevallard, 2015):
  $[S(X; Y; Q) \Rightarrow \{Q_1, Q_2, \ldots, Q_i, A^{\lozenge}_1, A^{\lozenge}_2, \ldots, A^{\lozenge}_j, W_1, W_2, \ldots, W_k, D_1, D_2, \ldots, D_l\}] \hookrightarrow A^{\heartsuit}$
  In the didactic system, X raises a question Q, which is *studied* with the guidance of Y in order to prepare an answer $A^{\heartsuit}$. The study of Q generates a research process involving a *didactic milieu M* made up of available "labelled" answers $A^{\lozenge}_j$ (already done, official), known and available works W_k (knowledge and objects), derived questions Q_i, sets of data D_l.

In the Herbartian schema, the *visiting of works* does not disappear. It corresponds to the case where X and Y have to study a set of "labelled" answers $A^{\lozenge}_j$. In the paradigm of questioning the world, this visit becomes a means to an end, which is the construction of the answer $A^{\heartsuit}$. The approach of this new paradigm is motivated by the attempt to escape from *traditional monumentalism*, where knowledge is presented as important works that need to be known —or to have visited at some point— and which turns curricular contents into difficult objects to question and without a clear *raison d'être*.

The new paradigm creates a strong change in the *didactic contract* that prevails in the first paradigm, that is, the kind of responsibilities assumed by teachers and students regarding the knowledge to be studied (Brousseau et al., 2020). In the new paradigm, teachers do not introduce the knowledge tools needed to answer the questions addressed. They act much more as leaders of the research process developed by the students. The change affects not only the pedagogical level, "How to teach?", but also the epistemological level, "What to teach?".

Knowledge is learned during the inquiry of relevant questions in the form of dynamic, provisional, and collective tools to address them.

The ATD proposes the implementation of *study and research paths (SRPs)* to investigate the conditions needed to foster the transition between pedagogical paradigms and the constraints that hinder it. SPRs consist of a specific proposal with its own methodology, in which an interesting and encouraging generative question is presented, which acts as a starting point for the inquiry process. In this type of proposal, moments of searching for information and experimentation are combined with moments of study where this information is analysed, validated and conclusions are drawn. Barquero et al. (2015) argue that teacher education proposals also need to be conceived within the paradigm of questioning the world. Thus, they consider the implementation of *study and research paths for teacher education* (SRPs-TE) "as a way to provide teachers with pertinent (theoretical and practical) tools to nourish their professional development" (p. 810).

In previous research on the ATD, we can find proposals of SRPs-TE for secondary preservice teacher education (Álvarez et al., 2016; Cid, 2016; Cid et al., 2020; Licera, 2017; Ruiz-Olarría, 2015), and for secondary in-service teacher education (Barquero et al., 2018; Otero & Llanos, 2019). The first investigation that introduced the SRP-TE proposal was Ruiz-Olarría (2015). It was then developed in different directions: Cid (2016) and Cid et al. (2020) to introduce the *raison d'être* of negative numbers in an algebraic environment, Álvarez et al. (2016) for the study of differential and integral calculus in functions of one variable, or Licera (2017) for the case of real numbers. Adaptations for in-service teacher education can be seen in two investigations: Barquero et al. (2018) focusing on the teaching of mathematical modelling, and Otero and Llanos (2019) focusing on the study of parables.

In Brazil, research about SRP-TE has been developed recently addressing different institutional and mathematical contexts. Five theses were identified focusing on teacher education in which an SRP-TE was implemented in different contexts. Freitas (2019) implemented an SRP-TE in the context of plane analytical geometry. Benito (2019) investigated an SRP-TE for preservice teachers to question, analyse, design, and experiment with teaching processes about conics. Valenzuela (2021) proposed an SRP-TE for Peruvian in-service teachers regarding the teaching of vectors in science and engineering courses to identify the mathematical and didactic praxeologies that teachers mobilise in this training process. García-Cuéllar (2021) proposed an SRP-TE for the teaching of quadrilaterals at the secondary level of the Peruvian educational system, to provide tools to in-service teachers for teaching this content and leading to learning supported by the paradigm of questioning the world. Gonçalves (2022) analysed the construction of a teaching proposal for the relative integers, resulting from a training for a group of in-service teachers using as methodological support some modules of SRP-TE. These investigations pointed to SRP-TE as a powerful device for the content of teacher education, but there are still conditions and constraints that need to be overcome for the diffusion of SRP at the secondary level. The main constraints focus on the dominant paradigm of visiting works in which we operate.

The shift from the paradigm of visiting works to the paradigm of questioning the world proposed by the ATD provides a crucial theoretical foundation for the implementation of the SRP-TE in statistics teaching. Traditional approaches to teacher education in mathematics, and particularly in statistics, often emphasise the transmission of fixed knowledge, treating curricular content as a set of works to be visited rather than as tools for inquiry. This monumentalisation of knowledge results in a disconnection between teachers' professional development and the real-world challenges of teaching statistics, reinforcing the double discontinuity described by Klein. By contrast, the paradigm of questioning the world, as operationalised through SRP-TE, encourages teachers to engage in statistics inquiry process, fostering a deeper understanding of both statistics as knowledge to be taught and statistics as knowledge for teaching. Through the ATD lens, the SRP-TE framework enables teachers to explore the praxeologies necessary for teaching statistics, bridging the gap between theoretical knowledge and classroom practice. In doing so, this approach aligns with the broader objective of transforming statistics education into a dynamic, investigative process that equips teachers with the tools to navigate and adapt to the complexities of contemporary data-driven society.

2.1.4. Reference epistemological model and the study and research paths for teacher education

In order to move towards the new paradigm of questioning the world, Chevallard (2015) suggests the implementation of a specific educational device, the study and research paths (SRPs), which places the *raison d'être* of school knowledge at the heart of the learning process. This approach fosters the conditions necessary to facilitate practical and functional mathematical activity and the progressive approach to this new paradigm inevitably entails the modification of the conditions in which teachers develop mathematics teaching and, especially, a profound reconsideration of the type of training required to teach mathematics (Barquero et al., 2018; Ruiz-Olarría, 2015).

Several studies within the field of ATD have questioned, with mathematical-didactical arguments, the purpose that the official curriculum assigns to specific domains (even individual concepts) of school mathematics as detailed in textbooks. This questioning usually arises from the analysis of a didactic phenomenon in which such a domain is involved and involves the presentation of a possible alternative purpose that is reflected in a *reference epistemological model* (REM) of the domain in question (Bosch & Gascón, 2006). If this alternative approach is seriously considered, it will end up significantly influencing the mathematical practice established in the school institution, specifically around the domain in question, as well as the interactions that this domain had with the other areas of school mathematics.

As a result of the school mathematics, and to establish a new interpretation of the field of mathematical activity at stake, in a way that responds to the alternative *raison d'être* proposed, the methodology of the ATD proposes, in each case, the creation of a REM and the implementation of SRPs based on this REM and adapted to the specific educational levels and conditions of each school institution.

The study and research paths (SRPs) are the fundamental tool that is introduced in the paradigm of visiting works to move towards the paradigm of questioning the world (Bosch, 2018). First, they try to give school mathematics a *raison d'être*, thus responding to constraints related to the incompleteness of mathematical organisations. As evidenced by Bosch et al. (2004), these are *punctual*, *rigid, and poorly articulated with each other*. On the other hand, the SRPs are proposed to overcome this *monumentalist pedagogy* typical of the paradigm of visiting works that we have described in the theoretical framework.

Four main owners of the SRP can be identified (Barquero, 2009):

1. The starting point is a *generative question*, which we will call Q_0. This must be a "living" question and of interest to those who will study it. The elaboration of Q_0 is a much-discussed process and still represents an open problem. Even so, what we can be sure of is that the goal of the SRP approach is *to study* Q_0 and should never be to arrive at a specific answer decided in advance.
2. An *arborescent* evolution is allowed. Based on Q_0 derivative questions will be generated, which will also allow Q_0 to be solved but which will also open new lines of research, perhaps away from the original. This property gives it a very open character: the evolution of the study depends on the whole group that studies Q_0, and not on the teacher. For this reason, an *a priori analysis* is of utmost importance, as we will mention in section 2.3, to be able to foresee the supply of the Q_0.

3. The construction of the answer $A^{\heartsuit}$ needs external answers $A^{\lozenge}_{j}$ that will have to be validated through the construction of the praxeological works W_k (mathematical or extra-mathematical). These $A^{\lozenge}_{j}$ will be found in the *media*, which may be means of communication or dissemination (textbooks, class notes, where these answers are available. These answers $A^{\lozenge}_{j}$, however, are focused to answer other questions independent of Q_i, therefore, will give rise to a *reconstruction* of them and a process of validating these answers, which will constitute the *milieu*.
4. The study of the answers requires the realisation of different *study and research activities (SRA)*. The SRAs do have the objective of reaching a certain mathematical organisation (MO). Therefore, during the process of finding an answer to Q_0, the study group will find different SRAs, which with the construction of the MO, will articulate this answer.

Barquero et al. (2015 and 2018) argue that teacher education proposals also need to be conceived within the paradigm of questioning the world. Thus, they consider the implementation of study and research paths for teacher education (SRP-TE) "as a way to provide teachers with relevant (theoretical and practical) tools to nurture and sustain their professional development" (Barquero et al., 2015, p. 810). An SRP-TE consists of five modules, which can be visualised in Figure 2:

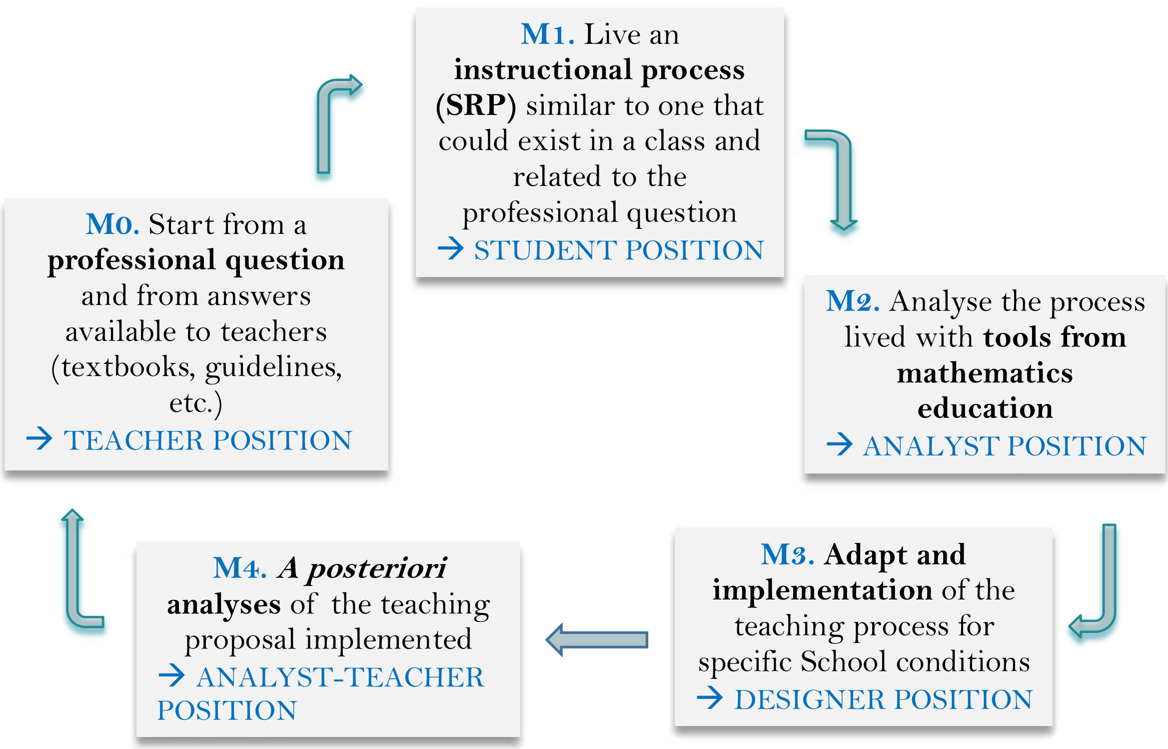


Figure 2.2. General structure of the SRP-TE (Barquero et al., 2015, 2018; Ruiz-Olarría, 2015)

- *Module 0:* This stage consists of proposing the open question ($Q_{0\text{-TE}}$ generating question) related to the topic to be discussed and to the teaching profession. In our case, our $Q_{0\text{-TE}}$ is: "How to teach statistics in secondary education and why to teach it?". Other derived questions will appear as research elements to answer the generating question, such as: *Why is there so little attention on teaching statistics in school? What are the conditions and constraints for teaching statistics in the final years of primary school? What statistics should we teach at this level of schooling, according to the curriculum? What kind of activities are proposed in Brazilian textbooks and official curriculum documents? What other proposals exist?* Partial answers to $Q_{0\text{-TE}}$ will appear throughout the SRP-TE (and it is expected that afterwards, it will also continue to appear), making it a transversal module of the educational process.
- *Module 1:* This module consists of proposing to the group of teachers an SRP with a relevant issue that could be addressed in a real classroom as if they were the students at the secondary school level. "The SRP can actually have been implemented in previous investigations or may simply have been designed by researchers for this purpose" (Barquero et al., 2015, p.810). With this, the group of teachers will be introduced and "lived" a study and inquiry process related to this SRP.
- *Module 2:* This phase is the moment for teachers to analyse the SRP they have experienced:

  > Three main phases are distinguished: (a) the mathematical analysis of the work done, including the elaboration of a reference epistemological model describing the modelling process involved (Bosch & Gascón, 2006); (b) a didactic analysis of the process, including a description of the differences between the contract established during the SRP to manage the modelling process, compared to the usual school didactic contract centred on the transmission of contents; (c) a more general study of the viability of SRP, including the identification of the institutional conditions and constraints affecting the development of […] practices in school settings. (Barquero et al., 2015, p. 810)

  In this module, we generally have the opportunity to transfer ATD didactic tools that enable mathematical and didactic analyses of the SRP experienced, in the same direction as presented by Barquero et al. (2018, 2022, and 2024) and which will be further detailed in section 2.1.5.
- *Module 3:* In this phase, teachers redesign or adapt and implement (if possible) an SRP for the school level under consideration. The new design is carried out based on the analyses made in the previous module, adapting, adding or excluding (if necessary) didactical tools which seem necessary —or at least useful— for the implementation of this adaptation of the SRP.

- *Module 4:* This last module consists in sharing teachers' experiences during all the implementations of the SRP to carry out an *a posteriori* analysis. Again, the mathematical and didactic tools used in Modules 2 and 3 are present and of great importance, "not only to provide some provisional answers to the question that was at the origin of the whole process ('How to teach …?'), but also as a means to analyse other possible alternative answers" (Barquero et al., 2015, pp. 810–811), such as the answers found in Module 0.

In conclusion, by shifting from the traditional paradigm of visiting works to the paradigm of questioning the world, SRP-TE emphasises the *raison d'être* of mathematical knowledge and fosters inquiry-based learning. Concerning the context of this dissertation, statistics teacher education, through its five modules, we advocate that SRP-TE equips preservice and in-service statistics teachers with both theoretical and practical tools, enabling them to critically analyse, design, and implement innovative statistics teaching practices. This approach not only enhances teachers' professional development but also redefines the institutional conditions and constraints of statistics education, paving the way for a more meaningful and functional engagement with statistical concepts in the classroom.

2.1.5. The analysis of the SRPs-TE and their ecology

SRPs and SRPs-TE can be described in terms of the extended version of the *Herbartian scheme* (presented in section 2.1.3). Bosch (2018) pointed out the importance of the *questions and answers* (*Q-A*) *dialectic* to ensure the dynamics of SRPs. The Q-A dialectic provides visible proof of the progress of the inquiry and contributes to the overall process management. It can also be used to analyse the inquiry process (Florensa et al., 2021). The *media-milieu* (*Me-Mi*) *dialectics* becomes crucial during the whole SRP. To analyse this dialectic, we examine the sources of external information, data, and answers, and how access to these is managed (media). We also consider how these sources are validated and transformed, and the materials used to develop final or intermediate answers (*milieu*). Ultimately, an SRP is a collective inquiry process wherein small groups (X_i) are formed, and individual work is also conducted. Groups X_i and Yj must collaborate effectively, necessitating a consistent sharing of responsibilities regarding which questions to study, the class's strategy, and the validity of answers. To analyse these responsibilities and roles assumed by X_i and Y_j during the development of an SRP, we will use the notion of *didactic contract* explained above.

In analysing an SRP, attention is directed towards three primary aspects (Barquero & Bosch, 2015). First is the *chronogenesis*, which involves how the teacher monitors the questions-answers dialectic over time. This aspect pertains to the temporal evolution of the inquiry process. The teacher's role in guiding the flow of questions and answers over time is critical. Effective *chronogenesis* ensures that the inquiry remains dynamic and responsive to emerging understandings and challenges (Bosch & Gascón, 2006). Second is the *mesogenesis*, concerning how the teacher stages the media-milieu dialectic. This focuses on the creation and structuring of the learning environment, including the tools, resources, and interactions that support the inquiry. The teacher's ability to manage the media-milieu dialectic ensures that students have access to necessary resources and can effectively transform information into knowledge (Barquero et al., 2019). Third is the *topogenesis*, which relates to the positions (*topos*) assumed by the teacher and students and their distribution of responsibilities. It involves the spatial and social positioning within the classroom. It addresses how responsibilities are shared between the teacher and students, determining who takes the lead in different phases of the inquiry process (Barquero & Bosch, 2015).

Finally, the analysis of the *ecology* of the SRP-TE consists of the study of the conditions and constraints that facilitate or hinder the application of the design. In ATD, ecological analysis looks at the relationship between the didactic system and its environment. This includes how the didactic system interacts with external factors such as educational policies, societal norms, and available resources. These factors can either support or constrain the implementation of innovative teaching designs like SRP-TE. To carry out this type of analysis, the *scale of levels of didactic codeterminacy* will be used as a fundamental methodological tool (Chevallard, 2002). By using the scale of levels of didactic codeterminacy, educators and researchers can develop a nuanced understanding of the ecological factors influencing teaching and learning processes. This understanding is crucial for designing effective educational interventions.

2.1.6. Addressing the ecological problem through SRPs-TE

The research presented in this dissertation can be seen as a direct development of the pioneering work of Barquero et al. (2018) when proposing SRPs-TE to introduce the ecological problem and institutional analysis in the teachers' professional development. This previous study aimed to investigate the institutional constraints that prevent mathematical modelling from becoming a standard teaching practice and the required stage of teacher professional development.

Therefore, designed an online SRP-TE for in-service teachers in a course at the Centre for Applied Research in Advanced Science and Technology (CICATA-IPN) in Mexico.

The ecological analysis proposed by Barquero et al. (2018) "[…] enables the detection of the constraints appearing at different levels, encompassing the specific ones related to how mathematical content is proposed to be taught and learned at school, the general ones regarding the general organisation of school activities, and the role assigned to schools in our societies" (Barquero et al., 2018, p. 33). They identified some curriculum constraints as teachers often feel compelled to align any activity with specific curriculum labels and content. This stems from institutional pressure to cover prescribed notions and procedures within limited timeframes. Time constraints were also evidenced as the course's design and implementation highlighted the issue of limited time availability for teachers. The extensive nature of modelling activities necessitated splitting the course into two separate parts for more in-depth analysis and design. Related to the local school context, specific institutional traditions, such as the format and terminology used in lesson plans, varied widely among teachers from different backgrounds. This variation made it difficult to standardise the course content and approach. It was evidenced that some institutional constraints are often so ingrained in the educational environment that they become "naturalised" and go unnoticed by those within the system.

In a recent investigation, Barquero and Ferrando (2024) uses the scale of levels of didactic codeterminacy "as as a common theoretical and methodological tool to categorise and locate the types of conditions and constraints identified by the participants into two different teacher education courses for modelling" (Op. Cit, p. 3). One of the courses analysed again was the one presented by Barquero et al. (2018). The authors show how the ecological analysis is refined when it is used the scale of levels of didactic codeterminacy (Figure 2.1). In the school level, constraints are related to the curriculum and school policies. At the pedagogy level, constraints arise from teachers' prior knowledge, beliefs, and teaching habits, as well as constraints due to students' prior knowledge, engagement, and attitudes towards modelling, etc. The course participants also had the opportunity to identify the constraints they related to the teaching and learning of mathematical modelling. Most of the constraints identified by them were placed at the levels of disciplines and pedagogies.

Addressing the ecological problem through the SRP-TE involves dealing with institutional constraints and promoting a shift towards inquiry-based teaching methods. Hence, SRP-TE can be a potential device to address ecological problems in education by helping teachers understand and overcome institutional constraints, promoting inquiry-based teaching, and fostering an environment of innovation and reflection. This approach not only enhances teachers' professional development but also contributes to a more dynamic and responsive educational system.

After presenting our theoretical background, we now present our hypotheses, research questions and general and specific objectives in the following section.

2.2. HYPOTHESIS, RESEARCH QUESTIONS, AND OBJECTIVES

According to Chevallard (1989), a didactic phenomenon refers to any occurrence or event that arises in the context of teaching and learning processes, which can be studied and analysed to understand the transformation and dissemination of knowledge within educational settings. Didactic phenomena are social and cultural constructs involving mathematical, pedagogical and sociological aspects of teaching and learning mathematics. In relation to "statistics teacher education", in our preliminary analysis (that will be presented in detail in Chapter III), we are evidencing two didactic phenomena.

The first didactic phenomenon in evidence concerns the process of didactic transposition (Chevallard, 1985) and the lack of a clear mathematical status for the data gathering and treatment in the scholarly institution. Although data processing —which we understand here as the aspects of collecting, recording, organising, cleaning, representing and analysing data— is increasingly important in society and professional statistical activities in the field of data science, these recent evolutions of the field have not reached the school institution. School statistical activities are often reduced to numerical calculations of statistical measures (frequencies, measures of central tendency, deviations, etc.) and the creation or interpretation of standardised graphical representations (pie charts, bar charts, histograms) (Burril & Ben-Zvi, 2019). Despite national curriculum guidelines addressing *data collection, planning, and executing surveys* (Brasil, 2018), textbooks rarely include these activities (Prestes, 2021). Short and Pigeon (1998) highlighted the neglect of planning and piloting phases in data collection, despite their acknowledged importance. Ben-Zvi and Garfield (2004) noted that students often equate statistics with mathematics, focusing on computations and single correct answers. Chevallard and Wozniak (2007) described this as "the reduction of statistical work to arithmetic work". Along the same line, as Ruiz-Higueras and Rodríguez-Fernández (1999), who discussed the phenomenon of invisibility of didactic facts in mathematics teaching, we refer to the didactic phenomenon of the *invisibility of the data treatment* related to statistical knowledge to be taught in secondary education (Verbisck et al., 2023).

A second didactic phenomenon is linked to an "applicationist" vision of both the domain of statistics and of statistics teacher education, in the same sense proposed by Barquero (2009) and Barquero et al. (2014). We thus speak of the manifestation of "applicationism" in two inseparable ways. The first deals with a single set of knowledge that is applied and taught without much variation, in the different training contexts ("statistics" for biology, engineering, psychology, economics, etc., and for teacher education). The second form of "applicationism" is manifested in the teacher education proposal itself, when it assumes that the training in statistics that preservice teachers must receive is similar to those of other specialisations and that this is a necessary and sufficient condition for its subsequent transposition to the classroom.

These two didactic phenomena give rise to two hypotheses that we are focusing on in this dissertation:

Hypothesis 1 (H1). Didactic transposition: An underdeveloped historical didactic transposition work leads to a limited dominant conception of mathematical statistics (dominant epistemological model of statistics) in secondary education, characterised by the invisibility of data treatment.

Hypothesis 2 (H2). Applicationism in teacher education: Concerning teacher education, two main types of constraints appear, one linked to a limited view of mathematical statistics (dominant epistemological model of statistics) and another linked to an "applicationist" conception of teacher education (dominant didactic model of teacher education).

The two hypotheses and the problem related to the teacher profession constitute the starting point of our research, which is situated within the ATD and, more specifically, addresses a general problem related to the *ecology of study and research paths in secondary teacher education*.

As stated before, the new formulation of the research problem within the ATD has been expressed as:

$P_{0\text{-}TE}$: *What praxeological needs do secondary school teachers have regarding statistics?*

We assume here a third hypothesis about the teacher education problem, which corresponds to the main statement to be defended in this dissertation:

Hypothesis 3 (H3). Study and research paths for teacher education: Study and research paths for teacher education (SRP-TE) are a potential device to lead preservice and in-service teachers question the knowledge to be taught and sometimes even related scholarly knowledge about statistics as data science. They are also a good starting point for the analysis of the transition towards the paradigm of questioning the world. Because SRPs-TE start from the consideration of teaching questions, they help overcome an "applicationist" perspective of teacher education.

Therefore, considering these three hypotheses, $P_{0\text{-TE}}$ gives rise to our reformulated research question:

$\boldsymbol{RQ}$: What conditions are needed for SRP-TE with secondary mathematics teachers to help preservice and in-service teachers design, analyse and implement new didactic processes for teaching statistics in secondary school that overcome the phenomenon of data treatment invisibility?

Based on this RQ, we propose a general objective for the development of the research:

General objective: Analyse the necessary conditions to implement SRP-TE in preservice and in-service secondary school teacher education, as well as the institutional constraints that hinder the transfer of tools to teachers for the design, analysis, and implementation of didactic processes of statistics in the transition to the paradigm of questioning the world.

In our case, we would address five specific research questions of this general problem, in connection with the SRP-TE and the didactic engineering research methods that will be explained in Section 2.3:

$\boldsymbol{RQ_1}$ **related to dominant conceptions of statistics for teacher education:** What are the dominant conceptions of statistics and statistical teacher education in institutions related to the knowledge to be taught at the secondary school level and preservice teacher education? How might these conceptions hinder the establishment of favourable conditions for statistical education that emphasise the study of data and their variability?

$\boldsymbol{RQ_2}$ **related to the design of a pilot SRP-TE:** What educational proposal based on SRP-TE can be designed and implemented with a group of preservice mathematics teachers in Brazil? How can this proposal contribute to providing future teachers with tools to design, analyse and implement new didactic processes for the teaching of statistics in lower secondary school?

RQ_3 related to the implementation of an SRP-TE for in-service teachers: What conditions implemented in an online SRP-TE can help teachers address and detach themselves from the phenomenon of the invisibility of data treatment in a context of online in-service teacher education? What constraints limit it?

RQ_4 related to the implementation of an SRP-TE for preservice teachers: What conditions implemented by a face-to-face SRP-TE can help preservice teachers address the phenomenon of the invisibility of data treatment and what constraints appear to limit it?

RQ_5 related to the ecology of SRP-TE and the dissemination of didactic research tools in teacher education: What collaboration mechanisms between teachers, researchers and educational institutions? What didactic tools would facilitate the dissemination of SRP-TE and research in statistics preservice and in-service secondary teacher education?

We can then introduce four specific objectives addressed in this dissertation. We will label them with letters, so as not to directly associate them with the research questions:

SO_A: Analyse the conception or model of statistics as data science that prevails through curricular documents (textbooks, national and international curricula, teaching resources, etc.) in the noosphere and in scholarly institutions. Study the consequences concerning the phenomenon of "applicationism" and that of the "invisibility of data treatment" in teacher education.

SO_B: Study the SRP-TE ecology through the design, implementation, analysis, and development of different modalities of SRPs-TE for preservice secondary school teacher education in the field of probability and statistics.

SO_C: Study the SRP-TE ecology through the design, implementation, analysis, and development of different modalities of SRPs-TE for in-service secondary school teacher education in the field of probability and statistics.

SO_D: Identify the general conditions that favour and the constraints that hinder the implementation of SRP-TE, comparing preservice and in-service modalities.

The relationship between the specific objectives of the research and the research questions is as follows:

	SO_A Statistics as data science	SO_B SRP-TE ecology in in-service teacher	SO_C SRP-TE ecology in preservice teacher	SO_D General traits
RQ_1: *Didactic transposition and statistics as data science*				
RQ_2: *Design of a pilot SRP-TE*				
RQ_3: *Ecology of SRP-TE in-service teacher*				
RQ_4: *Ecology of SRP-TE preservice teacher*				
RQ_5: *Ecology of SRP-TE and dissemination*				

Figure 2.3. Relationship between the specific objectives and the research questions

2.3. RESEARCH METHODS

This dissertation is structured into distinct chapters, each contributing to the overarching research framework. Notably, some of these chapters have been published as journal articles or book chapters, even if they globally correspond to a complete process of didactic engineering. While each RQ is examined through specific research methods tailored to its scope, these methods collectively align with a comprehensive didactic engineering process. This structure ensures that, despite their independent publication, the chapters maintain coherence within the broader research design, illustrating a systematic approach to investigating the topic.

2.3.1. Didactic engineering as a general research methodology

The general research methodology adopted will follow the principles of didactic engineering (DE), initiated by the theory of didactic situations (Artigue, 2014) and adapted with the tools of the ATD (Barquero & Bosch, 2015). The following sections explain the four main stages of DE. In each of these, it is briefly specified which theoretical and methodological tools will be used. As Artigue (2014) explains, about the origins of didactic engineering:

> The idea of didactical engineering (DE), which emerged in French didactics in the early 1980s, contributed to firmly establish the place of design in mathematics education research. Foundational texts regarding DE such as Chevallard (1982) make clear that the ambition of didactic research of understanding and improving the functioning of didactic systems where the teaching and learning of mathematics take place cannot be achieved without considering these systems in their concrete functioning, paying the necessary attention to the different constraints and forces acting on them. (Op. Cit, p. 203).

The adoption of DE as the overarching research methodology aligns with the objective of investigating the complex interplay between teaching practices and the institutional conditions that shape them. By situating the study within the framework of the ATD, this research acknowledges the inherent complexity of didactic systems and the need for a systematic approach to analysing educational phenomena. The combination of DE and ATD offers a robust methodological foundation, as it not only facilitates the design and implementation of educational interventions but also enables a thorough examination of the conditions and constraints that influence their outcomes. This dual perspective supports the development of reflective practices among educators and provides a comprehensive understanding of the didactic phenomena under investigation. By leveraging the tools of ATD, the study aims to move beyond surface-level observations and engage in a deeper exploration of the structural and epistemological dimensions that impact mathematics education, thereby contributing to both theoretical and practical advancements in the field.

In general, four phases can be distinguished in the didactic engineering process, neither independent nor with a predetermined order of development (see Figure 2.4). These four stages are explained below, and we take advantage of them to specify which type of analysis we will perform at each moment of our research.

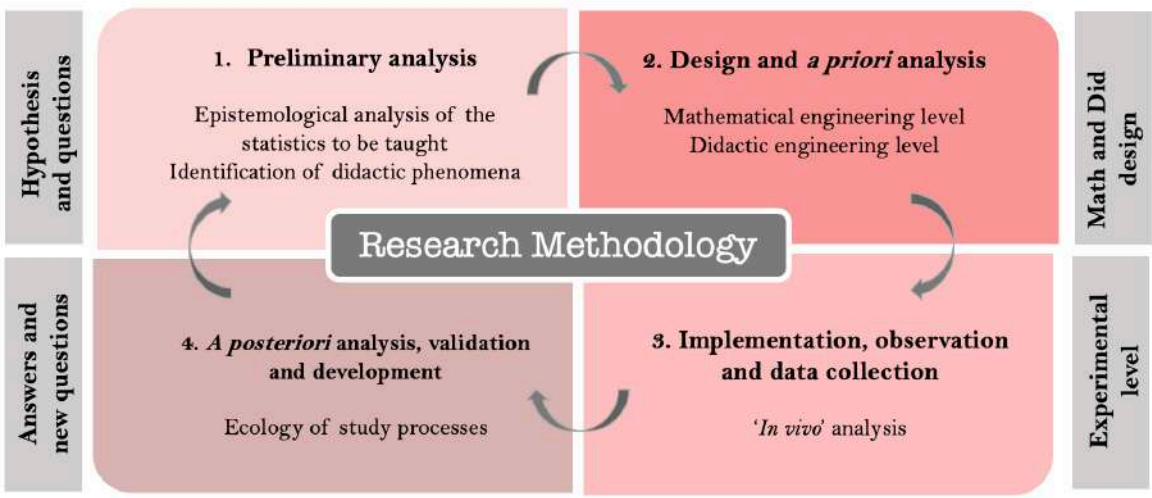


Figure 2.4. Stages of the didactic engineering process (Barquero & Bosch, 2015, p. 252)

Preliminary analysis or identification of didactic phenomena

Apart from the concepts specific to didactics in general and the knowledge of the discipline in question, we will make a preliminary analysis from three main dimensions:

- Epistemological analysis of the statistical content.
- Analysis of the conditions and constraints prevailing in the teaching of statistics at secondary level and in the corresponding teacher education processes.
- Analysis of previous studies in mathematics education about statistics teacher education.

In this phase, we some questions, hypotheses, and identify the didactic phenomena that may appear. In particular, we start with the three following general hypotheses:

- **H1**. An important didactic phenomenon concerning statistics education at secondary level (and beyond) is the *invisibility of data treatment* in both school and scholarly mathematics. This phenomenon is the consequence of specific didactic transposition processes that are to be studied more in-depth. In any case, we postulate that it should also be approached as a teacher education problem as it is central to the Klein's double discontinuity problem mentioned before.
- **H2**. Concerning statistics teacher education, two mean types of constraints appear, one linked to a limited view of mathematical statistics (dominant epistemological model of statistics) and the other linked to an "applicationist" conception of teacher education (dominant didactic model of teacher education).
- **H3**. (the main statement to be defended in this dissertation). From the ATD, it comes the assumption that study and research paths for teacher education (SRP-TE) have the capacity to lead teachers to question the knowledge to be taught and even, sometimes, the related scholarly knowledge (Barquero et al., 2022). SRP-TE can be also a potential device to address the didactic phenomena of the "invisibility of data treatment" and the presence of "applicationist" in teacher education. They are appropriate educational proposals to approach the limitations of the paradigm of visiting works.

Design and a priori analysis

In this second stage, the hypotheses raised in the previous phase are considered and the modules of the SRP-TE are used as a fundamental tool, following the proposal of Ruiz-Olarría (2015) and Barquero et al. (2018). The design and *a priori* analysis of the SRP-TE is carried out considering the planning of the experimentation with the concrete groups of preservice and in-service secondary school teachers.

Implementation and in vivo analysis

In this phase, direct observation is made in the classroom, and evidence will be collected (such as observation diaries, student productions, session recordings, questionnaires, etc.). The concept of *in vivo* analysis implies that there will be a simultaneous interpretation of the phenomena observed in the classroom to make decisions about the development of the SRP-TE.

A posteriori analysis and validation

In this last phase, the evidence collected during the experiments is considered and contrasted with the hypotheses and dissenting proposals that appeared in the first phases. During this phase, typically, new research sequences and proposals can be made for subsequent experiments. A typical type of analysis of didactic engineering in ATD appears, which is the study of the conditions and constraints that facilitate or hinder the application of the design or, in other words, the *ecological* analysis of the SRP-TE. To carry out this type of analysis, the *scale of levels of didactic codeterminacy* is used as a fundamental methodological tool (Chevallard, 2002). The ecological analysis of the study process will be conditioned by its categorisation within these levels of co-determination, following the example of Barquero and Ferrando (2024).

2.3.2. Data collection, analysis, and methodology adopted for each specific objective (SO)

Throughout the research process carried out in this dissertation, different plans and executions of empirical data collection were conducted to address the five research questions (RQ_i).

In the case of **RQ1** (Chapter III), the data collected took the form of six semi-structured interviews with lecturers who work in teacher education at three institutions in Brazil. The main methodology adopted in this study is qualitative research based on semi-structured interviews. The study involves interviewing six researcher-educators who teach the subject Probest at different Brazilian universities. The interviews follow a predefined script addressing five key topics, which explore epistemological and didactic models in statistics education. The study also employs content analysis, using *Atlas.ti* software to systematically categorise and code interview responses according to specific indicators of applicationism, a theoretical framework adapted from Barquero et al. (2014). The research methodology includes triangulation, where two co-supervisors review and validate the primary analysis to ensure reliability.

In the case of **RQ2** (Chapter IV), data was collected by recording the four online sessions of the pilot SRP-TE proposal carried out with a group of preservice teachers from the Federal University of Sergipe (UFS) and their productions, in collaboration with a UFS lecturer-researcher. The main methodology adopted in this study is the didactic engineering within the ATD framework, specifically through the SRP-TE model. The study involves designing, implementing, and analysing an inquiry-based teacher education proposal, where preservice mathematics teachers engage in problem-solving through a paradigm shift from visiting works to questioning the world. The methodology integrates qualitative research, including *a priori* analysis, pilot experimentation, and content analysis of teacher-student interactions, to investigate the development of statistical inquiry and teaching practices.

In the case of **RQ3** (Chapter V), the data gathered were the recordings of the fourteen sessions of the SRP-TE implemented in a virtual modality for in-service secondary school teachers in Brazil and the participants' written productions throughout the training, including discussion posts, collaborative documents, reflective journals and lesson-plan drafts. In the case of **RQ4** (Chapter VI), the data gathered consisted of audio recordings of the sixteen face-to-face sessions of the SRP-TE implemented for a group of preservice teachers at the Federal University of Mato Grosso do Sul, as well as the written productions of the participating teamwork and a questionnaire answered by the participants at the end of the training. These data gathering also took place at different periods and institutions from 2021 to 2023 and will be further detailed in each chapter. **RQ5** is also addressed in Chapter VI. Each methodology adopted will be detailed in each chapter. The main methodology adopted in both studies follow the didactic engineering phases applied to SRP-TE**.** It consists of four key steps: *a priori* analysis**,** implementation and *in vivo* analysis, and *a posteriori* analysis**,** focusing on the phenomenon of the invisibility of data treatment in statistics education.

Similarly, even though we have the ATD as our theoretical framework in this research, as we are dealing with empirical data gathered in different contexts, some specific elements of the ATD are used as tools for analysing the different RQs. For example, the notion of institutional relations, didactic transposition and the indicators of "applicationism" in the case of RQ1, a descriptive analysis using the question-answer map in the case of RQ2, the *questions and answers* (*Q-A*) and the *media-milieu* (*Me-Mi*) *dialectics* and the notion of *didactic contract* in the case of RQ3, and the scale of levels of didactic codeterminacy in the case of RQ4. Each theoretical element used as a tool for the analysis in the different studies that make up the chapters of this dissertation and address each RQ is detailed in the study itself.

In the conclusions chapter, we provide a comprehensive discussion and analysis of the entire research process, with a particular focus on developing detailed answers to RQ5, the overarching research question, as well as RQ1–RQ4. This chapter also articulates the specific objectives, methodologies, and findings of each study, highlighting their interconnections and broader implications.

In Figure 2.5 we present the organisation of this dissertation with the following topics: general research question, research question studied in each chapter, related specific objectives, methodological aspects adopted. Finally, the discussions and articulations of objectives, procedures and results achieved in this dissertation.

RESEARCH QUESTION

What conditions are needed for SRP-TE with secondary mathematics teachers to help preservice and in-service teachers design, analyse and implement new didactic processes for teaching statistics in secondary school that overcome the phenomenon of data treatment invisibility?

GENERAL OBJECTIVE

Analyse the necessary conditions to implement SRP-TE in preservice and in-service secondary school teacher education, as well as the institutional constraints that hinder the transfer of tools to teachers for the design, analysis, and implementation of didactic processes of statistics in the transition to the paradigm of questioning the world.

Specific research questions

CHAPTER III

RQ1: What are the dominant conceptions of statistics and statistical teacher education in institutions related to the knowledge to be taught at the secondary school level and preservice teacher education? How might these conceptions hinder the establishment of favourable conditions for statistical education that emphasise the study of data and their variability?

CHAPTER IV

RQ1: What educational proposal based on SRP-TE can be designed and implemented with a group of preservice mathematics teachers in Brazil? How can this proposal contribute to providing future teachers with tools to design, analyse and implement new didactic processes for the teaching of statistics in lower secondary school?

CHAPTER V

RQ3: What conditions implemented in an online SRP-TE can help teachers address and detach themselves from the phenomenon of the invisibility of data treatment in a context of online in-service teacher education? What constraints limit it?

CHAPTER VI

RQ4: What conditions implemented by a face-to-face SRP-TE can help preservice teachers address the phenomenon of the invisibility of data treatment and what constraints appear to limit it?

Specific objectives

SO_A: Analyse the conception or model of statistics as data science that prevails through curricular documents (textbooks, national and international curricula, teaching resources, etc.) in the noosphere and in scholarly institutions. Study the consequences concerning the phenomenon of "applicationism" and that of the "invisibility of data treatment" in teacher education.

SO_B: Study the SRP-TE ecology through the design, implementation, analysis, and development of different modalities of SRPs-TE for preservice secondary school teacher education in the field of probability and statistics.

SO_C: Study the SRP-TE ecology through the design, implementation, analysis, and development of different modalities of SRPs-TE for in-service secondary school teacher education in the field of probability and statistics.

SO_B: Study the SRP-TE ecology through the design, implementation, analysis, and development of different modalities of SRPs-TE for preservice secondary school teacher education in the field of probability and statistics.

Methodological aspects

A **qualitative research methodology** based on **semi-structured interviews** with six researcher-educators, analyzed through content analysis, and validated through **triangulation** .

Didactic engineering, specifically through the **SRP-TE device**, including *a priori* analysis, experimentation, and *a posteriori* including a content analysis of participants' interactions and productions.

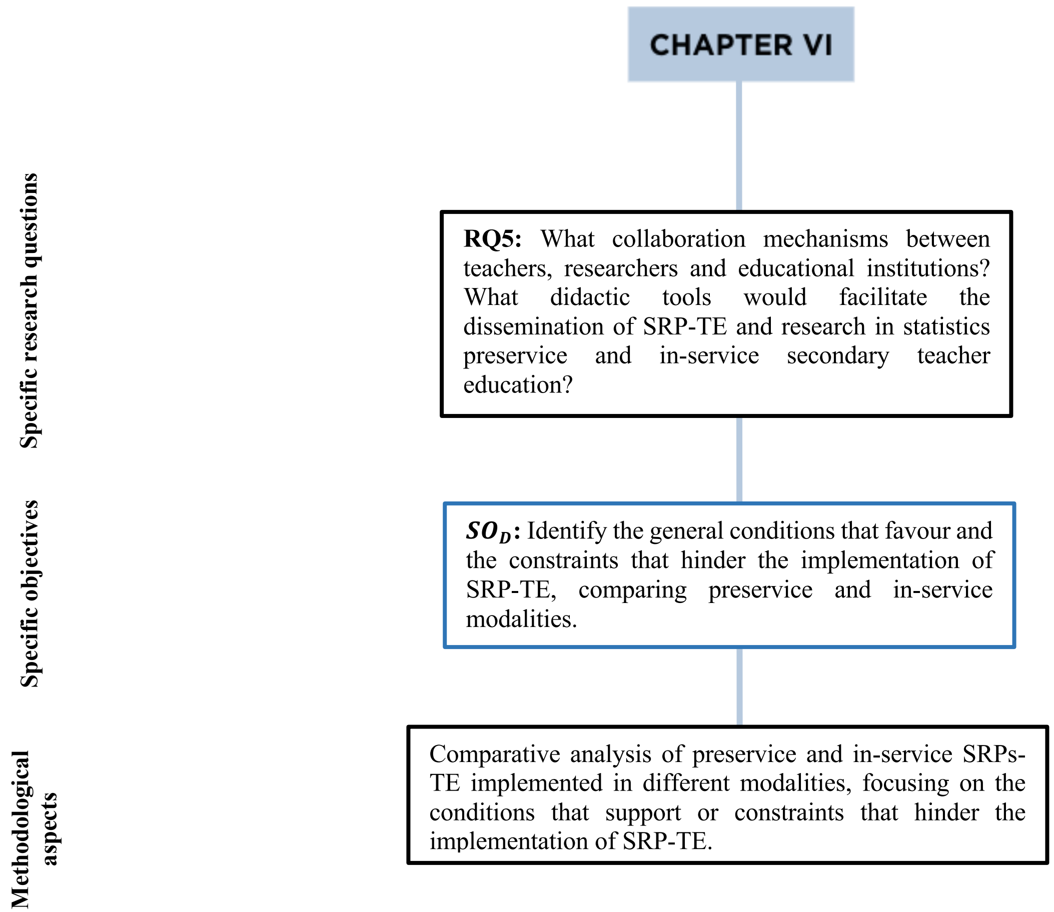


Figure 2.5. Organisation of the dissertation chapters according to the research questions, the general and specific objectives, and the related methodological aspects.

2.3.3. Data management plan and ethical considerations of research

This PhD research is conducted under a cotutelle agreement between the Federal University of Mato Grosso do Sul (UFMS), Brazil, and the University of Barcelona, Spain. As the doctoral student has been enrolled at UFMS since February 2020, and the research focuses on Brazilian teacher education, all data were collected from Brazilian preservice and in-service teachers in the different studies carried out. Given the involvement of human participants, the study adheres to strict ethical guidelines and data management protocols. The research project was approved by the Research Ethics Committee (CEP) of UFMS and authorised by the National Commission for Ethics in Research (CONEP) in 2021. This approval ensures that the study complies with Brazilian Resolution CNS n.466/2012, which governs research involving human participants. Ethical considerations include informed consent, confidentiality, voluntary participation, and the right to withdraw from the study at any time without consequences.

Before data collection, all participants received detailed information about the research objectives, methodology, data collection procedures, and their rights as participants. Each participant was required to sign an informed consent form, which will outline:

- The purpose and scope of the research.
- The methods of data collection and storage.
- The voluntary nature of participation and the right to withdraw at any time.
- The confidentiality and anonymity measures in place to protect their identity.
- How the data will be used, stored, and shared in compliance with ethical and legal standards.

The data collection process included audio and video recordings of online sessions (if was the case), written documents, and participant-generated content. These materials will be securely stored in encrypted digital files with restricted access limited to the principal researcher and the supervising research team. The storage and management of research data will follow these principles:

- Data Security: All digital files will be stored on password-protected servers and encrypted cloud storage.
- Access Control: Only the researcher, co-supervisors, and authorised members of the research team will have access to the data.

- Data Anonymisation: Any personally identifiable information (names, institutions, locations) will be removed or replaced with coded identifiers to ensure privacy.
- Backup and Redundancy: Regular backups will be conducted to prevent data loss, ensuring copies are stored in secure external drives and cloud repositories.
- Data Integrity: Version control and audit logs will be maintained to track modifications and access to the dataset.

Upon completion of the research, all collected data will be archived for a minimum of five years, as required by CONEP and in line with Resolution CNS n.466/2012. During this period:

- Data will remain securely stored and accessible only to the research team.
- If necessary, data may be reanalysed for further research within the ethical approval scope.
- After the five-year retention period, all data will be securely deleted following data disposal best practices (e.g., digital shredding, permanent erasure of files from servers).

While raw data will not be publicly shared due to ethical and confidentiality constraints, aggregated and anonymised findings will be included in:

- Journal publications, conference proceedings, and academic reports.
- Doctoral dissertation results and related academic outputs.
- Presentations at research seminars and teacher education workshops.

Any data-sharing request from external researchers will be considered on a case-by-case basis, ensuring compliance with ethical guidelines and requiring additional ethical clearance and participant re-consent if necessary.

2.4. OVERVIEW AND STRUCTURE

As we already mention in section 2.3, this dissertation is structured into distinct chapters, each contributing to the overarching research framework. Notably, some of these chapters have been published as journal article, book chapter, or proceedings, even if they globally correspond to a complete process of didactic engineering. The studies caried out and presented in each chapter address the specific research questions and objectives raised in section 2.2 and the didactic engineering phases (subsection 2.3.1) as the general research methodology. In this section, we present the organisation of these studies, to which phase of the didactic engineering they correspond, as well as the research questions they address.

Chapter I is structured into two main sections, outlining the transition from the teaching profession to the first formulation of the research problem in statistics education. The first section (1.1) examines the historical evolution of mathematics teacher education in Brazil, with a particular focus on statistics education. It includes a personal reflection on the author's journey from teacher education to professional practice, highlighting challenges related to the dichotomy between mathematicians and didacticians. Additionally, it presents the role of statistics in the Brazilian national curriculum, discussing key curriculum documents such as the National Curriculum Parameters (PCN), the National Common Core Curriculum (BNCC), and the international influences shaping these guidelines. The section concludes with the first formulation of the research problem, identifying gaps between university-level statistics education and the practical needs of secondary school teachers. The second section (1.2) explores statistics both as a subject to be taught and as specialised knowledge for teaching, emphasising the epistemological foundations of statistical literacy, reasoning, and thinking, and discussing how teacher education programmes can better integrate data analysis, inquiry-based learning, and digital tools to enhance teachers' professional development.

Chapter II establishes the theoretical framework and research methodology guiding this dissertation. The first section (2.1) provides the theoretical foundations, focusing on the ATD and its relevance to the teaching profession. It explores the ecological dimension of teaching, the need for a pedagogical paradigm shift, and how SRP-TE serve as a reference epistemological model to address these challenges. The chapter then examines how SRP-TE is analysed within its ecological context and how it can be leveraged to tackle institutional constraints in teacher education. The second section (2.2) presents the research hypotheses, questions, and objectives, framing the investigation. The third section (2.3) details the research methodology, identifying didactic engineering as the main research approach, followed by a breakdown of data collection, analysis strategies, and methodologies applied to each specific objective (SO). It also includes a data management plan and ethical considerations, ensuring compliance with ethical research standards.

Chapter III presented a study that aims to identify dominant epistemologies in the teaching of statistics and in secondary mathematics teacher education, through an analysis of interviews with lecturers. This chapter corresponds to the preliminary analysis phase of DE where the two didactic phenomena identified are highlighted: "the invisibility of data treatment" and the presence of “applicacionism” in teacher education. One publication related to this study was a communication paper presented at the *IX Cibem Congresso Iberoamericano de Educação Matemática*.

Chapter IV is about the *a priori* design of the SRP-TE and includes two different sections. The first one is the result of a communication paper presented at the CERME12. This paper presents the first design of an SRP-TE proposal starting from a school activity on the distribution of water in Brazil and its expansion to include aspects of statistical work such as data search, gathering, cleaning, and visualisation that were previously not included in secondary education. The second section of this chapter contains a book chapter that was presented as a communication at the CITAD7. This study presents the implementation of the SRP-TE with a group of preservice teachers. Both studies constitute the phase of the *a priori* analysis of DE. Two complementary publications of these studies were a communication paper presented at the *Eleventh International Conference on Teaching Statistics* (ICOTS11) and an article published in the *UNO: Revista de didáctica de las matemáticas*.

Chapter V is composed of two sections. The first is a result of a communication paper presented at CERME13. The second section contains an article accepted to be published in the special issue “Inclusive Statistics Education with Digital Resources” of the Statistics Education Research Journal (SERJ). Both articles present the SRP-TE implemented for in-service teachers in an online modality and the ecological analysis of this implementation regarding different aspects. They correspond to the phases of *a priori*, *in vivo*, and *a posteriori* analysis of DE.

Chapter VI contains part of a communication paper presented at the ICME15 and more data analysis of the SRP-TE implemented for preservice teachers. It focuses on the conditions established for the implementation of this SRP-TE in the context of preservice teacher education. It also corresponds to the phases of *a priori*, *in vivo*, and *a posteriori* analysis of DE.

Chapter VII presents the main reflections and conclusion included in the chapters, as well as in the studies that are in the process of being written and published. The structure of this chapter attempts to answer the different research questions posed. The chapter is finished with some open questions and some lines of future research. Finally, the appendices include the materials that have been used in the implementations, as well as the participants’ productions and other materials from the SRPs-TE.

In Table 2.1 we add the relationship between the specific research questions, objectives, the publications produced, and the corresponding DE phases.

Table 2.1. Relation of the studies carried out

<table>
<tr><td colspan="2">Preliminary analysis</td></tr>
<tr><td>RQ_1 related to didactic transposition: What are the dominant conceptions of statistics and statistical teacher education in institutions related to the knowledge to be taught at the secondary school level and preservice teacher education? How might these conceptions hinder the establishment of favourable conditions for statistical education that emphasise the study of data and their variability?</td><td>Chapter III
Related publication:
Verbisck, J., Bittar, M., Bosch, M., & Barquero, B. (2023). Transparencias en la estadística como saber por enseñar en la formación del profesorado de secundaria. In A. Manrique, & C. Groenwald (Eds.), Anais do IX Cibem Congresso Iberoamericano de Educação Matemática, (pp. 1136–1147). Editora Akamedy.</td></tr>
<tr><td colspan="2">The hypotheses following are evident:
H1. The main didactic phenomenon concerning statistics education at secondary level (and beyond) is the invisibility of data treatment in both school and scholarly mathematics. This phenomenon is the consequence of specific didactic transposition processes that are to be studied more in-depth. In any case, we postulate that it should also be approached as a teacher education problem as it is central to the Klein's double discontinuity problem mentioned before.
H2. Concerning statistics teacher education, two mean types of constraints appear, one linked to a limited view of mathematical statistics (dominant epistemological model of statistics) and the other linked to an "applicationist" conception of teacher education (dominant didactic model of teacher education).</td></tr>
<tr><td colspan="2">A priori analysis</td></tr>
<tr><td colspan="2">From the ATD, it comes the assumption that study and research paths for teacher education (SRP-TE) have the capacity to lead teachers to question the knowledge to be taught and even, sometimes, the related scholarly knowledge (Barquero et al., 2022). SRP-TE can be also a potential device to address the didactic phenomena of the "invisibility of data treatment" and the presence of "applicationist" in teacher education. They are appropriate educational proposals to approach the limitations of the paradigm of visiting works.</td></tr>
<tr><td>RQ_2 related to the design of a pilot SRP-TE: What educational proposal based on SRP-TE can be designed and implemented with a group of preservice mathematics teachers in Brazil? How can this proposal contribute to providing future teachers with tools to design, analyse and implement new didactic processes for the teaching of statistics in lower secondary school?</td><td>Chapter IV
Related publications:
Verbisck, J., Bittar, M., & Bosch, M. (2022a). Learning to teach statistics through study and research paths. In Hodgen, J., Geraniou, E., Bolondi,G. & Ferretti, F. (Eds.), Proceedings of the Twelfth Congress of the European Society for Research in Mathematics Education (CERME12) (pp. 982–989). Free University of Bozen-Bolzano and ERME. https://hal.science/hal-03754718v2

Verbisck, J., Bittar, M., Bosch, M., & Benito, R. (2024). Statistics teacher education at secondary school level in the paradigm of questioning the world. In Florensa, I., Ruiz-Munzón, N., Markulin, K., Barquero, B., Bosch, M. & Chevallard, Y. (Eds.), Extended Abstracts 2022. Proceedings of the 7th International Conference on the Anthropological Theory of the Didactic (CITAD7). Trends in Mathematics, vol 16 (pp. 555–572). Birkhäuser. https://doi.org/10.1007/978-3-031-55939-6_42

Verbisck, J., Bittar, M., Bosch, B., Barquero, B., Benito, R. (2022b). Study and research path for statistics teacher education at secondary school level: An exploratory study. In S. A. Peters, L. Zapata-Cardona, F. Bonafini, & A. Fan (Eds.),</td></tr>
</table>

	Bridging the Gap: Empowering & Educating Today's Learners in Statistics. Proceedings of the 11th International Conference on Teaching Statistics (ICOTS11 2022), International Association for Statistical Education. https://doi.org/10.52041/iase.icots11.T2A2 Verbisck, J., Bittar, M., & Benito, R. (2024). Undagando sobre la escassez de agua en Brasil: un proyecto de formación del profesorado de secundaria. *UNO: Revista de didáctica de las matemáticas*, *104*, 15–24. https://dialnet.unirioja.es/servlet/articulo?codigo=9650407
A priori, in vivo, and a posteriori analysis	
$\boldsymbol{RQ_3}$ **related to the implementation of an SRP-TE for in-service teachers:** What conditions implemented in an online SRP-TE can help teachers address and detach themselves from the phenomenon of the invisibility of data treatment in a context of online in-service teacher education? What constraints limit it?	**Chapter V** **Related publications:** Verbisck, J., Barquero, B., Bittar, M., & Bosch, M. (2023). A study and research path for teacher education in statistics: dealing with the transparency of data treatment. In Drijvers, P., Csapodi, C., Palmér, H., Gosztonyi, K., & Kónya, E. (Eds.), *Proceedings of the Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13)* (pp. 1078–1085). Alfréd Rényi Institute of Mathematics and ERME. https://hal.science/hal-04413704 Verbisck, J., Barquero, B., Bittar, M., Bosch, M. (2024). Addressing water scarcity through statistical inquiry in teacher education. *Statistics Education Research Journal*, *23*(2). https://doi.org/10.52041/serj.v23i2.722
$\boldsymbol{RQ_4}$ **related to the implementation of an SRP-TE for preservice teachers:** What conditions implemented by a face-to-face SRP-TE can help preservice teachers address the phenomenon of the invisibility of data treatment and what constraints appear to limit it? $\boldsymbol{RQ_5}$ **related to the ecology of SRP-TE and the dissemination of didactic research tools in teacher education:** What collaboration mechanisms between teachers, researchers and educational institutions, and what didactic tools would facilitate the dissemination of SRP-TE and research in statistics preservice and in-service secondary teacher education?	**Chapter VI** **Related publication:** Verbisck, J., Barquero, B., Bittar, M., & Bosch, M. (in press). A study and research path for teacher education in statistics to address data transparency. In *Proceedings of the Fifth International Congress on Mathematical Education (ICME15)*. ICME.

CHAPTER III: PRELIMINARY ANALYSIS

Chapter III addresses the question and specific objective:

RQ_1 related to didactic transposition: What are the dominant conceptions of statistics and statistical teacher education in institutions related to the knowledge to be taught at the secondary school level and teacher education? How might these conceptions hinder the establishment of favourable conditions for statistical education that emphasise the study of data and their variability?

SO_A: Analyse the conception or model of statistics as data science that prevails through curricular documents (textbooks, national and international curricula, teaching resources, etc.) in the noosphere and in scholarly institutions. Study the consequences concerning the phenomenon of "applicationism" and that of the "invisibility of data treatment" in teacher education.

And also, two other specific questions:

- *How to characterise the dominant epistemological and didactic models of statistics according to the positions adopted and the relations established by the interviewed subjects?*
- *What conditions and constraints are evidenced for the teaching of statistics in different training institutions, in particular in teacher education?*

This chapter explores the challenges and dominant epistemological models in the initial education of secondary school statistics teachers, focusing on the case of Brazil. It examines how statistics is taught in teacher education programmes and how certain dominant perspectives, particularly "applicationism", shape the way preservice teachers engage with statistical knowledge.

The chapter begins by identifying the gap between modern statistical education perspectives—which emphasise data collection, variability analysis, and real-world applications—and the current practices in schools and teacher education programmes, where statistics is often reduced to arithmetic calculations and graphical representations. The concept of "applicationism" is introduced as a key factor influencing this phenomenon, referring to a didactic approach that treats statistics as a set of tools to be applied rather than as a field of inquiry requiring critical engagement.

Through a theoretical framework rooted in the ATD and the notion of didactic transposition, the study examines how statistical knowledge is shaped and adapted as it moves from scholarly institutions to secondary schools according to the perspectives of university educators responsible for teaching statistics in teacher education programmes. The research methodology is based on semi-structured interviews with university educators responsible for teaching statistics in teacher education programmes. These interviews provide insight into the dominant epistemological and didactic models that influence teacher education, particularly how statistics is conceptualised, structured, and taught to preservice mathematics teachers.

The analysis reveals that teacher education in statistics often follows a uniform, applicationist model, where statistical knowledge is presented in a rigid, deductive structure, emphasising predefined concepts and formulas rather than encouraging exploration, data literacy, or real-world problem-solving. This lack of focus on the process of working with data—collecting, cleaning, and analysing it—leads to an "invisibility of data treatment", making it difficult for preservice teachers to develop a deep understanding of statistical reasoning.

The chapter concludes by discussing the implications of these findings for teacher education and curriculum reform, highlighting the need for a shift towards inquiry-based learning and a stronger integration of real-world data analysis in teacher education programmes. It calls for collaboration between statistics educators, didactic researchers, and policymakers to create conditions that support a more meaningful and conceptually rich approach to teaching statistics at the secondary level.

3.1. MANIFESTATIONS OF "APPLICATIONISM" IN SECONDARY SCHOOL STATISTICS TEACHER EDUCATION

3.1.1. The problem of teacher education in statistics

Statistics has evolved strongly in recent decades in line with the development of technological tools for data processing, giving rise to what is now known as "data science" (ASA, 2020; Cleveland, 2001). Research related to statistics education shows that there is agreement in the international research community on the importance of adopting a broad view of the field. This vision should include aspects such as searching for and collecting data in real contexts, selecting and organising these data, tabulating and visualising them, using specific software, simulation, reporting and communication, among others, within a perspective of solving open questions and studying variability (Ben-Zvi, 2020; Ben-Zvi & Garfield, 2004; Burrill & Biehler, 2011). Much research refers to this new vision under different terms, either with an emphasis in "statistical reasoning" (Ben-Zvi & Garfield, 2004), "statistical literacy" (Gal, 2002), "statistical thinking" (Wild & Pfannkuch, 1999), "statistical sense" or "stochastic sense" (Batanero et al., 2013; Burrill & Biehler, 2011), and "informal inferential reasoning" (Pfannkuch, 2006), among others.

Statisticians and didacticians alike advocate the need for teaching to incorporate the new perspectives of statistics, which are beginning to be included in the curricula of many European and American countries (ASA, 2020; Brasil, 2018). However, the reality in actual teaching is still far from this vision. School statistical activities are often reduced to numerical calculations of statistical measures (frequencies, measures of central tendency, deviations, etc.) and to the elaboration or interpretation of standardised graphical representations (pie charts, bar charts, histograms) (Burrill & Ben-Zvi, 2019). Even if national curriculum guidelines include aspects such as *data collection, organising and recording, planning and executing a sample survey* (Brasil, 2018), textbooks rarely present related activities (Prestes, 2021). One of the challenges highlighted by Ben-Zvi and Garfield (2004) regarding the teaching of statistics is that

> […] students equate statistics with mathematics and expect the focus to be on numbers, computations, formulas, and one right answer. They are uncomfortable with the messiness of data, the different possible interpretations based on different assumptions, and the extensive use of writing and communication skills. (p. 4).

We would add that, in many cases, teachers also feel the same discomfort. Chevallard and Wozniak (2007) described this situation in terms of "the reduction of statistical work to arithmetical work". This *arithmetisation of statistics* is often reinforced by the lack of a clear mathematical status for data treatment, which we have characterised in previous work as the didactic phenomenon of the “invisibility of data treatment” (Verbisck et al., 2023).

Teacher education appears as a critical variable to operate a change of perspective in teaching. Many authors (Batanero et al., 2011a; Ben-Zvi et al., 2018; Burrill & Ben-Zvi, 2019; Garfield & Everson, 2009) see a need to update the knowledge taught to teachers who received very limited training in the domain of statistics and probability. Building on the models established by Shulman (1986) and Ball et al. (2008) about the “pedagogical content knowledge” (PCK) and the “mathematical content knowledge for teaching” (MCKT) respectively, some authors propose more specific terminology to point at domain-specific aspects that require particular attention. Groth (2007) presents a conceptualisation of “statistical knowledge for teaching”, which considers both “common knowledge” (referring to the skills acquired in conventional mathematics courses) and “specialised knowledge” (developed through a careful approach to addressing specific mathematical tasks and problems that emerge in educational settings). These types of knowledge are articulated through the four components of statistical inquiry: “formulating questions”, “collecting data”, “analysing data”, and “interpreting results”. Henriques and Ponte (2014) propose that the professional development of statistics teachers should not only focus on specialised knowledge of statistical content but also on the use of exploratory data analysis and digital technologies. They argue that teachers need to understand how students develop statistical ideas and how to use technological tools to support their learning. Batanero and Díaz (2010) and Ponte and Noll (2018) provide a comprehensive review of the research literature on this topic and discuss various approaches to professional development for statistics teachers. Both reviews highlight that collaboration between mathematics educators and statisticians is essential to address the challenges of teaching statistics and teacher education. While statistics differs from other areas of mathematics, there are synergies between mathematics education and statistics education that can be leveraged in teacher education.

Despite the guidelines proposed by research to make teacher educational practices evolve, the changes needed to progress in this direction are far from assured. There is little research that addresses the problem of teacher education from an institutional perspective, to identify the conditions needed for change to take place in the entire *noosphere*[2] and, above all, to study the

[2] In ATD, The noosphere is the sphere of collective intellectual activity—comprising both formal and informal institutions—that exists outside the core of the education system yet plays a decisive role in legitimising and shaping the curriculum. It is essentially “the sphere where one thinks”, serving as a filter between societal knowledge and the strict confines of the teaching system by critically reflecting upon, debating, and transmitting

constraints that hinder the evolution of how statistics is conceived at school and teacher education institutions. In particular, we consider it essential to address the constraints that stem from the dominant epistemologies related to statistics in all the institutions involved in the teacher education process.

In this study, we focus on the initial training of secondary school teachers in statistics, considering the particular case of Brazil, and we aim to characterise the dominant epistemologies of statistics and teacher education in this particular domain. The empirical data considered focus on a set of six individual interviews with educators who have taught the subject Probability and Statistics (*Probest*, hereafter) in initial teacher education courses at three universities in Brazil. We aim to investigate the dominant conceptions of statistics and statistical teacher education in these universities to identify to what extent they might hinder the establishment of favourable conditions for teaching statistics more closely related to the study of data and their variability.

3.1.2. Theoretical framework and formulation of the research question

Our research is part of the anthropological theory of the didactic (ATD), developed by Y. Chevallard and colleagues since the beginning of the 1980s with the notion of didactic transposition (Chevallard, 1985). In ATD, an *object* is considered to be any entity, whether tangible or intangible, which has an existence for at least an *individual* or an institution (Chevallard, 1992). Everything can be considered an object, including all those works that are intentional products of human activity. Objects are not static entities but evolve over time and are influenced by social and cultural factors. Knowledge in the ATD is described by the notion of the personal (or institutional) *relationship* of an individual (or an institution) to an object. As described by Chevallard (1992):

> An object exists as soon as a person *X* or an institution *I* recognises this object as *existing* (for it). More precisely, it can be said that the object *O exists for X* (respectively, *for I*) if a relation exists, which I shall note $R(X, O)$ (resp. $R_I(O)$), and which I shall call the *personal relation from X to O* (resp. *institutional relation of I to O*). In other words, the object *O* exists if it exists for at least one person *X* or one institution *I*, that is if at least one person or one institution *relates to this object* (p. 142).

Institutions are social and organisational structures that influence the way individuals relate to objects. Institutions have defined positions. An individual becomes a *subject* of an institution

academic doctrines. In this role, the noosphere not only channels the flow of knowledge from the environment into educational practice through the process of didactic transposition but also ensures the compatibility of the teaching system with its broader socio-cultural context.

when she assumes certain institutional positions and, with them, certain relations to the objects *O* of *I*. it is postulated that what a person does, says, understands, etc. about an object *O* is conditioned (though not determined) by the institutional relations to *O* of the institutions in which *X* participates as a subject. At the same time, the participation of *X* in *I* contributes to making $R_I(O)$ evolve.

Pieces of knowledge (such as statistics, algebra, etc.) are objects that do not exist in a social vacuum but are always linked to at least one person or institution. Thus, certain conditions and constrain must be respected for a certain knowledge to exist in this institution (Chevallard, 2001). We consider schools (including the university, as a particular case) as institutions that impose conditions on those who occupy some position within them, such as that of teacher and student.

Many knowledge objects are created in specific institutions and then transferred to others, for instance to school institutions. These objects then follow a didactic transposition process (Chevallard, 1985), which postulates that:

> […] what is being taught at school (‘contents’ or ‘knowledge’) is, in a certain way, an exogenous production, something generated outside school that is moved — ‘transposed’ — to school out of a social need of education and diffusion. For this purpose, it needs to go through a series of adapting transformations to be able to ‘live’ in the new environment that school offers. For certain knowledge to be taught at school *transposition work* needs to be carried out so that something that was not made for school changes into something that may be reconstructed inside school. (Bosch & Gascón, 2006, p. 53)

In this study, we consider the didactic transposition processes of “statistics” that occur from and between different institutions. On the one side, there is the statistics that is created in universities and other research or professional institutions, what is called the “scholarly knowledge”, which recent evolution is transforming it at a high speed towards what is now called “data science”. On the other side, there is the secondary school institution where statistics is part of the knowledge organisation designed to be taught, what is called the “knowledge to be taught” at secondary school level. In between, we can consider the institution of teacher education, which is part of the school institution *noosphere* (Chevallard, 1985), where different organisations of scholarly statistics are selected for teacher education. In many countries, the institution of teacher education is also the university institution where statistics is created and used as scholarly knowledge (and also framed as “knowledge to be taught to future teachers”). In this case, a new type of “scholarly knowledge” is produced by research in statistical education, a type of knowledge that contributes to providing teachers with better strategies and perspectives about the teaching of statistics. Even if they cohabitate at the university, the positions can be clearly distinguished. In short, we can look at the statistical knowledge as it is elaborated and conceived from different institutional positions in the transposition processes. In this study, we consider three main kinds of knowledge organisation: *statistics as scholarly knowledge* (produced by statisticians), *statistics as knowledge to be taught at the university* (in mathematics studies or in teacher education) and *statistics as knowledge to be taught at secondary school.*

The relation $R_I(O)$ of an institution I to an object of knowledge O includes everything that is done in I with O. When O is a knowledge (such as statistics), we talked about the *dominant epistemological model* as a basic element of $R_I(O)$ that delimits the way O is understood and interpreted in I. Th study presented in this chapter aims to explore the *dominant epistemological models* of statistics as scholarly knowledge and as knowledge to be taught that become visible through the interaction with different subjects who adopt different institutional positions at the university. In particular, we aim to characterise what and how these subjects describe statistics as scholarly knowledge, as knowledge to be taught at the university, as knowledge to be taught in secondary education, and as knowledge to be taught in teacher education. We also seek evidence, through their discourses, of the *dominant didactic models* in these institutions, i.e. the most prevalent ways of organising the teaching and learning of this knowledge in certain educational contexts, as well as the possible existence of *alternative didactic models*.

From the interviews carried out, we can distinguish four main positions assumed by the interviewees in the university institution: that of “researcher in statistics” (RS), that of “researcher in statistics education” (RSE), that of “statistics lecturer at the university” (SLU) and that of “statistics educator in teacher education” (SETE).

This leads us to formulate the following research questions:

RQ1: *How to characterise the dominant epistemological and didactic models of statistics according to the positions adopted and the relations established by the interviewed subjects?*

RQ2: *What conditions and constraints are evidenced for the teaching of statistics in different training institutions, in particular in teacher education?*

By addressing these questions, we aim at contributing to research in statistics education by identifying the alignments and incoherences between the conception of statistics of two important institutions that intervene in teacher education processes: the scholarly institution responsible of the production, use and dissemination of statistical knowledge, the teacher education institution and the institution of research in statistics education.

3.1.3. Methodology

3.1.3.1. Topics of discussion and interviewee profiles

The interviews were conducted based on a semi-structured script consisting of five topics and corresponding questions:

(1) Describe *what statistics is? How would you define statistics?*

(2) Gather opinions on what statistics should be taught in secondary school and its *raison d'être*: *What do you think statistics should be taught in secondary school? What do these students need to learn statistics for?*

(3) Gather opinions on what statistics should be taught in a mathematics teacher education course: *What do you think should be included in the statistics syllabus in the mathematics education programme? What do you consider essential in the initial training of mathematics teachers in statistics?*

(4) Discuss differences between the way probability and statistics are taught in bachelor mathematics and mathematics education (only for those teachers who teach both university courses): *Are there any differences between what is taught in the bachelor mathematics course and the mathematics education course with respect to the subject Probest?* If so, *what are they?*

(5) Gather opinions on variations or new proposals for teacher education: *Do you think that teacher education in statistics could be different from what exists today?* If so, *what kind of education could there be, and what factors do you think restrict this kind of education?*

With this script, we seek to characterise, on the one hand, the *dominant epistemological models of statistics* as scholarly knowledge (topics 1), as knowledge to be taught in secondary education (topic 2), and as knowledge to be taught in secondary teacher education (topic 3). On the other hand, the two last topics address the dominant didactic models of statistics as knowledge to be taught in teacher education (topic 4) and the emergence of alternative didactic models in teacher education (topic 5).

A total of six semi-structured interviews were conducted with researcher-educators who have taught the *Probest* subject at four universities in Brazil. These were conducted individually, guided by the author of this study, in virtual mode between March and October 2022, with a duration between 30-90 minutes. All interviews were recorded and transcribed using *Google Meet* or *Microsoft Teams*. Table 3.1 summarises the characteristics of the six participants according to their doctoral training, research field and institutional link. We will treat them as forming a convenience sample that will help us refine our hypotheses without aiming at generalising them.

Table 3.1. Profiles of interviewees

Code	PhD	Research field	Institution
X_1	Biometrics	Statistics	Federal University of Mato Grosso do Sul
X_2	Statistics	Statistics	Federal University of Mato Grosso do Sul
X_3	Education	Statistics education	Pontifical Catholic University of Campinas
X_4	Mathematics education	Statistics education	Pontifical Catholic University of São Paulo
X_5	Education	Statistics education	University of São Paulo
X_6	Applied statistics	Statistics education	University of São Paulo

3.1.3.2. Characterisation of dominant models and research hypotheses: applicationism

Barquero et al. (2014) characterised "applicationism" as the dominant epistemological and didactic model related to the university teaching of modelling in Experimental Sciences. Under the influence of "applicationism", the modelling activity is reduced to an application of previously established knowledge, or even in its most extreme form, as a mere illustration of mathematical tools in non-mathematical contexts artificially created for that purpose. For their characterisation, the authors defined and empirically contrasted five indicators:

> I_1: *Mathematics remains independent of other disciplines* (epistemological purification): mathematical tools are considered independent of extra-mathematical systems and are applied in all cases in the same way.
>
> I_2: *The basic mathematical tools are common to all scientists*: all CCEE [Experimental Science] students must follow the same introductory mathematics course, no specificity is conceived (depending on their specificity).
>
> I_3: *The organisation of mathematical content follows the logic of concepts* (*deductivist logic*). Instead of being constructed by considering modelling problems originating in the various scientific disciplines, everything happens as if there were only one way of organising mathematical content.
>
> I_4: *Applications always follow basic mathematical training*: models are built from the concepts, properties and theorems of each subject and, once built, independently of any extra-mathematical system, possible applications are sought in each field of work.
>
> I_5: *Many extra-mathematical systems can be constructed without any reference to mathematics*. Ultimately, mathematics could be dispensed with in the teaching of CCEE [...]. (Barquero et al., p.89).

In our research, we assume two main types of constraints: the first ones are derived from the dominant epistemological model of statistics at the scholarly institutions, and the second ones are linked to the very conception of teacher education. We can thus speak of a double manifestation of applicationism. The first manifestation deals with applicationism in the university teaching of statistics, similar to the applicationism in the teaching of mathematical modelling in sciences. In this version of applicationism, prevails the assumption that there is a single set of knowledge —the statistical knowledge— that is applied and taught without much variation in different training contexts (biology, engineering, psychology, economics, etc., and for preservice teacher education). The second form of applicationism appears in the teacher education proposal itself, by assuming that the type of knowledge future teachers need is similar to those needed for other specialisations and that future teachers will know how to use (or "apply" it) when teaching it in the classroom. This second form of applicationism is not specific of statistics but is extensive to other mathematical areas such as calculus, algebra, geometry, etc.

This leads us to make a double adaptation of applicationism, on the one hand, in the case of the university teaching of statistics and, on the other hand, in statistics for teacher education. In our research, we propose to reformulate the initial indicators of Barquero et al. (2014) and consider three for each case: I_1, I_2 and I_3 referring to the dominant epistemological and didactic models in the teaching of statistics and I_{1TE}, I_{2TE}, I_{3TE} referring to the dominant models in teacher education. Table 3.2 presents their initial description.

Table 3.2. Indicators of "applicationism" for the case of statistics and teacher education (TE)

Indicator	Description
I_1 *Independence*	Statistical knowledge remains the same in all training courses. Statistical tools are considered to be constructed in order to be "applied" later on to different cases.
I_2 *Conceptualist statistics*	There is a *deductivist* organisation of the contents of statistics, following a logic of concept construction, which does not depend on the specific area where it is used (medicine, biology, engineering, economics, etc.)
I_3 *Invisibility of data treatment*	Aspects related to searching, cleaning and organising data are not considered as part of statistical knowledge.
I_{1TE} *Uniformity*	In the training of teachers (as in the training of scientists, engineers, etc.), the same kind of statistical tools are supposed to be needed and therefore different groups can be trained in the same way.
I_{2TE} *Formalisation and application*	Statistical *formalisation* precedes adaptation to school contexts in which it is "applied" without changing its structure.
I_{3TE} *Invisibility of data treatment*	Aspects related to finding, cleaning and organising data are not treated in teacher education as part of the statistical work and, if they are, they do not have a clear epistemological status.

It should be noted that the indicators I_3 and I_{3TE} have been added for the special case of statistics, being related to the phenomenon of invisibility of data treatment described in section 3.1.1.

Based on the proposed adaptation of the applicationism indicators, we formulate the following hypotheses that will be studied and discussed in this study:

- H1 *Applicationism in statistics*: The dominant epistemologies of statistics in the institution of secondary teacher education can be characterised as "applicationist" in that the construction of statistical models and tools precedes their use for data analysis.
- H2 *Applicationism in teacher education:* In some cases, the dominant didactic models in teacher education are applicationist to the extent that they are based on a single training for any specialisation, leaving under the future teachers' responsibility its subsequent transposition to the classroom.
- H3 *Relationship between both manifestations of applicationism*: The type of applicationism in statistics and applicationism in teacher education are linked to the profile of the lecturer in charge or teacher education courses or the their reference institution (research in statistics or in statistics education).

We analyse the validity of these research hypotheses using the results identified during the characterisation of the epistemological and didactic models and present this contrast in the discussion section.

3.1.3.3. Strategies for the analysis

To study the preceding hypotheses, we sought to identify evidence of the appearance of the indicators of applicationism in each topic of discussion. Table 3.3 shows the relationships between the interview themes and the indicators to be tested.

Table 3.3. Relationship between discussion topics, interview questions and indicators (use of brackets signifies lower anticipated presence)

Topics	**Related indicators**					
	I_1	I_2	I_3	I_{1TE}	I_{2TE}	I_{3TE}
(1)	**X**	**X**	**X**			
(2)	**X**	**X**	**X**			
(3)	**(X)**	**(X)**	**(X)**	**X**	**X**	**X**
(4)	**(X)**	**(X)**	**(X)**	**X**	**X**	**X**
(5)				**X**	**X**	**X**

From the transcription of the interviews, fragments of the speeches are grouped by topic of discussion and by reference to the different indicators. *Atlas.ti* is used to assign labels to the topic and indicator(s) being referred to[3]. Once the analysis by the first author had been

[3] In Appendix B we provide tables with all the fragments of the interviews' speeches labelled with the indicators.

completed, it was followed by triangulation by two of the cosupervisors, which involves comparing and contrasting the individual interpretations to identify agreements and divergences in the data.

3.1.4. Results

3.1.4.1. Topic 1: Dominant epistemological models in scholarly institutions

Table 3.4 presents a selection of the main excerpts in which the interviewees refer to the dominant epistemological models of statistics. The formulation of the interviewees has been respected, and our translation has been presented in English. In the case of this example, all the interviewees made explicit an institutional relationship in the position of RS (for the cases of X_1 and X_2) and RSE (for the cases of X_3, X_4, X_5 and X_6).

Table 3.4. *X*'s comments on their conceptions of statistics as scholarly knowledge (Topic 1)

Interviewees and comments	Classifications
$\boldsymbol{X_1}$: Statistics is that science that will help how to collect and, before collecting, how to plan a collection; how to collect, how to organise, how to analyse and, mainly, how to interpret any kind of data set. I like to summarise "*statistics*"*: how to plan, how to collect, how to organise, how to analyse and interpret any kind of data set, whether quantitative or qualitative*.	Not I_3. Non-invisibility of data treatment
$\boldsymbol{X_2}$: Basically, I *see statistics as a subfield of mathematics*. Some statisticians do not like that definition very much. But I see it as a subfield. A mathematician, for example, does statistics very well, he just has to adapt. *To me, the definition of statistics is just a sub-area of mathematics that allows you to draw conclusions from information, from data*. And you will draw those conclusions based on mathematical criteria. That is my definition of statistics [...] *Statistics is a mathematical process, with criteria and following the laws of mathematics that allow us, in the future, to draw conclusions*.	I_2. Conceptualist statistics
$\boldsymbol{X_3}$: *Statistics is a science of collecting, analysing and interpreting data* and is very important when we think about contemporary society and when we think, especially, about the use of technological resources, the speed and the amount of information that circulates in our society.	Not I_3
$\boldsymbol{X_4}$: For me, *statistics is the science of data, of numbers in context*. Which is Moore's definition. With that, I try to work, I think it is important that they understand all that data in the context in which it was extracted.	Not I_3
$\boldsymbol{X_5}$: Statistics (with the help of probability) will work with *risk and variability*, constant elements in all natural phenomena.	Science of risk and variability
$\boldsymbol{X_6}$: *Statistics is*, let's say, *observing a set of numerical information, processing this numerical information and drawing conclusions. Sometimes this involves an inferential process, i.e. if this numerical information comes from a sample, under the conditions of a sample that allow inference, inferences are made for the whole population*. It is often characterised as *the science of numbers*, which deals with numbers, i.e. collects numbers, and concludes about something (some object, some variable, some interest) from a larger set which would be the population.	Science of numbers

In the definitions of “what statistics is” presented by the interviewees, we note some aspects that characterise statistics in three main visions. A first vision (with its variants but predominant, present in the discourses of X_1, X_3, X_4 and X_6) is that of statistics as a science of data in the direction of research that we discussed in the introduction (American Statistical Association, 2020; Cleveland, 2001), with an explicit reference from X_4 (and X_6 also at other points of the interview) refers to Moore’s (1992) definition of statistics as “the science of data, numbers in context”. This first view emphasises the relevance and processes involved in the treatment of data (not I_3). A second view can be distinguished by characterising statistics as a subfield of mathematics (X_2) and relating it to the “arithmetisation” of statistics and its conceptual composition (I_2). The third view, considered by X_5, defines statistics as the science that works with risk and variability, which we had not considered in the first instance, but we believe it interesting to consider it.

The first view of statistics—data science—emphasises the aspect of data treatment, noting the presence of the negation of the indicator I_3, i.e. emphasising the non-invisibility of data treatment. Concerning what X_2 advocates, we note the presence of indicator I_2, which also strongly relates to the two manifestations of the “arithmetisation” of statistics and the invisibility of data treatment. With this discussion of the results around topic 1, we illustrate how we sought to identify the fragments of the interviewees’ discourse and the respective institutional relationship. Next, we present the main results for topics 2 to 5 (Table 3.2).

3.1.4.2. Topic 2: Dominant epistemological models of the statistics to be taught at secondary school

When asked about “what statistics to teach at secondary school level”, some changes in the interviewees’ positions are noticeable. In the comments of X_1 and X_2, very similar opinions are noted, and both assume the position of SLU to comment on statistics in secondary education, saying that they do not have experience with secondary education but that they can speak from the perspective of lecturers of students who have just left secondary education and are starting a bachelor’s degree. X_1 stressed that “some of the most important basic statistical skills in secondary education are how to work with the mean, the median and how to read a graph or a table” (excerpt from X_1’s speech). He complemented his comment by adding:

> Regardless of the field in which they are going to work, knowing how to correctly interpret a table or a graph and having at least notions of the main statistics, simple ones such as mean, median, what is a standard deviation, or how to work with the mean and standard deviation together, can be important for secondary school students. Because they are going to use it, no matter what field they are working in. (excerpt from X_1’s speech)

Indicator I_1 is evident here since, in the opinion of X_1, there is a basic knowledge of statistics that is maintained in any field or training. X_2 stresses the importance of secondary school pupils knowing how to summarise information "through the so-called 'central measures'. I think it is enough to know the mean, they do not even need to talk about what the median or mode is and, above all, to interpret graphs" (excerpt from X_2's speech). Both fragments are characterised as examples of the indicator of invisibility of data treatment (I_3).

The other interviewees speak from the perspective of researchers in statistics education (RSE). X_3 stresses the importance of promoting statistical and probability literacy from early childhood education as this develops citizens' critical reasoning, speeds up analysis and helps in decision-making. "But this literacy does not occur at a certain point in time as it is presented in curricula, for example, divided by school levels and what concepts to teach. That is, when it is studied from a more technical perspective: of rules, formulas, predefined concepts" (fragment of X_3's speech). We note a critique of conceptualist statistics (I_2) that characterises the organisation presented in curricula concerning the teaching of statistics. A criticism of I_1 is also noted when X_3 added: "The teaching of statistics that we advocate (that I advocate and that most researchers in this field advocate) is one in which it begins to be developed through activities related to real situations" (fragment of X_3's speech).

X_4 did not present many elements on statistics in secondary school but states that "from secondary school onwards I think it is fundamental that the student has descriptive statistics, probability. Do they need inferential statistics? No. But they can make some informal inferences while they are doing descriptive statistics" (fragment of X_4's speech). In this statement, the impact of the indicator of conceptualist statistics (I_2) is noticeable, as it assumes an organisation of knowledge following the logic of mathematical construction of concepts and states how far to go within this established conceptual organisation.

X_5 argued that starting the teaching of statistics from secondary school is essential to avoid deterministic thinking in students and to encourage a critical approach instead of an instrumental one as it used to happen: "There was only calculation and no reflection. From calculation to calculation, shifting from one chapter to another, and that was everything about statistics" (fragment of X_5's speech). It is suggested that statistics teaching should begin with the exploration of situations of uncertainty and data collection, promoting projects and debates from an early age. Students' participation in data collection and analysis is considered fundamental. A critique of conceptualist statistics (I_2) and attention to working with data (not I_3) were highlighted. An emphasis on the development of statistical modelling in secondary school is noted when X_5 stated that “you have to start young, that is, first analyse situations of uncertainty and variability in order to then be able to enter more directly into modelling, inference and more sophisticated things” (fragment of X_5’s speech).

In the opinion of X_6, the secondary school mathematics teacher should teach the difference between statistics and mathematics, “because statistics works with variability, with randomness, and this is what produces statistical methods so that we can decide things, without knowing the population” (excerpt from X_6’s speech). However, this difference often “goes unnoticed because of the teacher’s initial training, not understanding how to explore this issue and how to approach it with students” (fragment from X_6’s speech). According to his statement, students in secondary school should learn not only basic statistical concepts but also understand the fundamental nature of statistics: variability and chance. This implies that students should understand that statistics is not simply about performing deterministic calculations but about working with the uncertainty inherent in data and population, hence a critique of conceptualist statistics (not I_2).

3.1.4.3. Topic 3: Dominant epistemological models of the statistics to be taught in teacher education according to researchers in statistics education

When asked about statistics in teacher education, X_3 and X_4 take the positions of RSE. X_3 advocates that teacher education should include both the conceptual and procedural study of statistics, as well as its application to teaching. Concerning the teaching of Probest, X_3 proposes: “For example, classes could be divided into two phases. The first is devoted to conceptual and procedural study, with exercises to correct, solve and discuss. The second moment is to think about how these concepts will be worked on at school” (fragment of X_3’s speech), which emphasises the presence of the indicator of formalisation and application ($I_{2\mathrm{TE}}$).

X_4 states that teachers should be better prepared in statistics because, although this knowledge appears in the national curriculum on teacher education, teachers often express insecurities in surveys in dealing with this topic.

> I think, at the very least, they need to know descriptive statistics, probability and informal inference. But they need to know it in a deeper way. To go a little bit deeper into aspects of statistical thinking and reasoning. We have to remember that these teachers will make their pupil literate. They are the ones who are going to develop [statistical] literacy, so they need to have it very well developed. Because just knowing the content does not help them have the knowledge. For example, to do the changes of representation, to do the transnumerations, as Pfannkuch preaches. That kind of thinking, playing with transnumeration... they [teachers] will not have that vision. The teacher who only knows the content but has not developed the reasoning or statistical thinking does not do it (fragment of X_4’s speech).

This statement by X_4 highlights a critique of conceptualist statistics (I_2) and even mentions the work of Wild and Pfannkuch (1999). So, it includes a specificity in teacher education linked to the results of didactic research and the special perspective they propose for its teaching. Therefore, X_4 does not consider that the teacher education should be as equal as the others training, even if the same programme and structure are maintained.

The position of statistics educator in teacher education (SETE) is taken up by X_5 and X_6 and with complementary arguments. X_5 considered it essential that statistical training should receive more attention in teacher education and that data collection projects should be carried out throughout the undergraduate course. X_5 proposed introducing data collection and analysis from the first years of the degree course, starting with descriptive analysis and moving towards inferential analysis in later years (not I_{3TE}). With this education, the central aim is "to work with data and internalise the way of reasoning in Statistics. When students do this over the four years [of initial training] they will feel strong enough to do projects with their students" (excerpt from X_5's speech). According to X_6:

> [...] One of the things that I think is fundamental for preservice teachers to have the opportunity to experience is the collection of data and the analysis of the data collected, what I call projects. Because the idea is that this is developed with primary and secondary students, that they can collect data, either from their own class, school, neighbourhood or region, depending on the time they have to do it.
> [...] I think a mathematics teacher should be clear that statistics is not going to be treated in the same way as mathematics, with that equation-like determinism.
> [...] I think that randomness and variability are the main thing and that students can leave with these concepts clear in their heads. (Excerpts from X_6's speech)

In the same direction as X_5, X_6 emphasises the work on data traversal and data analysis in teacher education, as he believes that this can then be developed in his practice in mathematics classes (not I_{3TE}). Here, too, the strong "non-determinism" characteristic of statistics is emphasised as opposed to its focus on chance and variability. Finally, we note the presence of the uniformity indicator (I_{1TE}) in the last comment of X_6, stating that "then, of course, there is the filling in of contents because, when you open a subject, you open fronts: there are people who are going to be teachers, but there are people who are going to continue studying statistics in the future. So, you introduce a little bit of the fundamentals" (fragment from X_6's speech).

3.1.4.4. Topics 3 and 4: Dominant epistemological and didactic models of the statistics to be taught at the university and in teacher education according to researchers in statistics

As mentioned above, researcher-lecturers X_1 and X_2 are the only ones in our sample of interviewees who teach the subject Probest to all undergraduate courses at the university they are affiliated with. Therefore, in addition to asking them about "what statistics they believe to be important in a mathematics teacher education course" (Topic 3), we also asked them to discuss the differences (if any) between the way they teach this subject in the pure mathematics degree and in the mathematics teacher education (Topic 4).

When talking about statistics as knowledge to be taught in teacher education, X_1 assumes the position of SETE and, when talking about statistics as knowledge to be taught in the pure mathematics degree, he assumes the position of SLU. He does not present many elements when questioned about statistics in teacher education, he only argues that it is not necessary to "go too deep" into statistics for the training of teachers who will work in secondary education. Then, when talking about the differences between the subjects in the two courses, he switches to the SLU position and mentions that in pure mathematics the focus is more on theory, while in the teacher education course, the focus is on transmitting concepts to future teachers "once they are going to transmit what they have learned to their students" (fragment of X_1's speech). Here we observe the indicator of formalisation and application ($I_{2\mathrm{TE}}$) and the idea of "transmission", investigated within the TAD framework (Chevallard, 1997), that is intrinsically related to that of "applicationism".

Addressing statistics in teacher education, X_2 stated that:

> In teacher education, it is as if the student is a user. So, they have to know that the result is valid; they do not need to prove it. Because it is as a course for engineers... so that is the vision, they see it as a user. I am talking about engineers because a lot of times preservice teachers or engineering students take the course together. So, I think they have to know that it is valid and how to interpret the data. I focus on that approach. (Excerpt from X_2's speech)

This statement reflects an instance of the uniformity indicator ($I_{1\mathrm{TE}}$), where teacher education and engineering are perceived as equivalent. This perspective assumes that future teachers and engineers will be "users of statistics", primarily requiring skills in "interpretation of results" rather than theoretical understanding. When differentiating between the Probest subject offered at the pure mathematics degree and in teacher education, X_2 detailed the content covered in the teacher education and other related courses (such as engineering). This includes the definition of probability (from classical, frequentist, axiomatic, and geometric perspectives), random variables, probability distributions, descriptive statistics, estimation processes, confidence interval construction, and hypothesis testing, thereby illustrating the presence of indicators $I_{2\mathrm{FP}}$ and $I_{3\mathrm{FP}}$.

Additionally, X_2 provided a comprehensive description of the course offered within the pure mathematics degree, which is divided into distinct parts: one semester of probability and another semester of inference. Through this structure, X_2 highlighted the opportunity to work "*from a mathematical, theoretical perspective*", thus indicating the influence of conceptualist statistics (I_2). When distinguishing the approach between the pure mathematics course and the teacher education (licentiate degree), X_2 asserted:

> The difference between the two courses is this: in the pure mathematics degree, I focus more on proving things, on understanding why things are valid. In the licentiate degree, it is as if the student were a user. They need to know that the result is valid but do not need to prove it. Because it is a course for engineers... that is the perspective; they see it as a user. [...] In the pure mathematics degree, no. I focus on the construction itself, on the mathematical concepts that ensure its validity. (Excerpt from X_2's speech)

Both X_1 and X_2 described statistics education for future teachers and engineers as fundamentally the same, without the need for specific distinctions ($I_{1\text{TE}}$). However, they differentiate this education from that provided to pure mathematics students, emphasising a more "theoretical" nature of the latter (I_2).

3.1.4.5. Alternative didactic models of the statistics to be taught in teacher education: conditions and constraints

The last topic of discussion in the interviews was the alternative didactic models proposed by the interviewees in relation to statistics to be taught at undergraduate level. X_3 seemed to adopt the position of RSE and the others of SETE.

When asked to give their opinion on the current statistics education proposed to preservice teachers and on proposals for changes to this education, X_1 stated that "today the syllabus we have in the course is sufficient for the mathematics preservice teachers to be able to work in secondary school" (extract from the speech of X_1) and did not present ideas for changes. With this, we interpret the manifestation of uniformity ($I_{1\text{TE}}$) as predominant. Teacher education in statistics is the same as that offered to future engineers, chemists, physicists, etc., and is considered sufficient for their professional performance. According to this view, future teachers do not need specific training for their professional performance in secondary education and, as X_1 commented before, their job will be to "transmit" what they have learnt about basic statistics to their future students.

X_2 suggested that the teacher education (and engineering...) course should be less theoretical and more practical, where "*interpretation should be the main focus of the discipline*" (extract from X_2's speech). He presented factors that restrict teacher education and stated:

> As I said, I see statistics as a sub-area of mathematics. I think the problem is that most statisticians here in Brazil, at least the most respected ones, are mathematicians. And we, as mathematicians, try to mathematise everything; everything is a mathematical view. So, in statistics, what happens today is that people try to analyse all statistics. Forgetting the principles of statistics, the basic concepts, and the basic

> ideas of statistics came from practical problems. So, it is much easier for a mathematician to go and start talking about functions. Because they have been trained in it. That is what I was trained to do. To turn everything into functions and draw conclusions based on that. Without thinking about the student, from the point of view of his day-to-day life, what will he work on? What does that number mean in their day-to-day life? [...] For an undergraduate student, I see that the most important thing is ideas and interpretations. (Excerpt from X_2's speech).

With this statement of X_2, contrary to that of X_1, the fact that mathematicians are the educators in the undergraduate course appears as a constraint to the statistical teacher education (and of engineers as well, once X_2 characterises these two contexts as the same training). A certain criticism of the uniformity (not $I_{1\mathrm{TE}}$) of teacher education is also noted, but X_2 does not present alternative ways of solving this problem.

X_3 raised problems in the training of both lecturers who teach in pure mathematics and in teacher education, since "many times, educators do not work with didactics of statistics because they had statistical training in their undergraduate studies" (extract from X_3's speech), which limits the teaching of didactics of statistics. X_3 emphasised the importance of approaching statistics and probability from a perspective that can work on "analysing the performance of experiments, analysing real context issues, whether in the financial area or in the environment, or even in statistics working in various contexts" (fragment from X_3's speech), showing a criticism of uniformity (not $I_{1\mathrm{TE}}$).

Also in this sense, X_4 highlighted the constraint about the limited time in the subject Probest, but which could be solved with conditions such as "redistribution, joint work with Teaching Practice [a subject in the teacher education course], for example. Transforming a discipline into 'projects' to work on various aspects of mathematics in an articulated way, for example" (extract from X_4's speech). In addition, a teacher education different from the current one is proposed in the sense that "it would go deeper into aspects that today we still call 'didactics of statistics'. This includes this part of developing thinking, reasoning and statistical literacy" (extract from X_4's speech). Similarly to X_3, X_4 stated that the factor that most constrains change in statistics education is the training of the educators themselves and stressed the importance of having educators who consider the aspect of didactics of statistics, in addition to academic training in mathematics or statistics.

X_5 highlighted the need for closer collaboration between education and statistics departments to address the teaching of statistical concepts in the educational context. For her, this becomes crucial when Probest educators are faced with questions about how to apply these concepts in education. It is recognised that this process can be complicated, and constraints are often encountered, including a lack of interest on the part of the statistics department.

Finally, X_6 presented proposals and constraints very similar to those of X_5.

> Many of the educators who teach statistics come from the statistics department, and their training was, shall we say, more mathematical. Often, they see statistics almost as a part of mathematics. So, when they come and teach a course in undergraduate or wherever, they go and present it theoretically, which for them is intuitive because, at the end of the day, they have a theoretical background. [...] An important battle is that these undergraduate courses effectively consider the fact that the student will be a teacher (extract from X_6's speech).

With the exception of the first interviewee (X_1) who did not present any problem with teacher education and did not propose any changes, the others focused on conditions and constraints arising from the fact that current teacher education is characterised by a vision of uniformity, in which future teachers receive a type of training in statistics that is the same for several other training courses. This vision does not consider any kind of specificity for the case of future teachers.

3.1.5. Discussion

Table 3.5 presents an overview of the manifestations of the interviewees' institutional relations:

Table 3.5. Characterisation of the manifestations expressed by the interviewees

Topics	Manifestations		
	X_1 and X_2	X_3 and X_4	X_5 and X_6
(1) *Dominant epistemological model of statistics as a scholarly knowledge*	I_2 (1) Not I_3 (1)	Not I_3 (2)	"Science of risk and variability" (4) "Science of numbers" (1)
(2) *Dominant epistemological model of statistics as a knowledge to be taught in secondary education*	I_1 (1) I_3 (2)	Not I_1 (1) Not I_2 (1) I_2 (1)	Not I_1 (1) Not I_2 (2) "Statistical modelling" (1) Not I_3 (1)
(3) *Dominant epistemological model of statistics as knowledge to be taught secondary teacher education.*	I_1 (2) $I_{2\text{-FP}}$ (1)	$I_{2\text{-FP}}$ (1) Not I_2 (1)	$I_{1\text{-TE}}$ (2) $I_{2\text{-TE}}$ (1) Not $I_{2\text{-TE}}$ (1) Not $I_{3\text{-TE}}$ (2)
(4) *Dominant didactic model of statistics in secondary teacher education.*	I_2 (1) $I_{2\text{-TE}}$ (1) $I_{3\text{-TE}}$ (1)		
(5) *Alternative didactic model for statistics in secondary teacher education.*	I_1 (2) Not I_2 (1)	Not $I_{1\text{-TE}}$ (2)	Not I_2 (1) Not $I_{1\text{-TE}}$ (2)

Based on these characterisations, we will now discuss the aspects that most caught our attention in this empirical study in relation to the three research hypotheses put forward. The findings of this study illustrate a complex interplay between the epistemological and didactic models that shape the teaching of statistics in secondary teacher education. Our analysis has identified notable inconsistencies between educators' conceptions of scholarly knowledge and its transposition into educational contexts. In particular, the results support our initial hypotheses regarding the presence of an applicationist approach in both the teaching of statistics and teacher education.

Regarding hypothesis H1, we note that the interviewees explicitly stated different dominant epistemological models depending on whether they refer to the different types of knowledge (scholarly knowledge, knowledge to be taught in secondary school, at university and in teacher education). We identified three main views of statistics as "scholarly knowledge": statistics as "data science", statistics more closely linked to what we could consider "conceptualist statistics" based on the calculation of probabilities (statistics as a sub-area of mathematics), and statistics as the "science of risk and variability". Consequently, mainly in the cases of the non-didacticians interviewees, inconsistencies can be observed when they express their views on scholarly knowledge, the knowledge to be taught and their practice as educators. For example, in the statements of X_2, he stated that he was aware that the problem observed in teacher education in relation to statistics was that "mathematicians mathematise statistics", but that he himself was an agent of this practice. And in the case of X_1, who presented the definition of scholarly statistics as data science, but in his practice did not take the dimension of working with data as part of the training.

Three dominant epistemological models to be taught in secondary school appear: one is strongly characterised by the indicator of invisibility of data processing (I_3) which considers basic concepts such as "central measures and interpretation of graphs and tables" (for X_1 and X_2); then there is the idea of working on "descriptive statistics and informal inference" (for X_4) and statistics through projects and working with data, as opposed to the indicator I_3.

Regarding hypothesis H2, an "applicationist" view of the epistemological models of statistics as knowledge to be taught in mathematics teacher education is also observed, predominantly when considering teacher education as the same as the training of engineers. With mathematicians, more attention is paid to theoretical aspects; with preservice teachers, didactic aspects are considered. It should also be noted that didactic aspects are always mentioned at the end of the course, after the theoretical study, partially confirming the indicator of formalisation and application (I_{2TE}). In most cases, educators describe a subject structured according to a deductive logic: the principles of probability go first, followed by distributions, descriptive statistics, sampling and inference. We can therefore observe the existence of a vision that we can characterise as "applicationist" of this area of knowledge in relation to the teaching needs

(H2). This applicationism is based on proposing the same organisation of the knowledge to be taught for the different professional fields, which is only subsequently specified (or "applied") to each specific field. Finally, educators did not link their discourses in the two cases (contradiction of hypothesis H3). Those who defined statistics as the data science did not seem to place greater emphasis on this type of training, nor did they mention data treatment as a crucial aspect of the training.

We also observe that there is a limitation in the way of referring to the different forms of knowledge (or dominant models). Researchers in statistics education had more resources to talk about statistics because, even if implicitly, they have alternative models grounded in research in statistics education. This also allowed them to distinguish between the knowledge to be taught in secondary education and the knowledge to be taught in teacher education (teachers' professional knowledge). Finally, the indicator model has been effective in highlighting these disparities between the dominant models of educators according to the institution in which they are primarily located (scholarly knowledge institution or didactic research institution).

Our study highlights the interest of studying the different institutions involved in the process of didactic transposition: scholarly statistics, university teaching, teacher education, and secondary education. Even if the individuals assume different positions at the same time, their discourse includes specificities of the positions assumed, even leading sometimes to contradictions, for instance about what statistics should be taught in teacher education and what statistics they actually teach. We have characterised as "applicationism in teacher education" the strategy that consists of giving the same training to all future professionals, who will then have to "apply" and specify this training to their professional field. The analogy with Barquero et al. (2014) is that there exists one possible organisation of elementary statistics (or mathematics) and that this is the kind of statistics (or mathematics) that is to be taught to all students regardless of their speciality (teacher education, engineers, chemistry, biology or geology).

3.1.6. Conclusions

In conclusion, our study highlights the importance of the indicators of "applicationism" as tools for distancing oneself from the various institutions involved in the process of didactic transposition, an aspect that has been scarcely addressed in approaches based on the professional development of teachers of statistics (Batanero & Díaz, 2010; Ponte & Noll, 2018).

However, limitations are recognised and future lines of development are identified. As an exploratory study, our research has only considered a small sample of university profiles. The hypotheses obtained must be tested – and maybe extended – to a broad sample of interviewees to include a diversity of university profiles, professionals (applied statistics *versus* theoretical statistics *versus* other areas of mathematics), professional profiles (data science), trainer profiles (researchers in non-statistics education, trainers who do not do research in didactics, agents of the noosphere who make decisions on teacher education, etc.). Also, it is important to establish systematic dialogues with PCK and MCKT-based approaches to develop shared tools that facilitate the inclusion of transposition process analysis in research methodologies, in the line proposed by Thorstein and Bosch (2024).

This exploratory study requires further fieldwork to confirm and develop these observations, which we believe are crucial for better understanding the constraints that currently weigh on secondary teacher education and the scope for action that they open for its possible transformation. We stress the importance of considering the processes of didactic transposition (Chevallard, 1985) through which the knowledge to be taught is shaped and updated, in our case in the field of statistics. Teacher education appears as a key moment in the development of the processes of didactic transposition since teachers are fundamental agents in the implementation and dissemination of curricular reforms and represent an intermediate space in which knowledge institutions (in mathematics, didactics of mathematics, among others) interact with the teaching profession.

CHAPTER IV: *A PRIORI* DESIGN OF AN SRP-TE

Chapter IV addresses the issues related to the research question RQ_2 and specific objective SO_B regarding the design of a pilot SRP-TE.

RQ_2 related to the design of a pilot SRP-TE: What educational proposal based on SRP-TE can be designed and implemented with a group of preservice mathematics teachers in Brazil? How can this proposal contribute to providing future teachers with tools to design, analyse and implement new didactic processes for the teaching of statistics in lower secondary school?

SO_B: Study the SRP-TE ecology through the design, implementation, analysis, and development of different modalities of SRPs-TE for pre-service secondary school teacher education in the field of probability and statistics.

This chapter explores the pilot design and implementation of an SRP-TE as an innovative approach to preparing preservice mathematics teachers for the challenges of teaching statistics in secondary education. The chapter is structured into two main 1s.

Section 4.1 examines the educational gaps in statistics teacher education, highlighting the disconnect between university-level statistical education and practical classroom teaching. The research discusses how statistics is often marginalised in school curricula and how preservice teachers struggle with integrating statistical reasoning into their pedagogy. Drawing on the ATD, the study proposes SRP-TE as a methodology to engage teachers in an inquiry-based learning process, shifting from the traditional paradigm of visiting works to the paradigm of questioning the world.

Section 4.2 presents the design, experimentation, and analysis of an SRP-TE pilot study conducted with a group of preservice mathematics teachers. The study begins with an analysis of a textbook exercise on water distribution in Brazil, illustrating how conventional activities often fail to promote critical inquiry. Through collaborative questioning, data exploration, and interdisciplinary discussions, participants developed a more holistic approach to statistical problem-solving. The findings reveal both the strengths and limitations of the SRP-TE model, particularly in how teachers engage with real-world data, statistical literacy, and interdisciplinary connections.

This chapter ultimately argues that SRP-TE can serve as an effective framework for enhancing statistical inquiry in teacher education, yet it also highlights the challenges in fostering a deeper integration of data literacy and statistical reasoning within preservice teacher education.

The studies in sections 4.1 and 4.2 address RQ_2 by designing and implementing a pilot SRP-TE aimed at equipping preservice mathematics teachers in Brazil with the necessary tools to design, analyse, and implement new didactic processes for teaching statistics in lower secondary schools. The research follows the ATD and the paradigm of questioning the world, moving away from the traditional approach of passive learning. Through the pilot study, preservice teachers engaged in an inquiry-based learning process centred on real-world statistical challenges, specifically focusing on the distribution of water resources in Brazil. This process allowed them to experience firsthand the complexities of statistical investigation, such as data collection, validation, and modelling, thereby enhancing their ability to critically approach statistical education. The study revealed that while teachers were able to generate meaningful questions and interdisciplinary connections, there remained a tendency to overlook explicit statistical reasoning, highlighting the continued invisibility of data treatment in education.

In addressing SO_B, the research explored the ecology of the pilot SRP-TE, implementing and analysing its effectiveness with preservice teachers. The results indicated that, although teachers embraced the paradigm shift, they struggled with integrating structured statistical methodologies into their inquiries. The study highlighted the need for additional support in statistical reasoning and data validation, reinforcing the argument that SRP-TE is a valuable framework for enhancing statistics education.

4.1. LEARNING TO TEACH STATISTICS THROUGH STUDY AND RESEARCH PATHS

4.1.1. Introduction: Research context

In this study we present our PhD research proposal that is being developed at the *Universidade Federal of Mato Grosso do Sul* (Brazil) in partnership with the *Universitat de Barcelona* (Spain). The questions for the elaboration of the research project came from the initial education process followed by the first author during her degree in mathematics education. In the case of statistics, it seems that the double discontinuity described by Klein (Isaev & Eichler, 2017) appears with even greater prominence. The subject *Probability and Statistics* does not provide future mathematics teachers with a practical vision of the area. It is then common to hear reports from new mathematics teachers expressing difficulty in working with this theme in Brazilian compulsory education[4] when they take up a classroom, as happened with the first author of this study. An important gap is perceived between the education received and the teaching practice. In her master's dissertation, Verbisck (2019) focused on the case of probability and the teaching proposals of different textbook collections from grade 1 to grade 12. In all the cases, only a small part of the textbooks is devoted to probability and statistics, with a tendency to locate it in the last chapters of each book. More than ten years ago, Lopes and Ferreira (2004, p.12, our translation) stated that "until the implementation of [...] PCNs[5] (MEC, 1998), the teaching of statistics at elementary and high school grades was very restricted and marginal. The topics covered were included in the mathematics discipline, in the most advanced grades and, generally, it was one of the last topics in the textbook [...]". However, this is a provision that persists today, at least in more than half of the textbooks approved by the Brazilian Ministry of Education in 2017.

We believe that this and other results observed in this analysis reflect the practice of mathematics teachers, who need to look for other sources to overcome the difficulties found

[4] In Brazil, the compulsory education is composed by preschool (for children between three and five years old); primary school (for children between six and ten yards old); secondary school (for children between eleven and fourteen years old); and high school (for teenagers between fifteen and seventeen years old).

[5] *Parâmetros Curriculares Nacionais* (National Curriculum Parameters), official documents used in Brazil as a curricular tool for educators and educational institutions. These curriculum documents were published in 1997 and 1998 and have recently been replaced by the document: BNCC, *Base Nacional Comum Curricular* (Common National Curriculum Base). The BNCC was approved by the Brazilian Education Minister on December 20, 2017. With this document, "school systems and public and private educational institutions now have a mandatory national reference for the preparation or adaptation of their curricula and pedagogical proposals" (Brasil, 2018, p. 5, our translation).

when teaching probability and statistics. Therefore, for the PhD research proposal, we decided to focus only on statistics and restrict it to the final two grades of lower secondary school (students between 12 and 14 years old).

Thus, our PhD research objective is to investigate the possibilities and contributions of implementing a teacher education proposal for preservice mathematics teachers as a way to focus their reflections on these and other issues surrounding statistics teaching. It also provides teachers with experience in carrying out an inquiry, with the search and construction of means to overcome difficulties that future teachers may face when teaching statistics at these levels of schooling. Thus, the research question that we will seek to answer with the development of this dissertation is: *What educational proposal is it possible to implement with a group of preservice mathematics teachers in Brazil and in what aspects does this proposal contribute to provide future teachers with tools to design, analyse and implement new didactic processes for the teaching of statistics in the final grades of the lower secondary school?* This investigation is based on the Anthropological Theory of the Didactic (ATD), within the paradigm of questioning the world (Chevallard, 2015), which will be detailed in the next section.

4.1.2. Theoretical framework: study and research paths for teacher education

Chevallard (2015) states that, in our societies, teaching mathematics —and teaching in general— participates from what he calls the *paradigm of visiting works*. In this paradigm, the role of students is to study ready and finished knowledge organisations built in topics, areas, domains, and disciplines. It is up to the students to "look and admire" these works without necessarily questioning their validity or their value. So, in this paradigm, topics and subjects are like monuments: students cannot change them, they do not need to know their *raison d'être*, they just have to study them. Questions about the knowledge value and validity are, in any case, posed by the teacher, characterised as "the one who has the knowledge".

In contrast to this first paradigm, Chevallard describes the *paradigm of questioning the world* in which knowledge organisations change into *questions*. Teaching and learning processes become inquiry processes aiming at answering the questions. In other words, students are the inquirers of the generating questions Q proposed by the teachers (or by the students themselves). In the search for answers to a question, there may be times when *visiting works* is required, but with a specific *raison d'être* (answering Q). They also have to search for answers in media (such as the internet, books, experts, etc.), and they have to validate these answers and check their utility to answer Q. This paradigm creates strong changes in the teachers' and students' roles. The first are no longer the "holders of knowledge" and the students raise questions, investigate, search or elaborate answers and even validate them. To study the conditions needed for the transition to this second paradigm, Chevallard (2015) proposes a general inquiry format called *study and research paths* (SRP). The interplay between questions and answers plays a crucial role in the dynamics of SRPs. Students, helped by teachers, address an initial question Q, display Q into derived questions Q_i, search or elaborate answers A_i, find new questions during the process which, in turn, call for new answers, etc. Bosch and Winsløw (2015) point out the importance of such dialectic between questions and answers to ensure the dynamics of SRPs, which is usually represented using *questions-answers maps* (Figure 1).

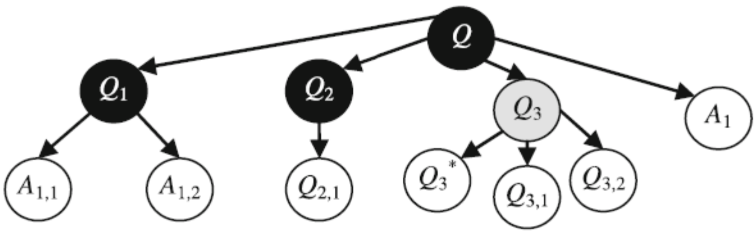


Figure 4.1.1: Example of Questions-Answers maps (Q-A) (Winsløw et al., 2013, p. 271)

Barquero, Bosch and Romo (2015) argue that teacher education proposals also need to be conceived within the new paradigm of q*uestioning the world*. Thus, they consider implementing *study and research paths for teacher education* (SRP-TE) "as a way to provide teachers with pertinent (theoretical and practical) tools to nourish and sustain their professional development" (Barquero et al., 2015, p. 810). An SRP-TE consists of five modules, which will be described below.

Module 0: This stage is to propose the open question (generating question $Q_{0\text{-TE}}$) related to the subject to be discussed. In our case, our $Q_{0\text{-TE}}$ is: "How to teach statistics in the final grades of lower secondary school?". Other derived questions will appear as search elements to answer the generating question, such as: Why is there so little attention on statistics at school? What are the conditions and constraints[6] for teaching statistics in the final grades of the lower secondary school? What statistics should we teach at these levels according to the curriculum? What kind of activities are proposed by official textbooks? What other proposals exist? Partial

[6] Conditions can be understood as limitations (or rules) that can be overcome and constraints are those that cannot be overcome. The school imposes conditions and constraints on those who occupy some function in this institution, such as the teacher and the student. For example, the time/class is a condition that the teacher cannot change, so it is characterized as a restriction (Chevallard, 2007).

answers to $Q_{0\text{-TE}}$ will appear during the entire SRP-TE (and hopefully also afterwards), making it a transversal module of the educational process.

Module 1: This module consists of proposing the group of teachers an SRP with a relevant question that could be approached in a real classroom as if they were the "students" of the final grades of the lower secondary school. "The SRP can actually have been implemented in previous investigations or may simply have been designed by researchers for this purpose" (Barquero et al., 2015, p.810). With this, the group of teachers will be introduced to an inquiry process.

Module 2: This stage is the moment for teachers to analyse the experienced SRP:

> Three main phases are distinguished: (a) the mathematical analysis of the work done, including the elaboration of a reference epistemological model describing the modelling process involved (Bosch & Gascón, 2006); (b) a didactic analysis of the process, including a description of the differences between the contract established during the SRP to manage the modelling process, compared to the usual school didactic contract centred on the transmission of contents; (c) a more general study of the viability of the SRP, including the identification of the institutional conditions and constraints affecting the development of modelling practices in school settings. (Barquero et al., 2015, p. 810)

Module 3: At this stage, teachers design and implement (if possible) an SRP for the schooling level under discussion. The new design is carried out based on the analyses made in the previous module, adapting, adding, or excluding (if necessary) didactic tools that seem necessary — or at least useful — for the implementation of this new SRP.

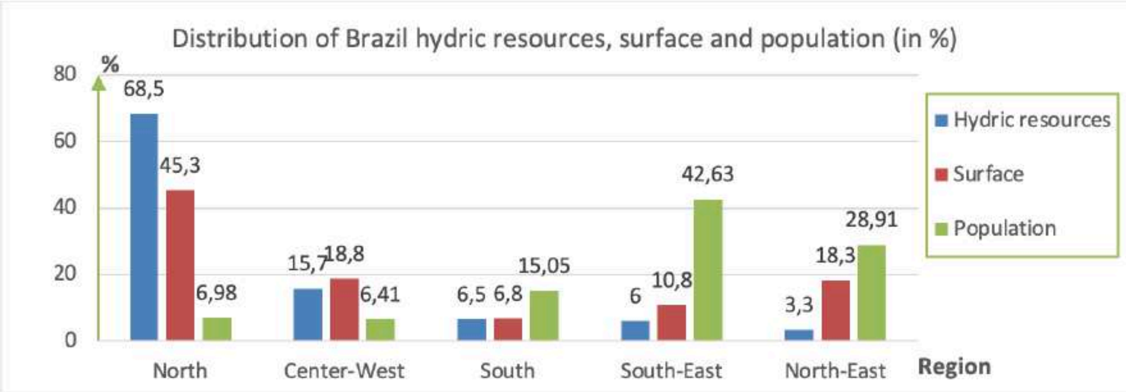
Module 4: This last module consists in the teachers sharing their experiences during all the implementations of the SRP to carry out an *a posteriori* analysis. Again, the mathematical and didactic tools used in modules 2 and 3 are present here and have great importance, "not only to provide some provisional answers to the question that was at the origin of the whole process ('How to teach …?'), but also as a means to analyse other possible alternative answers" (Barquero at al., 2015, pp. 810-811), as those answers found in Module 0.

These are the modules that structure the SRP-TE. We present below an exercise identified in a Brazilian textbook of the eighth grade of secondary school that gave us the initial idea of a generating question to be studied in the SRP when we are developing module 1 with the group of teachers.

4.1.3. From a school exercise to a study and research path

The proposed SRP for the teachers starts from a school activity about "the water distribution in Brazil". Looking at the approach to statistics in some textbooks from the eighth and ninth grades of secondary school, we find the exercise below:

Brazil has about 13.7% of the total fresh water in the world, being considered a territory rich in water terms. However, the country is experiencing serious problems, related to both the degradation of water quality,



especially in the urban areas' proximity, and the lack of control of excess and insufficiency of water, which affect several Brazilian locations. It is not just the floods that affect Brazilian cities: water scarcity also imposes serious restrictions and high costs on economic and social development of large cities in Brazil. Looking at the chart below, answer in your notebook:

Information obtained in: MINISTÉRIO DO MEIO AMBIENTE. Available in: <http://wwww.mma.gov.br/estruturas/ sedr_proecotur/_publicaco/140_publicacao09062009025910.pdf>. Accessed: 1st July 2018.

a) What kind of chart is this?
b) Indicate the Brazilian region:
• with the largest surface;
• with more water resources;
• with the second lowest population concentration.
c) Which region has the lowest percentage rate of water resources in our country?
d) In which region is there the greatest concentration of population?
e) Can it be said that the larger the surface of the region, the greater the number of inhabitants? Justify your answer.
f)How many percent of the world's fresh water is in the Southeast region of Brazil? Explain how you elaborate your answer.
g) Can it be said that the region with the most water resources is the one with the largest population? (Giovanni Júnior & Castrucci, 2018, p. 26, our translation).

This type of exercise, which is rather common in the textbooks, is clearly located in the paradigm of visit works. Although it presents a relevant topic to be discussed (and even includes a link to the source of the data), it does not clearly pose an open question, nor does it encourage students to search the data, organise and summarise them, and investigate other issues related to the topic. The questions raised in the textbook only require students to look at the graph, identify the largest or smallest bar, and perform some simple percentage calculations. If we look at the official curriculum guidelines, we can observe that this is not the kind of activity that encourages students to develop the skills assigned to this level of education, which are:

Assess the suitability of different types of charts to represent a survey dataset.

Classify the frequencies of a continuous variable of a survey into classes so that they summarise the data in a way that is suitable for decision making.

Obtain the values of measures of central tendency of a statistical survey (mean, mode and median) with an understanding of their meanings and relate them to the data dispersion, indicated by the range.

> Select reasons of different natures (physical, ethical, or economic) that justify conducting sample and non-census surveys and recognise that sample selection can be done in different ways (simple random, systematic, and stratified sampling).
>
> Plan and execute a sample survey, selecting an appropriate sampling technique, and write a report that contains the appropriate graphs to represent the data sets, highlighting aspects such as measures of central tendency, range, and conclusions. (Brasil, 2018, p. 315, our translation)

Despite the limitations of the exercise proposed, the topic presents many interesting questions that could generate a potential SRP within an SRP-TE. For this, we need to move from the paradigm of visiting works to the one of questioning the world and *question* the piece of information given. We will see, at the same time, how the inquiry process it generates can incorporate dimensions of the statistical work that used to be absent from secondary education, like the search, collection, cleansing and representation of data, together with a critical reading of quantitative information. Thus, for the SRP to be developed with the group of preservice teachers in Module 1, we propose to start with Q_0 and initiate a questioning that can lead to the following derived questions (starting from the same text and graphical information as before):

Q_0: *How can we explain the contradiction between the abundance of water resources and the water problems (scarcity, quality degradation, lack of control, etc.) of Brazilian locations? How are hydric resources distributed in Brazil, compared to the population and surface?*

Q_{1_Water}: *What do we know about water distribution in Brazil?*

- $Q_{1.1}$: *Are there studies about the water problem in Brazil? Where can we find them?*
- $Q_{1.2}$: *What disciplines are involved in the studies: geography, politics, geology?*
- …

Q_{2_Graph}: *What information can we draw from the graph?*

- $Q_{2.1}$: *What variables appear in the graph? What others could it be interesting to consider?*
- $Q_{2.2}$: *What are hydric resources? How are they measured?*
- $Q_{2.3}$: *Why are the variables in percentages? How are these percentages calculated?*
- $Q_{2.4}$: *How would be the graph if we use units instead of percentages?*
- $Q_{2.5}$: *Can we improve the graph using another type or adding/omitting information?*
- …

Q_{3_Data}: *What data is used to make the graph?*

- $Q_{3.1}$: *Is it available? Where? [The presented link does not work.]*
- $Q_{3.2}$: *Is the data available also reliable? How is it obtained?*
- $Q_{3.3}$: *What are the units of the different variables in the data source?*
- $Q_{3.4}$: *Can we use the available data to reproduce or update the graph?*
- $Q_{3.5}$: *Are there other interesting variables with available data to consider?*
- …

$Q_{4\_\mathrm{Working\ with\ data}}$: *How to download the data to start working with it?*

$Q_{4.1}$: *How to clean the table of data to make it ready-to-use?*

$Q_{4.2}$: What tools are available for data processing and which ones can we use?
$Q_{4.3}$: What types of numerical and graphical summaries are appropriate?
...

We started from the main subject of the text presented in the textbook exercise and elaborated a relevant open question (Q_0) that could be worked on in a classroom. As the exercise presented a graph with the distribution of water in Brazil, as well as the population and surface area by region, this graph will also serve as a possible answer for a derived question, since its source is the Ministry of the Environment. In this initial work to list our generating question and possible derived questions, we identified two large groups of questions: those related to the topic "water" (Q_1, $Q_{1.1}$, $Q_{1.2}$, ...) and those related to the topic "graph" (Q_2, $Q_{2.1}$, $Q_{2.2}$, ...). Still, within the topic "graph", another large group of questions appears, which is the topic related to "data" (Q_3, $Q_{3.1}$, $Q_{3.2}$, ...). By raising the questions related to "data", we entered the topic "working with data" (Q_4), which also made us think of derived questions ($Q_{4.1}$, $Q_{4.2}$, ...). With this, we obtain an *a priori* questions-answers map (Figure 4.1.2) that is part of SRP-TE.

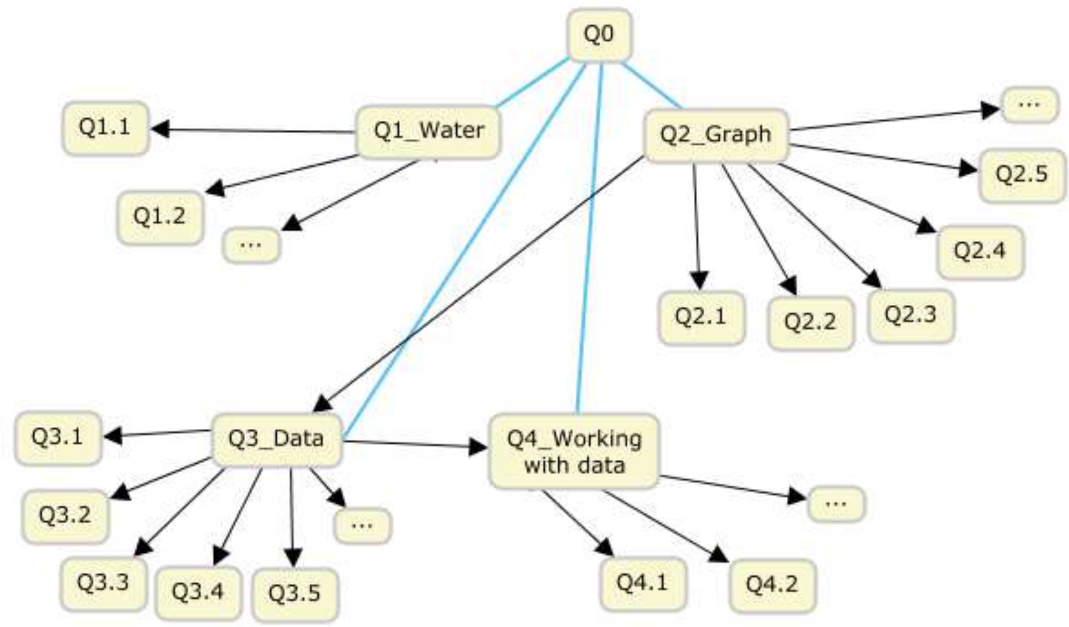


Figure 4.1.2. Questions-answers maps about water distribution in Brazil

We started from an exercise proposed in a Brazilian textbook and reworked it to display the initial and derived questions that may arise during the inquiry process. This questions-answers map serves as a possible SRP to be developed with the group of preservice mathematics teachers. The answers are still being developed since we have not yet worked directly with the group. It is an *a priori* study about the proposed theme: distribution of water in Brazil and, with this SRP in development, we believe it will be possible to work with dimensions of the statistical work that used to be absent from the final grades of the lower secondary education, like the search, collection, cleansing and representation of data. It is likely that the inclusion of provisional answers will enrich the map with more questions and potential maps to follow. It will also introduce more connections between the derived questions and the different branches they form.

The *a priori* questions-answers map shows the productivity of the change of attitude from *visiting works* to *questioning the world*. It first gives visibility to many interrogations that are usually hidden in the way school tends to present (quantitative and also qualitative) information, giving no room for doubts or questioning. It also shows how these interrogations go beyond the strict statistical reading and analysis of data to merge with other disciplines or areas concerned by the question addressed. This is not specific to statistics, even if it plays a crucial role there. Teaching modelling also suffers from the same phenomenon of disciplinary confinement. It makes many connections between different topics of statistics that tend to be presented separately even if they nourish each other (like the definition and measurement of variables, the reliability of data, the ambiguity of percentages, the pertinence of the type of graphs chosen, etc.). Finally, it provides future teachers with a broad — and more real — vision of statistics that can help them detach from the narrow vision proposed by textbook exercises and might facilitate the introduction of richer statistical activities in the classrooms.

4.1.4. Partial conclusions

Our hypothesis is that the design and implementation of an SRP-TE in statistics with a group of preservice mathematics teachers will provide them with tools to reflect, develop, analyse and implement new statistics teaching proposals when working in the final grades of the lower secondary school, as well as developing a critical posture when acting in their profession. We emphasise that the theoretical (ATD) and methodological (SRP-TE) frameworks adopted are essential for this purpose. The change of paradigms proposed by Chevallard (2015) makes us, researchers, also adopt a critical stance towards the social, educational, political and epistemological dimensions that involve the themes of teacher education and statistics teaching.

4.2. STATISTICS TEACHER EDUCATION AT SECONDARY SCHOOL LEVEL IN THE PARADIGM OF QUESTIONING THE WORLD

4.2.1. Introduction

We present a PhD project in progress at the Universidade Federal de Mato Grosso do Sul (Brazil) and the Universitat de Barcelona (Spain). The research topic arose from the first author's perception of a deficiency in preservice teacher education, especially in the domain of statistics, where a strong double discontinuity (Winsløw & Grønbæk, 2014) was observed between the statistics taught at university and the needs she experienced as a novice teacher. Our starting point is the teacher education problem:

$P_{0\text{-}TE}$: What praxeological needs do lower secondary school teachers have with regard to statistics?

That gives rise to our research question:

RQ: What educational proposal can be implemented with a group of preservice mathematics teachers in Brazil, and how can this proposal contribute to providing future teachers with tools to design, analyse and implement new didactic processes for the teaching of statistics in lower secondary school?

The strategy followed starts with the consideration of a mathematics textbook exercise about the distribution of water resources in Brazil that focuses on a highly topical and paradoxical issue: Brazil has a lot of water, yet a lot of water scarcity. However, the exercise only points at trivial aspects of the graph summarising the situation. We decided to use this case as the origin of a *study and research path for teacher education* (SRP-TE, Barquero et al., 2018) to address a twofold problem. On the one hand, the SRP-TE assumes the need for teachers to experience an SRP where data play an important role as a basis to introduce them into the didactic analysis. On the other hand, the exercise represents an illustrative case of the consequences of the pedagogical paradigm of visiting works, where critical questions are cast aside for the sake of the answers expected. We here present the design of the SRP-TE and a pilot study implemented with a group of secondary school preservice teachers in Brazil at the end of 2021.

4.2.2. Hypothesis and SRP-TE design

Our first hypothesis corresponds to the assumption that SRPs-TE have the capacity to lead teachers to question the knowledge to be taught and even, sometimes, the related scholarly knowledge (Barquero et al., 2022). A second hypothesis, sustained by our previous work (Verbisck, 2019; Verbisck et al., 2024), and also by other authors (Sorto, 2006; Batanero et al., 2011a; Gould et al., 2018; Zapata-Cardona & Escobar, 2019), affects the nature of school statistics and its dominant model in secondary education institutions. We assume that school statistics is mainly composed of numerical calculations of statistical measures (frequencies, means, medians, modes, deviations, ranges, quartiles, etc.) and standardised graphical representations of distributions (pie charts, bar charts, histograms). Moreover, these measures and graphs are generally introduced to summarise or represent already available clean data, without systematically linking them to the need to solve open problems.

From the ATD, we propose a reference epistemological model that, within the statistical activity, incorporates aspects such as:

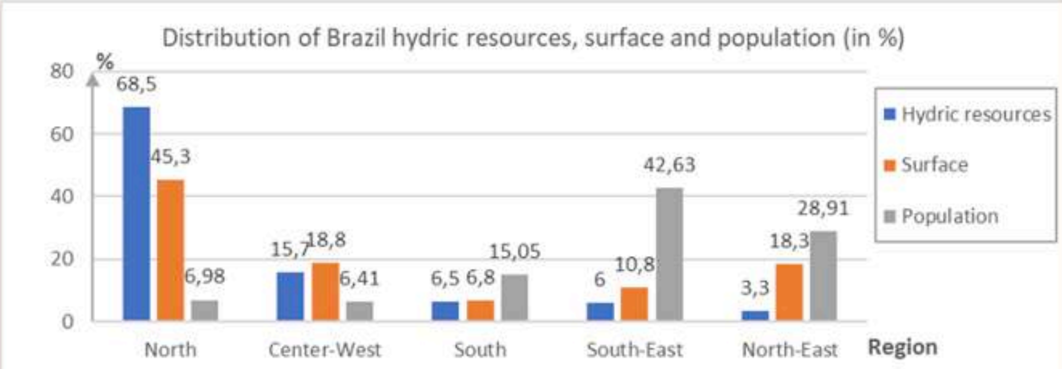
(1) Collecting, cleaning, organising, summarising, representing, and exploring data as a strategy for studying problematic questions;

(2) The study of data variability and the use of probabilities as theoretical distributions of observed frequencies, that is, as models of certain characteristics of the observed frequencies (Wozniak, 2005);

(3) The role of conjecture production and a validation tool including the idea of "confidence level".

When looking at Brazilian lower secondary school textbooks, it is clear that the established statistical tasks primarily consist of completing calculations (frequencies, means, medians, etc.) and reading graphs and tables. Figure 1 depicts one of these activities, which is very common in Brazilian textbooks, and this activity in particular, clearly fits within the paradigm of visiting works. It does not ask any open questions, nor does it require students to examine, organise and summarise any data, or investigate other issues linked to the topic, although it presents a relevant topic for discussion. Students just have to glance at the graph, pick the largest or smallest bar, and perform some percentage calculations to answer the questions. Our goal was to turn the educational activity into an investigative problem, moving it away from the paradigm of visiting works toward the paradigm of questioning the world.

Brazil has about 13.7% of the total fresh water supply in the world, being considered a territory rich in terms of water. However, the country is experiencing serious problems related to both the degradation of water quality, especially in the vicinity of urban areas, and the lack of control of excess and shortage of water, which affect several Brazilian locations. Not just floods affect Brazilian cities, water scarcity also imposes serious restrictions and high costs on economic and social development of large cities. Looking at the chart below, answer in your notebook:



Information obtained from: Ministério do Meio Ambiente <http://wwww.mma.gov.br/estruturas/sedr_proecotur/_publicaco/140_publicacao09062009025910.pdf>. Accessed: 1st July 2018.

a) What kind of chart is this?
b) Indicate the Brazilian region:
- with the largest surface;
- with the most water resources;
- with the second lowest population concentration.

c) Which region has the lowest percentage rate of water resources in our country?
d) Which region has the highest population concentration?
e) Can it be said that the larger the surface of the region, the greater the number of inhabitants? Justify your answer.
f) What percentage of the world's fresh water is found in the Southeast region of Brazil? Explain how you elaborate your answer.
g) Can it be said that the region with the most water resources is the one with the largest population?

Figure 4.2.1. Brazilian exercise (Giovanni Júnior & Castrucci, 2018, p. 26, our translation)

Therefore, in your *a priori* analyses of this school exercise, for the SRP to be developed with the group of preservice teachers, we propose to start with a generating question Q_0:

Q_0: How can the contradiction between the abundance of water resources and the water problems (scarcity, quality degradation, lack of control, etc.) of Brazilian regions, as illustrated in the graph, be explained?

In our *a priori* analysis of Q_0, we included four initial derived questions with their related sub-questions:

Q_{1_Water}: What do we know about water distribution in Brazil?

Q_{2_Graph}: What information can we draw from the graph?

$Q_{3\_\text{Data source}}$: What data are used to produce the graph?

$Q_{4\_\text{Data work}}$: How to obtain data to start working with them?

We can see how the enquiry process incorporates dimensions of statistical work that are typically absent from secondary education, such as data collection, cleaning, debugging, organising, summarising, and representing, as well as questioning the reliability of data taken from real media, i.e., a critical reading of the given quantitative information by placing ourselves in the paradigm of questioning the world. A new statistical perspective emerges, less influenced by the school institution and more in line with the supposed reference epistemological model.

However, this was an *a priori* analysis of a possible generating question concerning the chosen school exercise. We did not know its potential with preservice teachers or secondary school students in terms of other possible derived questions. Nor did we know what possible constraints could arise during the study (access to data, interesting conclusions, motivating themes for the group of trainee teachers, etc.). We decided to carry out a pilot study with preservice teachers from the Pedagogical Residency Programme of the Federal University of Sergipe (Brazil).

4.2.3. Experimentation: Results and discussion

4.2.3.1. Conditions of the Pedagogical Residency Programme

The Pedagogical Residency Programme (PRP)[7] is an initiative that integrates the National Policy for Teacher Education in Brazil and the *Coordenação de Aperfeiçoamento de Pessoal de Nível Superior* (CAPES) and is aimed at strengthening the initial training of secondary school teachers. The PRP regulations are set out in Ordinance No. 82/2022 (Coordenação de

[7] The PRP was not officially abolished but underwent significant restructuring in 2024. From that year, the PRP was merged with the *Programa Institucional de Bolsas de Iniciação à Docência* (PIBID), in English: Institutional Teaching Initiation Scholarship Programme, keeping the name PIBID. The aim of this merger was to unify the teacher education initiatives, extending the participation time of the scholarship holders and optimising the programme management processes. It is important to emphasise that this change did not represent the extinction of pedagogical residency activities, but an integration aimed at improving teacher education in Brazil. The activities previously carried out under the PRP continue to be carried out under the PIBID structure, guaranteeing the continuity of the practical immersion of preservice teachers in secondary education schools.

Aperfeiçoamento de Pessoal de Nível Superior, 2022), after its regulation by Ordinance No. 38/2018 (Coordenação de Aperfeiçoamento de Pessoal de Nível Superior, 2018). Implemented through institutional projects led by *Instituições de Ensino Superior* (IES)[8], the programme seeks to integrate theory and practice, immersing undergraduates in the daily life of public compulsory education schools, known as "schools of practice". In this way, students experience real teaching and learning experiences, improving their professional teaching identity.

The main objectives of the PRP include strengthening the theoretical and practical training of preservice teachers, valuing the experience of compulsory education teachers in the preparation of preservice teachers and inducing collaborative research based on classroom experiences. The preservice teachers are encouraged to, amongst other activities, classroom management and pedagogical interventions, accompanied by an experienced schoolteacher and supervised by a lecturer from the university implementing the programme. In addition, the programme establishes co-responsibility between IES, education networks and schools in the teacher education process. To structure this process, the PRP relies on:

- *Institutional Project*: Project presented by the IES to develop teaching residency activities.
- *Sub-projects*: Divisions of the institutional project organised by area of knowledge.
- *Nuclei*: Groups made up of supervisor (lecturer of the teacher education programme), "preceptors" (schoolteachers) and residents (preservice teachers).
- *"School of practice"*: Public secondary school where the pedagogical residency activities are carried out.

The participants in the PRP have specific roles: the institutional coordinator is responsible for running the project at the IES; the supervisor plans and monitors the residents' activities; the preceptors guide and supervise the preservice teachers during their practical work. To encourage participation, scholarships are offered to those involved, with the following amounts (in reais, Brazilian currency): R$400.00 for residents; R$765.00 for the preceptor; R$1,400.00 for the supervisor; and R$1,500.00 for the institutional coordinator. However, preservice teachers must meet requirements such as having completed at least 50 per cent of their teacher education. CAPES monitors and evaluates the projects through periodic reports, technical visits

[8] In English: Higher Education Programmes, and here are represented by the teacher education programmes.

and performance assessments, guaranteeing the continuity of activities and identifying possible irregularities.

The PRP provides a favorable environment for implementing the SRP-TE due to its strong emphasis on integrating theory and practice, fostering collaboration between teacher education programmes and secondary schools. The PRP immerses preservice teachers in real classroom settings, allowing them to engage directly with students, observe pedagogical challenges, and apply innovative teaching approaches. This structure aligns well with the SRP-TE device, which encourages teachers to explore real-world questions, engage in statistical inquiry, and challenge traditional didactic models. By participating in an SRP-TE within the PRP, preservice teachers can develop a deeper understanding of the role of data treatment in statistics education, actively applying and reflecting on their learning in a school context. The presence of experienced preceptors and university supervisors further supports the inquiry process by facilitating discussions, guiding pedagogical decisions, and helping residents navigate the transition from traditional methods to inquiry-based teaching.

Additionally, the PRP's collaborative structure creates conditions that promote the sustainability and scalability of SRP-TE initiatives. The co-responsibility between universities, education networks, and schools allows for the adaptation of SRP-TE principles to different educational realities, making it possible to address specific institutional constraints and challenges. In this way, the PRP serves as an ideal condition for developing, testing, and disseminating SRP-TE approaches, contributing to the transformation of teacher education in Brazil.

4.2.3.2. Organisation of the sessions

In this context, the SRP-TE activity was proposed to a group of 16 preservice teachers in four 90-minute sessions, which consisted of tutorials offered by the university teacher. They were organised virtually (through Google Meet) due to the COVID-19 pandemic. We used the last four sessions of the academic year, in November and December 2021. The first author and the regular lecturer led the sessions, while the other two authors acted as observers. At the time, the students were already familiar with some elements of the ATD, especially the SRP device, as the lecturer had already implemented it in different contexts with this group. The students were also used to autonomous teamwork outside the classroom, between the different sessions. The development of the four sessions is shown in Table 4.2.1.

Table 4.2.1. Organisation of the SRP-TE sessions

1st session – 11/11/21 (three moments)
The educators present the school exercise of Figure 1, containing only the introductory statement and the graph. In groups of 4, the student teachers are asked to discuss the following proposal: *Based on the situation presented in the school exercise, what questions can be raised about the theme considered, and about the data used?*
In teams, they write down the issues raised and organise them to be presented; they elect a communicator for the moment of sharing ideas.
Each team of students presents the questions raised and discusses them with the educators and others.
2nd session – 25/11/21 (two moments)
The researcher presents the complete school exercise (with the textbook questions and expected answers), comments on the strong limitations of the statistical work proposed (especially when compared to the questions raised by the students) and presents a report of the questions raised in the previous sessions, which consists of a list of questions grouped by themes and a question-answer map that summarises them (Figure A in Annex 1, subsection 3.2.7).
The researcher distributes a main derived question to each team and asks them to start studying them: - Group 1. $Q_{\text{Resources}}$: How much water is there in the five regions of Brazil? How is it measured? - Group 2. Q_{Climate}: Where does the water come from in the five Brazilian regions? Rainfall? Other water sources? - Group 3. $Q_{\text{Consumption}}$: What is water consumption like in the five regions of Brazil? What factors cause scarcity? - Group 4. Q_{Sharing}: What strategies exist to take water from one region to another? The students are asked to note down all the sources consulted, both those that were used and those that were discarded; they appoint a secretary who notes down everything that is being done and a communicator who prepares a summary including the main results to be presented.
There was no time to share results at the end of the session.
3rd session – 08/12/21 (two moments)
Each team presents the question addressed and the proposed answer elements, along with one or two questions arising from the study (to complete the map). The other groups are asked to pose questions to the presenting group: both clarifications and questions they consider the most important.
Teamwork to propose a project activity adapted to lower secondary school students, which may be a project or a longer activity. The design of the project/ activity must answer the following questions: - What is the initial question to approach? - How is the question posed, what is proposed to be done, what tools are available to the students? - How much time is the educational proposal going to take up in class: what is the approximate length of the activity? - How is the activity expected to be completed? - Which content themes, domains, or areas (not only in mathematics) are addressed in the activity proposed?
The students started the work during the session and continued it at home, ending up with a presentation to share their results in the last session.
4th session – 16/12/21 (one moment)
Presentations of the teaching proposals of each group. Final discussion and didactic analysis of the students' proposals and the initial activity proposed by the researcher.

4.2.3.3. Sessions 1 and 2: Questions raised by the student teams

The first session was aimed at raising questions related to the theme presented in the exercise in the book. To prepare the next session, we organised a Q-A map with the questions grouped into six topics: water resources (distribution of these resources), sharing water resources (from one region to another), quality of water resources, climate conditions (rainfall), consumption of water resources (scarcity, deforestation, etc.), and policies (citizen actions, responsibility of governments). However, questions related to the data presented in the exercise such as, "What variables do the exercise present and how were they measured? Where were these data taken from? Is the source reliable? How to validate?" did not appear.

At the beginning of the second session, we presented the school exercise (Giovanni Júnior & Castrucci, 2018), including the questions proposed in it. It was interesting to see that the preservice teachers were surprised by the poverty of the textbook questions as opposed to the richness of the questions they had prepared. Afterwards, we presented their questions grouped into topics that were the most repeated, and questions they had not considered, to incorporate them and enrich the work done so far. We included these and other data-related issues into the Q-A map. They are highlighted in red in Table 4.2.2 and Figure 4.2.2.

Table 4.2.2. Questions organised by topics

Q_0: How can the contradiction between the abundance of water resources and the water problems (distribution, scarcity, quality, lack of control, etc.) of Brazilian regions be explained?
$Q_{1\text{Sharing}}$: How are water resources distributed in Brazil? Where to find the data? How to summarise and analyse them? Are they reliable?
$Q_{1.1\text{Sharing}}$: Is there a poor distribution of water resources?
$Q_{1.1.1\text{Sharing}}$: Can water distribution provide water to the entire population?
$Q_{1.1.2\text{Sharing}}$: Do geographical factors influence water distribution in each region of the country?
$Q_{1.1.3\text{Sharing}}$: How does the state-owned water distribution company work and why is the quality of the service considered poor in the eyes of the population?
$Q_{1.1.4\text{Sharing}}$: What are the factors affecting water distribution in the northeast region?
$Q_{1.1.5\text{Sharing}}$: How to better distribute water resources?
$Q_{1.2\text{Sharing}}$: In what context does pollution affect the distribution of water?
$Q_{1.3\text{Sharing}}$: What are the consequences related to the expansion of water distribution?
$Q_{1.3.1\text{Sharing}}$: What measures should be taken to reduce the lack of water distribution?
$Q_{1.3.2\text{Sharing}}$: Is there any form of water sharing between regions (as most of the water is in the northern region)?
$Q_{1.3.3\text{Sharing}}$: Is it worthwhile taking freshwater from regions with plenty of water resources to the regions where water is scarce? Is this possible? Would it harm the environment?
$Q_{1\text{Quality}}$: What is the quality of water like in Brazil? How is it measured? Where to find the data? How to summarise and analyse them? Are they reliable?
$Q_{1.1\text{Quality}}$: What causes degraded water quality?
$Q_{1.1.1\text{Quality}}$: What are the consequences of degraded water quality?
$Q_{1.2\text{Quality}}$: Do regions with lower water resources have the worst water quality? If so, why?

$Q_{1.3\text{Quality}}$: Are urban regions the most affected by water quality degradation problems?

$Q_{1.3.1\text{Quality}}$: Why are urban regions the most affected by water quality degradation problems?

$Q_{1.4\text{Quality}}$: How much untreated sewage is returned to rivers?

$Q_{1\text{Sharing\&Quality\_Relation}}$: Is there a relationship between the distribution and the quality of the use of water resources in Brazil?

$Q_{1.1\text{Sharing\&Quality\_relation}}$: What is the relationship between the distribution and the quality of the use of the water resources in countries that have less water than ours?

$Q_{1\text{Resources}}$: How many water resources do the five Brazilian regions have?

$Q_{1.1\text{Resources}}$: How are water resources measured?

$Q_{1.2\text{Resources}}$: Where to find the data? Are they reliable?

$Q_{1.3\text{Resources}}$: How to summarise and analyse them?

$Q_{1\text{Climate}}$: Where does the water come from in the five Brazilian regions? Rainfall? Other water sources?

$Q_{1.1\text{Climate}}$: How are water resources that come from rainfall measured? Where to find the data? How to summarise and analyse them? Are they reliable?

$Q_{1.1.1\text{Climate}}$: How does pollution interfere in the replacement by rainfall of water resources?

$Q_{1.1.2\text{Climate}}$: Is there water storage for dry periods?

$Q_{1.1.2.1\text{Climate}}$: Is the storage sufficient? If not, what is needed to make it sufficient?

$Q_{1.1.3\text{Climate}}$: Can you make it rain more in places that lack water?

$Q_{1.2\text{Climate}}$: How much do climatic conditions positively or negatively affect a certain region?

$Q_{1\text{Consumption}}$: What factors cause water scarcity? How is it measured? Where to find the data? How to summarise and analyse them? Are they reliable?

$Q_{1.1\text{Consumption}}$: Is there more water shortage in rural areas than in urban areas?

$Q_{1.2\text{Consumption}}$: Does water scarcity influence the amount paid for treated water?

$Q_{1.3\text{Consumption}}$: Can water scarcity cause water shortage in the future (depletion of water resources in Brazil)?

$Q_{1.3.1\text{Consumption}}$: If no action is taken regarding water scarcity, when will Brazil run out of water?

$Q_{1.3.2\text{Consumption}}$: What measures can be taken to diminish the impacts caused by water shortage?

$Q_{1.4\text{Consumption}}$: Is there any relationship between deforestation and water scarcity in urban areas? How is deforestation measured?

$Q_{1.5\text{Consumption}}$: What are the constraints and costs to the country's economic and social development related to scarcity?

$Q_{1.6\text{Consumption}}$: Do regions with a greater presence of agriculture and livestock have a greater impact on the scarcity of water resources?

$Q_{1.6.1\text{Consumption}}$: What is the share of agriculture and livestock in this scarcity? How much water does a cattle farm consume?

$Q_{1\text{Sharing\&Consumption\_Relation}}$: What are the relationships between distribution and scarcity of water resources in Brazil?

$Q_{1\text{Politicy}}$: What can be done to make better use of available resources?

$Q_{1.1\text{Politicy}}$: Could limiting monthly water consumption in each region solve some problems?

$Q_{1.2\text{Politicy}}$: Is it worth taking salt from sea water to turn it into fresh water?

$Q_{1.3\text{Politicy}}$: Is the government the only and main body that should manage and preserve the country's water resources?

$Q_{1.3.1\text{Politicy}}$: What can we as citizens do to stop the degradation of these resources?

Figure 4.2.2. Q-A water map of sessions 1 and 2

The questions formulated by teacher-students demonstrated an ability to expand beyond a strictly mathematical or statistical lens to encompass environmental, social, economic, and political dimensions. Their inquiries included topics such as water distribution inequalities, environmental degradation, and government policies—indicating an awareness of the complexity of the issue. This interdisciplinary perspective is an essential step toward developing a holistic understanding of real-world problems. However, while students effectively explored various domains, they did not fully integrate statistical reasoning into their investigative process.

Despite engaging with various aspects of water distribution, teacher-students did not immediately consider statistical concerns such as data validity, measurement errors, or biases in sources. This omission highlights a critical challenge in teacher education: statistics, while a fundamental tool for analysing real-world problems, is not always perceived as central to interdisciplinary inquiries. The absence of questions addressing how data were collected, processed, and interpreted suggests that statistical literacy needs to be explicitly developed alongside thematic investigations.

A pivotal moment in the SRP-TE occurred when teacher-students compared their self-generated questions with those presented in the textbook. They expressed surprise at the superficial nature of the textbook's inquiries, which primarily focused on simple numerical calculations rather than meaningful investigation. This contrast underscores the limitations of conventional school exercises and the necessity for a more robust, data-driven approach.

To strengthen statistical reasoning, future iterations of the SRP-TE should integrate structured guidance on data collection, cleaning, organisation, and representation. Teacher-students reported difficulty in finding reliable and recent data, revealing a fundamental challenge in statistical inquiry. This challenge can be reframed as a teachable moment, highlighting the complexities of working with real-world data. By embedding explicit statistical frameworks within the SRP-TE, facilitators can ensure that students develop the skills necessary to critically engage with quantitative information.

The initial Q-A map predominantly centered on content-related questions rather than statistical inquiries. In future implementations, facilitators should introduce prompts to deepen statistical engagement, such as: *How can we validate the reliability of the water distribution data? What statistical methods can we use to analyse regional differences in water scarcity? How can we visually represent disparities in water quality across regions? How do we determine whether government interventions in water distribution are effective?* Encouraging students to systematically incorporate these statistical questions will foster a more comprehensive approach to problem-solving and improve their ability to critically assess data.

The analysis of teacher-student-generated questions in the SRP-TE reveals both strengths and areas for improvement. While they demonstrated strong interdisciplinary thinking, their engagement with statistical concepts was limited. Addressing this gap requires explicit instructional strategies that integrate statistical reasoning into investigative inquiries.

Due to the time constraint, it was not possible to work on finding answers to all the questions raised, although they were very interesting and relevant. Instead, we chose one question for each group to research (Table 4.2.1 – second moment of session 2), collect some elements of answers and raised other questions.

4.2.3.4. Sessions 3 and 4: Elements of answers and didactic design

In the third session, one representative from each group presented their elements of answers, and possible derived questions, to the question assigned to them in the previous session. We organised these questions and answers in the Table 4.2.3 and into a new Q-A map, presented in Figure 4.2.3.

Table 4.2.3: Elements of answers

$Q_{Resources}$: How much water is there in the five regions of Brazil? How is it measured?
$Q_{1Resources\_of\_the\_Earth}$: How is the planet's water distributed?
$A_{1Resources\_of\_the\_Earth}$: There are 1,358,099.876 km^3 of water on the planet, 97.24 percent being salt water and 2.76 percent fresh water. As a result, we have 1,320,409,125 km^3 of ocean and sea water, and 37,794,876 km^3 of fresh water.
$A_{1Resources}$: Given that the entire volume of fresh water on the globe is 37,794,876 km^3 and that the volume of fresh water in Brazil corresponds to 13.7 percent of the total fresh water on the planet, we estimate that the amount of fresh water in Brazil is roughly 5,177,898.012 km^3.
We calculated the percentage of fresh water distributed in the Brazilian areas using this conclusion and the percentages shown in the graph from the same exercise, and the results are as follows:
- North: 3,546,860.138 km^3
- Mid-west: 812,929.987 km^3
- South: 336,563.370 km^3
- Northeast: 170,870.6344 km^3
- Southeast: 310,673.8807 km^3
$A_{2Resources}$: Rivers in Brazil produce an average of 168.790 m^3/s.
The North region: The Amazon River mouth, the coast of *Pará*, and the west of the region have a total annual precipitation of over 3,000 mm; in the northwest-southeast direction, these values range from 1,500 to 1,700 mm.
The Northeast receives between 300 and 2,000 mm of rain every year. The peak season is autumn-winter, while the lowest season is spring-summer, along the eastern coast and on the plateau slopes from *Rio Grande do Norte* to *Bahia*.
The Southeast region: Rainfall on the coast ranges from 1,700 to 2,400 mm, with peaks of more than 3,000 mm in the Southeast. Rainfall in the rest of the Southeast region attains 1,500 mm.
Rainfall on the coast ranges from 1,700 to 2,400 mm, with peaks of more than 3,000 mm in the Southeast. Rainfall in the rest of the Southeast area reaches 1,500 mm, with roughly 900 mm in the *Jequitinhonha* and *Doce* valleys.
The South region: The average annual rainfall in the southern region is between 1,250 and 2,000 mm, with the exception of the *Paraná* coast and the *Santa Catarina* wetlands, where it reaches 2,000 mm.
The Centre- West: The territory of the Centre-West is divided into two parts: Rainfall in this area is virtually entirely determined by the atmospheric circulation system. The average annual rainfall in northern *Mato Grosso* is from 2,000 to 3,000 mm, dropping east and south to 1,500 mm east of *Goiás* and 1,250 mm in *Pantanal do Mato Grosso*.

$Q_{Climate}$: Where does the water come from in the five Brazilian regions? Rainfall? Other water sources?

$A_{1Climate}$: The total amount of water on the planet is about 1.35 million cubic kilometres. Unfortunately, just 2.5 percent of this total is fresh water, and most of it is frozen. Human consumption only accounts for 0.3 percent of the total.

Rainwater is also used. Environmental imbalance and climate change have altered these cycles, causing damage to natural rainfall storage times. We are depleting this valuable service that nature provides us with every day by combining the absence of rain with the process of contamination of water resources and environmental degradation.

$A_{2Climate}$:

Northeast: The *Atlântico Nordeste Ocidental, Atlântico Nordeste Oriental, Atlântico Leste, Paranaíba, São Francisco*, and *Tocantins-Araguaia* hydrographic zones bathes in subterranean and river waters.

Tocantis-Araguaia, Atlântico Norte, and *Amazônic*a are the three hydrographic zones that surround it in the **North**. The latter is made up, amongst other things, of the *Rio Amazonas Hidrográfica* (Amazon River Basin), which has the world's most extensive hydrographic network, covering 6,110,000 km^2. The Amazon basin alone consumes one-fifth of the world's fresh water, accounting for nearly 60 percent of the country's total water availability, according to the Ministry of the Environment.

Five hydrographic zones supply the **Centre-West**. *Tocantis-Araguaia, Paraná, Amazônica, Paraguai*, and *São Francisco* are the names of the cities.

South: The hydrographic zones of the *Atlântico Sul*, *Uruguai*, *Paraná*, and a tiny part of the *Atlântico Sudeste* provide it with water.

Southeast: It is surrounded by the *Paraná* and *Atlântico Sudeste* hydrographic zones. *São Francisco*, *Paraná*, *Atlântico Leste*, and *Atlântico Sudeste* are the four hydrographic zones that cover the state of *Minas Gerais*.

$Q_{Sharing}$: What strategies exist to take water from one region to another?

$A_{1Sharing}$: We discovered feasible techniques to store water in particular places using hydrographic basins and dams, as well as desalination of seawater, throughout our investigation. The only way to get water from one location to another was to transpose rivers, such as the San Francisco River.

The *Projeto de Integração do Rio São Francisco com Bacias Hidrográficas do Nordeste Setentrional (PISF)*, as the Brazilian government calls it, is a project that involves the displacement of a portion of the São Francisco River's waters.

- The *São Francisco* River flows through states such as *Alagoas*, *Bahia*, *Distrito Federal*, *Goiás*, *Minas Gerais*, *Pernambuco*, and *Sergipe*, and reaches over 520 cities in Brazil.

- The project will assist 390 cities in *Pernambuco*, *Paraná*, *Ceará*, and *Rio Grande do Norte*.

$Q_{Consumption}$: What is water consumption like in the five regions of Brazil? What are the main factors?

$A_{1Consumption}$:

- The average daily water consumption in the country is 152.1 litres per inhabitant; - The State of *Rio de Janeiro* consumes the most water, around 207.0 litres per inhabitant;

- According to the UN (United Nations), 110 litres per day is enough water to meet a person's basic needs;

- 7.5 percent of children and adolescents have water at home, but it is not filtered or from a safe source;

- In 2017, the average daily water use was 420.1 litres, however there was significant regional variation: in general, water consumption was greater in the North and lower in the Northeast.

$A_{2Consumption}$:

- Only 63 litres of treated water are consumed in Brazil for every 100 litres produced, with the remaining 37 litres is lost. Leakage, uneven connections, absence of or improper metering, and theft are all causes of losses.

- The UN recommends an average daily consumption of 110 litres per person per day. According to studies, this amount is sufficient to cover a person's fundamental needs. But that is not the case; according to the *Trata*

Brasil Institution, the average daily usage in Brazil is 166.3 litres per inhabitant. This is 51percent more than what is suggested.

$A_{3\text{Consumption}}$: The **Northeast** is the region that consumes the least water in the country, with an average of 112.5 l/inhab, almost 60 percent less than the 179.7 l/inhab. consumed by the **Southeast, leader of the list**.

$A_{4\text{Consumption}}$: Each family member in the **Southeast** uses an average of 143 litres of water each day. This equates to a spending level that is 70 percent higher than what has been validated in the north. Each person in this region consumes an average of 84 litres of water each day. It is worth noting, however, that individual usage accounts for a minor percentage of total water consumption in the country. **Agriculture, cattle, forest production, fishing, and aquaculture consumed 97.4 percent of the 329.8 trillion litres spent in Brazil in 2017.**

$A_{5\text{Consumption}}$:

Main factors

Despite the fact that water has the ability to regenerate itself, its usage exceeds this capacity.

Water usage is increasing in Brazil as a result of population, industrial, and agricultural growth. According to the *Agência Nacional de Águas* - ANA - (National Water Agency), **72 litres of water are utilised for agricultural irrigation for every hundred litres consumed**.

Water waste: As we have seen, agricultural irrigation accounts for a significant portion of Brazil's water use. However, it is also one of the leading sources of water waste. Waste can also be seen in people's daily lives, such as leaving taps open for too long, bathing for too long, and leaking.

Reduced rainfall: Deforestation in the Amazon forest contributes directly to the country's lack of rainfall. But what about the "flying rivers" phenomena, which supplies moisture to various parts of South America? The following is how it works: water vapor from the Atlantic Ocean's tropical waters meets and is fed by moisture from the Amazon jungle. All this moisture travels through the Amazon until it reaches the Andes Mountains' wall. Part of the fluid evaporates and falls as rain, feeding the springs of big rivers like the Amazon. The rains are caused by the other component, which is aimed at Brazil's Centre-West, Southeast, and South areas.

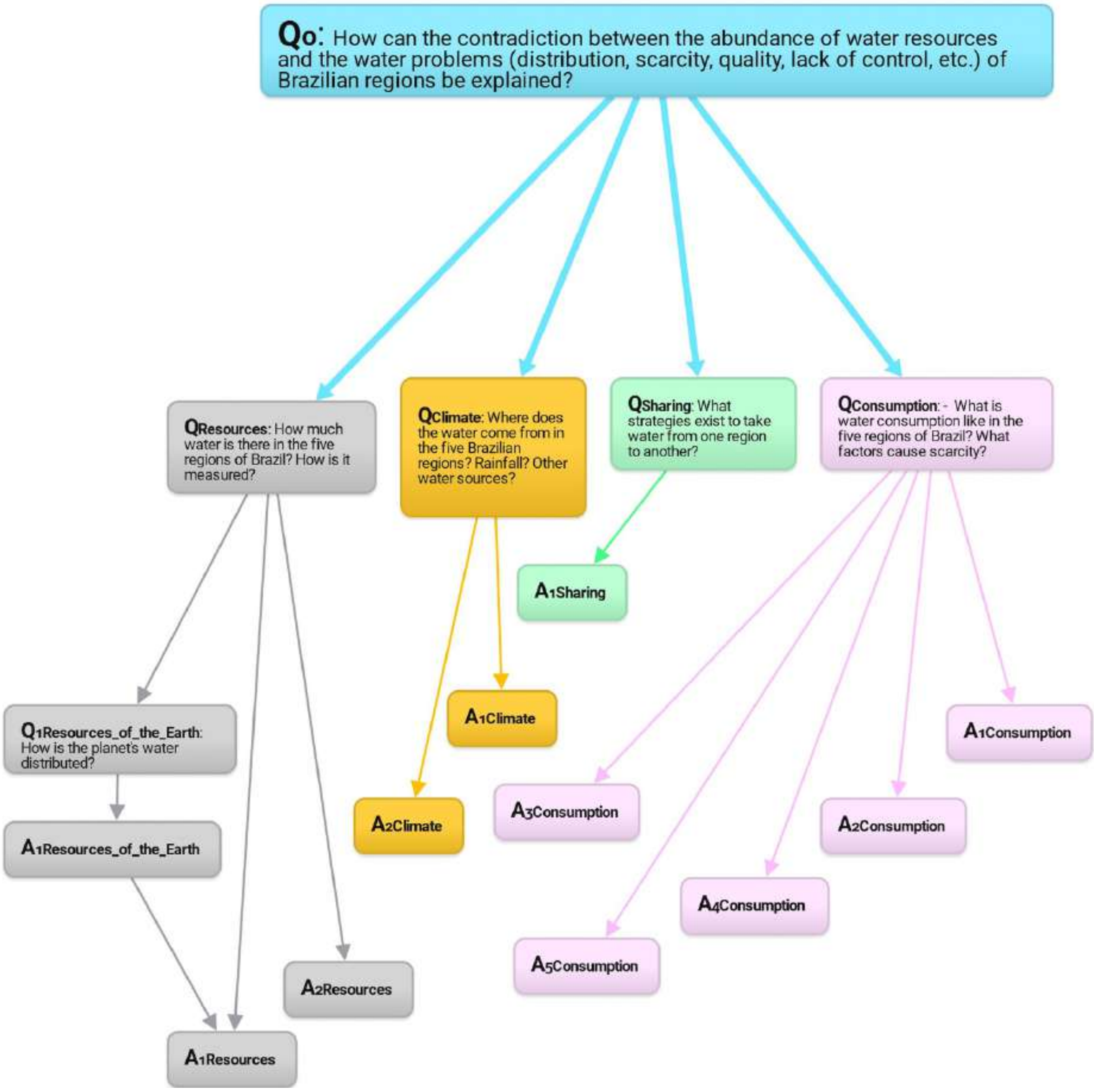


Figure 4.2.3. Q-A map with elements of answers

The elements of answers and Q-A Map in Table 4.2.3 and Figure 4.2.3 provide a structured synthesis of the teacher-students' investigative work. This section of the SRP-TE serves as a pivotal moment in the inquiry process, as it reflects not only the teacher-students' ability to seek answers to their own questions but also the limitations and challenges they encountered when engaging with real-world data.

The responses formulated by them demonstrate a broad engagement with the inquiry proposed, covering aspects such as water resources, climatic influences, consumption patterns, and policies. These responses reflect a progression from initial questioning to the construction of knowledge, a key goal of the SRP-TE approach. However, while some responses include quantitative data, others remain descriptive and qualitative, lacking statistical validation or critical evaluation of data sources.

A critical observation from the Q-A Map is the teacher-students' struggle with data acquisition and analysis. While they successfully gathered information on water availability and policies, there was limited engagement with statistical methodologies to validate their findings. Questions related to data measurement and reliability were not extensively addressed, and while data sources were cited, there was little discussion on potential biases or inconsistencies. This suggests that while they were able to collect and summarise data, they did not fully critically analyse its validity or reliability—highlighting the need for stronger statistical literacy interventions in future SRP-TE implementations.

The preservice teachers reported difficulties when searching for information on the theme related to the question they were trying to answer, as well as difficulties in finding information on water resources in the five Brazilian regions and a lack of recent studies on the investigated themes. A lot of the information is repetitive and very old (from decades past) and looks as if it needs to be updated. These challenges reinforce the necessity of teaching preservice teachers students how to critically assess data sources and introduces strategies for data collection, cleaning, and interpretation.

The Q-A Map effectively organises the inquiries and responses into thematic categories, fostering systematic thinking and helping students visualise relationships between different aspects of the problem. However, some limitations remain: the Q-A Map focuses more on content-driven responses than on methodological reflections. Future iterations should integrate more explicit discussions on how data were obtained, processed, and validated and introduce deeper analytical techniques such as correlation, trends, or statistical modelling.

After the presentations, we proposed one last activity for the groups of student teachers. As we mentioned earlier, our goal for this SRP-TE activity was to develop an attitude (change of paradigm) converting a textbook exercise into an investigative activity, an enquiry problem, with a group of preservice teachers. Bearing in mind the themes discussed in the previous sessions, the last task for each group was to design an instructional proposal (a project or a longer activity) adapted to students at the lower secondary school level. For the design, the students were asked to use the following question as their guide:

$Q_{0\text{-FP}}$: What is the initial question to approach? How is the question posed, what is proposed to be done, what tools are available to the students? How much time is the educational proposal going to take up in class: what is the approximate length of the activity? How is the activity expected to be completed? Which content themes, domains, or areas (not only in mathematics) are addressed in the activity proposed?

The different groups started to outline their proposals in this session but, due to the short time remaining, they had to continue working on them after the session. Sharing results was left for the last session.

In Appendix C we provide the entire SRP designed by the four groups of preservice teachers, maintaining these productions in the original language: Portuguese.The four groups presented different teaching proposals including the following generative questions:

> $Q_{0\_Group\_1}$: Why is water consumption limited in certain regions of Brazil?
>
> $Q_{0\_Group\_2}$: Is it possible to collect rainwater to use at home?
>
> $Q_{0\_Group\_3}$: Given the problems related to water quality degradation, what is the percentage rate of degraded water resources?
>
> $Q_{0\_Group\_4}$: If water resources are abundant in Brazil, why is there water scarcity and/ or rationing in all regions of the country?

Due to a lack of time, we focused on presenting one of the teaching proposals that interested us the most, as shown below:

Table 4.3.4: SRP proposal by Group 2

Q_0: Is it possible to collect rainwater to use at home?
Context:
Using the context of the water distribution SRP, we start from how water is distributed in our city, making students realise that in our region there is relatively, little water. The idea of trying to save water is hence put forward, since water distribution is a process that is hardly used in Brazil. To do this, we will talk about the climate and how rainfall can help alleviate this problem. We then introduce our Q_0 to them. Throughout the SRP, the students can use whichever tools available, such as books, mobile phones, etc.
NOTE: Considering most of the students are from rural areas, they usually use rainwater, as piped water supplies are not always available in the homes in those areas.
Questions that may arise when answering Q_0
- How to store/ treat rainwater and distribute it for use in most household activities? - Can rainwater be used for drinking purposes? Is it possible to convert it into drinking water? - How to collect as much rainwater as possible? Should a rain barrel be used? - How much rainwater can be harvested? - Would this save money on water bills? - How can I clean rainwater to make the most of it? - How many gallons of chlorine would have to be used if 300 gallons of water were collected? - Is the amount of rainfall in my region enough for a person to live on rainwater alone? - How much water does an average person use in their daily life? - Etc.

Subjects and contents

Considering the previous questions, we expect to enquire about the topics pertaining to the area of chemistry, biology, mathematics and geography. We initially thought about the following contents:

- Mathematics: ratio, magnitudes and measures, functions, areas of figures, volume
- Geography: climate of the region
- Chemistry: stoichiometry
- Biology: micro-organisms and bacteria, water cycle

[…].

Estimated time

We believe it will take about a month of class time to complete this enquiry, since we anticipate two weeks of discussions, one to learn about the issues that arise in the process and another to prepare and present the final answer.

Conclusion

We hope that the study will conclude with an answer to the initial question, in which the students use the knowledge acquired throughout the lessons to say whether it is possible to carry out what is asked or not. If the answer is yes, it would be necessary to show a project that in practice fulfils what is asked in Q_0 and estimates how much water can be collected. If it is not possible, the students should argue why and under what conditions it could be feasible.

It is interesting to note in this proposal from Group 2 that they took the question about the problem of water scarcity that exists in the region where they live (northeast of Brazil) seriously. They posed a provocative question—collecting rainwater for household use—demanding both theoretical and practical responses. They listed derived questions that followed the logic of the generating question rather than the knowledge they used to answer it. As they themselves pointed out, they included several fields of study such as biology, geography, and chemistry. This generating question appears to be strengthening interdisciplinarity. However, they never considered the search for data to be statistical work. Although mathematics is present, statistics does not appear to be part of this investigation, even though data searching is undoubtedly a process that would appear in different derived questions. It is surprising given that the initial problem was taken from a statistics textbook, and they knew that. They also searched data during the second module of the SRP-TE, when they addressed the initial generating question on their own.

4.2.4. Conclusions

We are aware of the fact that the pilot study was carried out under restricted conditions, including few sessions and using the very last classes of the academic year, as well as online interactions due to the COVID-19 pandemic. However, they provided us with interesting learnings about the teacher education proposal and its possible future use. The change of paradigm proposed by Chevallard (2015) seems particularly appropriate in the case of statistical enquiries because of the specific descriptive tools it provides.

The virtual sessions brought to light the difficulty to check on the work performed within the teams, and the inability to monitor the work done outside the classroom. However, we were able to identify some favourable conditions for the development of the activity. First of all, the group of trainee teachers already knew about the ATD and had participated in other SRPs. The students were used to generating questions without being restricted to mathematical content. In this respect, the students did better than in our *a priori* analysis. They broadened the range of questioning from geographical (resources and climate) to social, political and environmental aspects (sharing and quality), allowing, as Chevallard said: "A *co-disciplinary* symphony in which mathematics contributes with other disciplines to elucidating the conditions and constraints of all kinds that determine the production of answers A to questions Q" (Chevallard, 2004b, p.12).

However, to our surprise, amongst the contents activated by the SRP, aspects related to statistics and data processing appeared only tangentially. The students did not focus on the search and use of data, as we did in the *a priori* analysis. Having little time to undertake the activity may explain the students' choices to focus on topics for which they found easily accessible studies. More generally, we can also point at the fact that the statistical work that underpins many studies is not very visible, and difficult to access. This raises the problem of defining "what it is to study a question Q_0" more clearly, which experimental field is the most appropriate, and why we should not be satisfied with "copying" directly accessible answers. Setting up an appropriate media-milieu dialectic seems to be an important challenge to be addressed.

Finally, the invisibility of data processing reappears when the students are asked to associate curricular knowledge with the development of an SRP. This is a return to the invisibility of many statistical activities they incorporated in the proposal but did not identify as part of the knowledge to be taught (while they did identify many others). We relate it to the *invisibility of knowledge objects* discussed by Margolinas (2014). This points to yet another challenge for the implementation of future SRPs-TE for teachers who teach statistics.

CHAPTER V: AN ONLINE SRP-TE FOR IN-SERVICE TEACHERS

Chapter V addresses the issues related to the research question RQ_3 and specific objective SO_C about the ecology of an SRP-TE implemented for in-service teachers.

RQ_3 related to the implementation of an SRP-TE for in-service teachers: What conditions implemented in an online SRP-TE can help teachers address and detach themselves from the phenomenon of the invisibility of data treatment in a context of online in-service teacher education? What constraints limit it?

SO_C: Study the SRP-TE ecology through the design, implementation, analysis, and development of different modalities of SRPs-TE for in-service secondary school teacher education in the field of probability and statistics.

This chapter examines the implementation and analysis of an SRP-TE in statistics, focusing on in-service teacher education in Brazil. The chapter is divided into two main sections, each exploring different perspectives of the same SRP-TE experience.

Section 5.1 investigates the invisibility of data treatment in secondary school statistics education. It highlights how statistical work is often reduced to numerical calculations and standardised graphical representations, neglecting essential aspects of data handling such as collection, organisation, cleaning, and interpretation. Through the SRP-TE, in-service teachers engaged in an inquiry-based activity that required critical engagement with real-world data, specifically analysing claims about water resource scarcity in Brazil. The study reveals the challenges teachers face when integrating data treatment into their teaching practices and examines the institutional constraints that contribute to the lack of emphasis on data literacy in secondary education.

The study of section 5.2 also addresses some specific questions:

- In what sense can a teacher education proposal, based on an SRP-TE, help teachers promote inclusive statistics? What kind of activities and through what interaction with digital tools facilitate teachers' reflections on inclusive statistics?
- What conditions implemented in an SRP-TE can help teachers address and detach themselves from the phenomenon of the "invisibility of data treatment" in a context of online in-service teacher education? What constraints limit it?

Section 5.2 explores the role of statistical inquiry in addressing real-world problems, with a particular focus on inclusive statistics education. The SRP-TE was designed to encourage teachers to use digital tools and real-world contexts to make statistics more engaging and meaningful for diverse student populations. A case study of an in-service teacher implementing an inquiry-based project on rainfall levels and drought conditions in Brazil illustrates how statistical reasoning can be used to address environmental and social issues.

We emphasise that inclusive statistics is not one of the central research focus of this dissertation, but in this section, it appears as a topic for discussion because it was an article published in a special issue —inclusive statistics education with digital resources— of the *Statistics Education Research Journal*. Together, these two perspectives provide a comprehensive analysis of how SRP-TE can transform the teaching and learning of statistics, bridging the gap between theoretical knowledge and classroom practice. The chapter underscores the need for teacher education programmes to prioritise data literacy, inquiry-based learning, and interdisciplinary approaches to better equip future educators for the demands of modern statistics education.

The study presented in both sections addressed RQ_3 and SO_C by designing and implementing an SRP-TE that actively engaged in-service teachers in an inquiry-based approach to statistics education. The SRP-TE framework was designed to counteract the invisibility of data treatment in secondary education by encouraging teachers to engage in the full cycle of statistical inquiry, including data collection, processing, and interpretation. The study highlighted teachers' initial struggles in recognising data treatment as a core component of statistical knowledge, as well as the constraints imposed by traditional pedagogical norms and the online learning environment.

Despite these limitations, the SRP-TE demonstrated how inquiry-based teaching could help teachers transition toward a paradigm of questioning the world, fostering deeper engagement with data-driven decision-making. It also revealed the challenges of breaking away from the dominant mathematical paradigm that reduces statistics to computational techniques, as evidenced by a teacher's comment undervaluing data analysis without measures of central tendency. The findings reinforce the need to provide teachers with epistemological and didactic tools to redefine the role of data treatment in statistics education. While the SRP-TE showed promise in fostering more comprehensive statistical reasoning, it also underscored the persistence of institutional constraints that limit the recognition and integration of data treatment in school mathematics.

5.1. A STUDY AND RESEARCH PATH FOR TEACHER EDUCATION IN STATISTICS: DEALING WITH THE INVISIBILITY OF DATA TREATMENT

5.1.1. Introduction

Over the past decades, statistics has strongly evolved in line with the development of technological resources for data processing, resulting in what is known today as “data science” (Holmes, 2017). A review of statistics education research shows the importance of taking a broader view of this field, including aspects such as searching and collecting data in real-world contexts, selecting, organising, tabulating, and visualising them, using specific software, simulation, and reporting. All of them directed toward the study of open questions involving variability (Batanero et al., 2011a; Burrill & Ben-Zvi, 2019; Garfield & Ben-Zvi, 2008). Bringing this broader perspective into the classroom is no easy task, and it requires at least the complicity of the teaching and teacher education institutions. However, what Felix Klein called the “double discontinuity” between school and university mathematics (Eichler & Isaev, 2023) also seems to apply in the case of statistics.

When considering the kind of statistics introduced to preservice secondary school teachers, the perspective concerning “dealing with data” as part of the statistical knowledge for teaching varies significantly. For instance, in Brazil, most courses for preservice secondary school teacher education offer a single subject, *Probability and Statistics*. This subject does not differ from the one proposed for other university degrees like mathematics, engineering, or physics. The subject contents are organised following the logic of the axiomatic construction of concepts about, e.g. descriptive statistics, probability, random variables, models of distributions, inference with one and more samples, simple linear regression and correlation. Therefore, a significant disassociation should be highlighted between the statistics education received at the university and the statistics to be taught at school.

Related to the dominant conception of statistics at the secondary school level, data processing, although it is becoming more and more important in society, still does not have a clear status in secondary school. There, statistical activities are often reduced to numerical calculations of statistical measures (frequencies, means, medians, deviations, quartiles, etc.) and to the elaboration or interpretation of standardised graphical representations (pie or bar charts, histograms and scatter plots) (Batanero et al., 2011a; Burrill & Ben-Zvi, 2019). This was already evidenced many years ago by Short and Pigeon (1998) when affirming that, although statistics educators agree that data gathering and analysis steps are valuable, the planning and piloting phases of data collecting are frequently overlooked. Even if recent curriculum guidelines include aspects such as *data collection, organisation* and *recording*, *planning* and *executing a sample survey*, rarely do textbooks include related activities. Using the terminology of Margolinas (2014), we refer to this phenomenon as the *invisibility of data processing and analysis* concerning the statistical knowledge to be taught at secondary school (Verbisck et al., 2023).

When considering Klein's double discontinuity and the assign a better place to data processing and management in secondary school statistics, two research questions can be drawn. First, how to evidence the invisibility phenomenon in a teacher education context. Second, how to approach it by involving teachers in inquiry processes as instructional activities. In this study, we address these research questions through a case study based on a teacher education proposal for statistics we designed and implemented in an online course in Brazil. The proposal pursues three main aims: first, getting in-service teachers involved in inquiring into a question that requires the collection and management of quantitative data; second, providing teachers with design tools to adapt and implement a similar activity in secondary school; third, helping teachers approach instructional resources more critically and productively, especially in what concerns the treatment of quantitative data.

5.1.2. Research theoretical framework

Our research is developed within the framework of the Anthropological Theory of the Didactic (Chevallard, 2015) and the tools developed to analyse the paradigm shift in mathematics education, from the currently prevailing *paradigm of visiting works* (PVW) to the *paradigm of questioning the world* (PQW). The new paradigm affects the transformation in mathematics education not only at the pedagogical level, about "how to teach?", but also at the epistemological level, about "what to teach?". In the paradigm of questioning the world, knowledge is learnt during the study and inquiry of relevant questions and appears in the form of dynamic, provisional and collective tools to address them. In this context, data analysis appears as an essential tool for inquiry, whereas in the paradigm of visiting works, only notions and strategies related to the calculation of central and dispersion measures use to seem valued as official statistical knowledge.

To study the conditions needed to transit towards the paradigm of questioning the world, Chevallard (2015) intends a teaching proposal called *study and research paths* (SRPs), which can be described through the *Herbartian schema*: $S(X; Y; Q_0) \hookrightarrow A^{\heartsuit}$. This can be considered as a model to design, describe, and analyse any study process (not only in the PQW). A student x (or a group of students X), helped by a teacher y or teachers Y, addresses an initial question Q_0 to provide a final answer $A^{\heartsuit}$. In the process from Q_0 to the collective elaboration of $A^{\heartsuit}$, the didactic system $S(X; Y; Q_0)$ displays Q_0 into derived questions Q_i, searches already available "labelled" answers $A_j^{\lozenge}$, elaborates and adapts them to Q_i, finds new questions during the process which, in turn, call for new answers, and so on. Bosch (2018) points out the importance of the *questions and answers* (*Q-A*) *dialectic* to ensure the dynamics of SRPs. The Q-A dialectic provides visible proof of the progress of the inquiry and contributes to the overall process management. To elaborate $A^{\heartsuit}$, the didactic system creates a didactic *milieu Mi*: $[S(X; Y; Q) \rightarrow Mi] \hookrightarrow A^{\heartsuit}$. This *milieu* is composed of the derived questions Q_i, the "ready-made" answers $A^{\lozenge}_j$ that seem helpful to answer Q_i, works and other objects W_k, and the sets of data D_m of all natures gathered during the inquiry. The extended *Herbartian schema* is symbolised as $[S(X; Y; Q) \rightarrow \{Q_i, A^{\lozenge}_j, O_k, D_m\}] \hookrightarrow A^{\heartsuit}$. The *media-milieu* (*Me-Mi*) *dialectic* becomes crucial during the whole SRP. To analyse this dialectic, we look at where external information, data and answers come from, and how their access is managed (*media*). We also ask how they are validated and transformed, and with what materials are the final or intermediate own answers developed (*milieu*). Finally, an SRP is a collective inquiry process during which small groups X_i are generated and individual work is also carried out. X_i and Y_j must organise themselves to work together. To analyse this *individual-collective* (*I-C*) *dialectic*, we focus on the roles assumed by X_i and Y_j during the SRP.

The proposal of the SRP was extended to the *study and research paths for teacher education* (SRP-TE) to provide teachers with pertinent (theoretical and practical) tools to nourish and sustain activities close to the paradigm of questioning the world (Barquero et al., 2019). An SRP-TE consists of five modules. *Module 0* (M0) starts with a professional question (e.g. how to teach proportionality, algebra, or statistics). In *Module 1* (M1), the educators let the teachers experience an SRP close to what could exist in their classes and related to the M0 professional question. The experienced SRP is then analysed using epistemological and didactic tools provided by the educators in *Module 2* (M2). In *Module 3* (M3), teachers design and implement an SRP under specific school conditions to finally, analyse it and share their experiences in *Module 4* (M4).

5.1.3. Methodology and the online SRP-TE on statistics for in-service teachers

The methodology follows the didactic engineering research principles applied to the SRP-TE (Barquero & Bosch, 2015). They consist of four major steps. The first step is to identify the didactic phenomena to be addressed. In our case, it corresponds to the invisibility of knowledge related to data treatment at the secondary school level. The second step (the *a priori* analysis) entails designing an inquiry activity related to the phenomenon under consideration—in this case, an SRP-TE course—to make the phenomenon visible and test the potentialities of the proposed instructional activity. The third step is the implementation, observation, and *in vivo* analysis of the activity. Finally, the fourth step is the *a posteriori* analysis based on the validation of the teacher's educational activity, as well as the development of knowledge about the initially identified didactic phenomenon.

Our case study focuses on an SRP-TE implemented as an online modality course from September to December 2022 for in-service secondary school teachers in Brazil (voluntary participation). The course took place in 14 sessions on Saturday mornings, each session lasting three hours, with a short break. Participants were required to attend the course sessions and assume some after-class work, especially to implement a teaching proposal some weeks in November/December and, in the end, to write a final report about the different phases of the SRP-TE. We used the *Microsoft Teams* platform as part of the infrastructure for online synchronous sessions, some in small groups and others all together. *Microsoft Teams* was also used as a document repository. A *Whatsapp* group with all educators and students supported the exchanges, especially between sessions. All *Teams* sessions were recorded. The data gathered include educators' and teachers' productions and the transcripts of their exchanges in the course sessions. In this study, we analyse the teachers' behaviour during the different modules of the course, paying special attention to the difficulties found in using the ATD tools introduced to describe, design and implement inquiry processes related to data treatment. In fact, we will interpret these difficulties as constraints coming from the prevailing paradigm of visiting works and the related phenomenon of invisibility of data treatment in secondary school mathematics.

In M0, our $Q_{0\text{-TE}}$ was about *How should we teach statistics in secondary school?* In this module, the educators presented $Q_{0\text{-TE}}$ as a cross-cutting issue in all other modules. Our didactic system was composed of three groups working in parallel and carrying out different SRP under the guidance of three educators. Group 1 is the only one considered in this study. It was composed of five in-service teachers, working in lower secondary education, coordinated by the author of this study (y_1) and the other authors as observers. The initial generating question $Q_{0\text{-SRP1}}$ was about a newspaper headline about "*Brazil has lost 15% of its water resources in 30 years, a loss of almost twice the water surface area of the entire Northeast*". *How to analyse the veracity of this news*? In *Microsoft Teams,* we created four permanent rooms: a general room for discussions and sharing with the three groups together, and a room for each group to work on the SRP-TE modules based on the different SRPs. The sessions had a general structure, as shown in Table 5.1.1.

Table 5.1.1: Organisation and structure of the SPR-TE sessions

Sessions	Description of the work	Room
1st: 3/09/22 M0 (2 parts)	Researchers introduced the course (modules, chronogram, group dynamics). *PVW and PQW were introduced.*	General
	Participants elaborated professional questions on a *Padlet.* General discussion and organisation according to the different paradigms.	
2nd: 10/09/22 M0-1 (2 parts)	Researchers reintroduced the course and the paradigms: PVW and PQW.	General
	$Q_{0\text{-SRP}}$ was proposed by coordinators in each X_i. Teamwork on specifics $Q_{0\text{-SRP}}$: elaboration of other questions, searching on the internet. *Q-A dialectic introduced.*	Groups Room
3rd: 17/09/22 M1 (2 parts)	Coordinators of each X_i shared the derived questions of their respective $Q_{0\text{-SRP}}$.	General
	Inquiry into $Q_{0\text{-SRP1}}$ and Q_i: finding a database and working with data in *Excel*.	Groups Room
4th: 24/09/22 M1 (2 parts)	Researchers' intervention in the aspects of data treatment and the Brazilian secondary education curriculum. Coordinators of each X_i shared the data.	General
	Study on specifics $Q_{0\text{-SRP}}$ and Q_i: elaborating $A^{\heartsuit}$ to $Q_{0\text{-SRP}}$.	Groups Room
5th: 1/10/22 M1-2 (2 parts)	Finalisation of $A^{\heartsuit}$ to $Q_{0\text{-SRP}}$. And elaboration of a *Q-A map* on a *Padlet.*	Groups Room
	One participant of each X_i presented their $A^{\heartsuit}$ to $Q_{0\text{-SRP}}$ to the whole group.	General
6th: 8/10/22 M2 (2 parts)	Researchers introduced the *Herbartian schema*, *Me-Mi dialectic*, and *I-C dialectic* as didactic tools to analyse the SRP each X_i experienced.	General
	Collective analysis of the SRP experienced, using the didactic tools introduced.	Groups Room
7th: 15/10/22 M2-3 (2 parts)	Coordinators of each X_i shared and discussed their analyses.	General
	Design and adaptations of an SRP to be implemented in the real classroom.	Individual work
8th: 22/10/22 M3	Collective design and adaptations of an SRP to implement in the classroom.	Groups Room
9th: 5/11/22 M3 (2 parts)	Researchers introduced the notion of *didactic contract*.	General
	Finishing the collective design of an SRP to implement in the classroom.	Groups Room
10th: 2/11/22 M3 (2 parts)	One participant of each X_i presented the SRP designed.	General
	Teamwork on the activity: "From a school exercise to an SRP".	Groups Room

Sessions	Description of the work	Room
11th: 19/11/22 M3 (2 parts)	Participants x_1 and x_2 shared the first lessons they implemented during the week.	General
	Teamwork on the activity: "From a school exercise to an SRP"	Groups Room
12th: 26/11/22 M3-4 (2 parts)	Participants x_1 and x_2 shared the further lessons implemented during the week; participant x_3 shared the first lessons implemented during the week.	General
	One participant of each X_i presented the SRP designed in the activity "From a school exercise to an SRP".	
13th: 3/12/22 M4	Each participant started to elaborate a final report.	Individual work
14th: 10/12/22 M4 (2 parts)	Participants x_1, x_2, and x_3 shared the finalisation of the SRP implemented in class.	General
	Collective *a posteriori* analysis, final discussions.	

5.1.4. Results and discussion of the SRP-TE on Brazil's water resources

The generating question $Q_{0\text{-SRP1}}$ presented to Group 1 comes from a previous research work presented in Verbisck et al. (2022), where we carried out an *a priori* analysis of an SRP-TE that starts from a school activity about water resources in Brazil. We saw how considering this activity within the paradigm of questioning the world leads to its extension by incorporating dimensions of the statistical work that tend to be absent from secondary education, such as the search, collection, cleaning, and representation of data. In M1, y_1 proposed to begin with a Brazilian newspaper related to water resources: "Brazil has lost 15% of its water resources in 30 years, a loss of almost twice the water surface area of the entire Northeast". $Q_{0\text{-SRP1}}$ was about *How to analyse the veracity of this news*? The maps of questions-answers were presented to the in-service teachers as a tool for analysing the development of the inquiry process. Initially, the team raised many questions, however, they chose some of them to search for answers:

- $Q_{1\text{-SRP1}}$: What data are presented in the news?
  - $Q_{1.1\text{-SRP1}}$: Are there official databases that provide these data? Can we access these data?
  - $Q_{1.2\text{-SRP1}}$: What information can we get from it (the database)?
    - $Q_{1.2.1\text{-SRP1}}$: How are the data from this official database produced?
    - $Q_{1.2.2\text{-SRP1}}$: Which data do we use to perform the analysis?
- $Q_{2\text{-SRP1}}$: How can we corroborate the diagnoses mentioned in the news story?
  - $Q_{2.1\text{-SRP1}}$: What is the water situation of each Brazilian region?

In the corresponding answers to $Q_{1.2.2\text{-SRP1}}$, $Q_{2\text{-SRP1}}$, and $Q_{2.1\text{-SRP1}}$, the group downloaded the data from the *MapBiomas* database (the one presented in the news) and worked with *Excel*. To elaborate $A^\heartsuit$, they organised the data from (1985 to 2020) into a table and constructed bar graphs and line graphs of the five Brazilian regions to make the comparisons among losses. They were unsure about the main tools to elaborate tables and graphs. They also had difficulties in choosing between a bar graph or a line graph to represent these data. They raised some questions concerning these difficulties:

$Q_{\text{Construction of tables}}$: What are the main elements of a table?
$Q_{\text{Statistical\_graphs}}$: What type of graphs to choose to represent the data? What are the main elements of a graph? Is it better to construct bar graphs or line graphs to represent this data? Why? Series graphs take different values, how to compare?

Four sessions were set aside for the development of this SRP and we (researchers) were already aware that it would not be enough time to investigate in depth all the issues raised. The aim of M1 was for in-service teachers to realise, albeit superficially, how an inquiry activity based on an SRP can work.

In M2 with the analysis of the SRP, the main discussion was about the statistical knowledge used for the elaboration of $A^{\heartsuit}$. The group pointed at the *construction of the table with data, the construction of graphs, comparisons of loss in percentages, and the calculation of the annual variation (absolute and in percentages).* We might notice here how much of the previous work—database search and access and its organisation—was not considered part of the statistical knowledge. In M3, a teacher x_1 volunteered to implement an SRP with sixth-grade students (aged 11-12). She was interested in working on the topic of rainfall levels in the region (*Agreste* – Northeast of Brazil). She found a newspaper article entitled “Pernambuco decrees emergency due to drought in 61 cities of *Agreste*”. So, participants worked on the *a priori* design of an SRP that took this news as a starting point for the Q_0: *What is an emergency about drought? When is it decreed?* The teacher implemented this SRP over three weeks, with a total of nine hours. In her class, the 26 students were organised in pairs or triads. In the first session, the class carried out a collective reading of the text (newspaper), the teacher formulated Q_0 and the students, working in groups, began to elaborate new questions Q_i:

- Q_0: What is an emergency about drought? When is it decreed?
- Q_1: What are the water sources in our region? What is the name of the river that runs near the school (Pesqueira town)?
- Q_2: Are the waters from rivers and wells drinkable?
- Q_3: Where is the town of Salgadinho?
- Q_4: What are the rainfall levels in our region?

In the second class, the students searched different websites to elaborate answers $A^{\lozenge}_j$ to Q_i. Students incorporated some $A^{\lozenge}_j$ into their *milieu* and wrote down the “source” in which they found the answer because the teacher told them to mention it. In some moments, students also used the teacher as a media, writing in one of the answers: “source: the teacher”. Question Q_4

prompted the teacher to hand out a table with data on average monthly rainfall for the town where the students live and for eight surrounding towns. These data were collected from an online database. So, to seek answers to Q_4, pairs and triads began to construct bar graphs of the nine cities they had selected (guided by the teacher). In the following lessons, with these data, students mainly considered bar graphs. The teacher noticed that students had difficulties constructing the graphs, as she believes this is the first time they had elaborate graphs on their own. In the last two lessons, she proposed a final analysis of the graphs, guided by the questions: *What comparisons can we make between the graphs? Which months are there less rainfall in the municipalities observed? What conclusions can be drawn?*

In M4, we had the opportunity to share comments about this experience. Participants with educators jointly analysed the implemented SRPs. We could identify how the teacher succeeded in managing the different dialectics of the inquiry process. In the case of *Q-A dialectic*, when she proposed an initial question to start the inquiry and encouraged her students to investigate by elaborating on other questions and searching for answers in different media. For the *Me-Mi dialectic*, she mobilised different media, such as the internet, maps of the region or a database that she pre-selected. All these elements are achieved to incorporate new objects into their *milieu*. As for the *I-C dialectic*, she tried to work on a different classroom dynamic: organising students in pairs and trios, proposing to go to the technology room, as well as collective graph constructions. And she tried to adopt a new role and responsibilities in guiding the inquiry process without giving the answers to students' questions.

Finally, we notice a critical episode in the course that highlights the strength of the phenomenon of *data treatment invisibility* that the SRP-TE could not really overcome. It appeared in the comment of another participant x_3 regarding the implementation of x_1:

> x_3: What you did seems interesting, but *statistics* is something different because it is about data analysis, the calculation of measures of central tendency, and even more so. But you [x_1] did not have the opportunity to do this part which, in my opinion, is the most important.

Although x_1' students elaborated several graphs and used them as relevant means to extract information about the question addressed, interpreting, comparing graphs, and summarising conclusions, x_3 expressed once more the invisibility of these aspects in statistical activity.

5.1.5. Conclusion

The online SRP-TE on statistics for in-service teachers made visible some conditions and constraints related to the phenomenon of data treatment invisibility. First of all, we observed difficulties for the groups in starting the inquiry of their respective SRPs in M1, especially when searching for empirical data and organising and analysing them. The educators needed to encourage the participants in much of this work, many times providing more hints than were initially expected. The online modality did not help, but similar difficulties are found in face-to-face teacher education (as it is being implemented at this moment). Second, the invisibility of statistics knowledge in what concerns data gathering, processing, and reporting emerged strongly. In M2 when participants did not value some important aspects when analysing their work in the SRP (e.g. the collection of data in a dataset and its organisation), or in M4 as we saw in one example of a teacher's comment [x_3]. The exception of the teacher x_1 with her students is a piece of hope and motivation to go on with the research.

These findings reinforce the prevalence of the invisibility of the knowledge at stake related to data treatment, in alignment with the results presented by Newton et al. (2011) who highlight the little importance given by teachers when proposing data collection to their students in statistical processes. Our results aim to bring the discussion further and focus more on the unclear *status* of mathematical knowledge related to data treatment in teachers' (and consequently students') knowledge and know-how, despite its recent introduction in many countries' curricula. We postulate that bringing to school a broad vision of statistics and data treatment requires granting it a specific status as part of the "official school mathematical knowledge". Although the implemented SRP-TE evidences the difficult challenges we might address, it also opens some lines of hope and future development. What seems totally necessary from statistics education is that we (researchers and educators) provide tools that give visibility and a clearer status to data treatment in mathematics school teaching, as it would not be enough that the curricula include the stages of the statistical cycle. Future research should go in this direction to change the culture and, in particular, provide teachers with epistemological and didactic tools to question and build a common "understanding" of what statistics is.

5.2. ADDRESSING WATER SCARCITY THROUGH STATISTICAL INQUIRY IN TEACHER EDUCATION

5.2.1. Introduction

Brazil, as a nation characterised by significant social and geographic inequalities, has implemented educational strategies to address cultural, social, and economic inequalities. The goal of inclusive education is to ensure that everyone can receive an education. It assumes equal opportunities for all and the value of individual differences, considering, among other things, the ethnic, social, cultural, intellectual, physical, sensory, and gender diversity of people (Kollosche et al., 2019; OECD, 2023). To guarantee access, participation, development, and learning for everyone implies the reform of the education culture, practices, and policies currently in place. Despite the current legislation, many challenges remain, like the restrictions due to poor structural conditions of public schools, the lack of specialised professionals to monitor students, and students' learning diversities (Alquati Bisol et al., 2015; Cury, 2005).

We adopt the perspective of inclusive statistics education that focuses on ensuring that all students have access to statistics education, recognising the diversity of students in the quest to adapt the teaching of statistics so that it is accessible, relevant, and meaningful for all (Kollosche et al., 2019; Witmer, 2021). We assume that statistics might play an important role to confront inequalities in the classroom and develop critical thinking and decision-making skills when working on an inquiry-based approach using open questions in real contexts. Over recent decades, statistics has evolved in line with the development of technological resources for data processing, giving rise to what is known today as "data science" (Loy et al., 2019). Statistics education has responded similarly adapted in line with the changes. A review of the literature revealed the emphasises put on the importance of taking a broad view of this field, including aspects of searching and collecting data in real-world contexts; selecting, organising, tabulating, and visualising data; using specific software, simulation, and reporting to answer open-ended questions; and study the variability of specific phenomena (Burrill & Ben-Zvi, 2019; Shaughnessy, 2007). This field is consolidating in terms of "statistical reasoning" (Ben-Zvi & Garfield, 2004), "statistical literacy" (Watson, 2016), and "statistical thinking" (Wild & Pfannkuch, 1999).

The problem of how to address inclusive statistics in teacher education has been developed as an important line of research (Casey & Ross, 2022; Dogucu et al., 2023; Kollosche et al., 2019;

Ni Bhroin & King, 2020; Witmer, 2021). Although data gathering and processing are becoming increasingly important in society (Ben-Zvi, 2020), they still do not have the same status in school mathematics. As stated by Batanero et al. (2011b) and Burrill and Ben-Zvi (2019), statistical activities are often reduced to numerical calculations of statistical measures (e.g., frequencies, measures of central tendency, deviations) and the creation or interpretation of standardised graphical representations (e.g., pie or bar charts, histograms, and scatter plots). This was made clear some decades ago by Short and Pigeon (1998) when they noticed that, although statistics educators seem to agree that data gathering and analysis steps are valuable, "the planning and piloting phases of data collection are often neglected" (p. 1). Even though recent national curricula include aspects such as data collection, organisation, recording, planning, and executing a sample survey (see Brasil, 2018), rarely do textbooks include related activities (Prestes, 2021; Verbisck & Bittar, 2021). Fewer still are textbooks that promote working with big and real data sets that call for initial steps of selecting, cleaning, and organising data to be analysed.

Ben-Zvi and Garfield (2004) emphasised that "students equate statistics with mathematics and expect the focus to be on numbers, computations, formulas, and one right answer" (p. 4). Similarly, Chevallard and Wozniak (2007) described and analysed this situation in terms of the prevalence of "the reduction of statistical work to arithmetical work." This reduction is part of a broader didactic phenomenon we call the *invisibility of data treatment*. It refers to the school's difficulties in providing an official role and status to all the work with data that is not seen as arithmetical work, for example, data gathering, processing, management. This phenomenon sets up important constraints for the teaching and learning of statistics, in particular, in secondary school (Verbisck et al., 2024).

The research reported in this study focuses on designing, implementing, and analysing a teacher education proposal for inclusive statistics education designed to overcome constraints derived from the phenomenon of the invisibility of data treatment. On the one hand, we aim to promote an online teacher education proposal based on the so-called *study and research paths for teacher education* (SRP-TE), as proposed by the Anthropological Theory of the Didactic. It is proposed, the SRP-TE progresses with teachers in the development of inclusive statistics education to address relevant societal questions and use digital resources. In this case, the context of the classroom inquiry undertaken was the environmental problem of water scarcity in some regions of Brazil. On the other hand, we aim to analyse the conditions facilitating and the constraints hindering the construction and transference of an official status to data treatment in teacher education and in secondary school. The research questions addressed in this study are:

RQ1: In what sense can a teacher education proposal, based on an SRP-TE, help teachers promote inclusive statistics? What kind of activities and through what interaction with digital tools facilitate teachers' reflections on inclusive statistics?

RQ2: What conditions implemented in an SRP-TE can help teachers address and detach themselves from the phenomenon of the "invisibility of data treatment" in a context of online in-service teacher education? What constraints limit it?

We address these research questions through a case study based on a teacher education proposal for statistics we designed and implemented in an online course in Brazil. The course sought to pursue three main goals. First, it involved in-service teachers in enquiring into a problem about water scarcity using statistical knowledge. Second, it provided teachers with design tools to implement a similar activity to be transferred to secondary school. Third, it aimed to help teachers approach teaching resources more productively, particularly in terms of their understanding of statistics. The study focuses on one of the cooperating participants, who worked in a rural school with few resources and poor infrastructure in an impoverished district of the Northeast. The training helped her to implement an inquiry project with her sixth-grade pupils, as well as use digital resources for data visualisation and graph representation. This case study contributed to understanding how the SRP-TE enhanced teacher professional development and provided aspects for teaching inclusive statistics using digital resources.

5.2.2. Theoretical framework

5.2.2.1. The anthropological theory of the didactic and the change of paradigms

The Anthropological Theory of the Didactic (ATD) is an educational theory developed by Chevallard, a French mathematician and researcher. In the 1980s, the theory arose as an attempt to comprehend the process of knowledge construction and acquisition in the field of mathematics education. The ATD theorises how knowledge is produced, disseminated, communicated, and received in social institutions, and how these processes affect the interactions between teachers, students, and the subject matter being taught. It started in the field of mathematics education but has been used in a wide range of different subjects and educational settings. The need to develop further didactic tools and devices for teacher education, as well as to study their functionality, impact, and diffusion, is supported by recent research in the ATD in terms of a change of pedagogical paradigm from the *paradigm of visiting works* towards a new *paradigm of questioning the world* (Chevallard, 2015).

According to Chevallard (2015), school education mainly takes part in what he referred to as the paradigm of visiting works in contemporary societies. Students' responsibility in this paradigm is to study content prescribed by rules and according to ready-made categories of different sizes: topics, areas, domains, and disciplines. The students must study and value these works without necessarily challenging their validity or worth. Hence, in this paradigm, themes and subjects are like monuments: students cannot change them; they do not need to understand why they exist; all they must do is study them. The instructor, who is referred to as "the one who knows," is the one who presents the works to study and proposes tasks for the students to get familiar with them and acquire a certain mastery of their use in different types of tasks. In contrast to this first paradigm, Chevallard (2015) described the *paradigm of questioning the world* in which content is organised by *questions,* and teaching and learning result from inquiry processes aimed at answering these questions. In other words, students are the inquirers of the generating questions (Q) proposed by the curriculum. There may be times when exploration and research are required in the search for answers to a question, but with a specific *raison d'être*: elaborating and justifying an answer to Q.

The new paradigm creates a strong change in the *didactic contract* that prevails in the first paradigm, that is, the kind of responsibilities assumed by teachers and students regarding the knowledge to be studied (Brousseau et al., 2020). In the new paradigm, teachers do not introduce the knowledge tools needed to answer the questions addressed. They act much more as leaders of the research process developed by the students. The change affects not only the pedagogical level, "How to teach?", but also the epistemological level, "What to teach?" Knowledge is learned during the inquiry of relevant questions using dynamic, provisional, and collective tools.

The change of paradigm introduced by the ATD promotes different changes. First, it gives the school the mission to address important social problems that affect all students, similarly. It then gives priority to the study of the questions and problems raised and not to the tools needed to answer them. In this context, data analysis appears as an essential tool for inquiry even if it is not always valued as official statistical knowledge or does not have any official status in school. In this context, inclusion means taking students' real problems seriously and showing them that a school is a place where these problems can be studied and addressed, to bring new knowledge about what matters to society (Casey & Ross, 2022; Kollosche et al., 2019; OECD, 2023; Skovsmose, 2019; Witmer, 2021). In this same line, steps towards the paradigm of questioning the world might help to establish better conditions for inclusion and inclusive statistics: important questions can be addressed in the school with all students are concerned, who can be given the chance to address these questions in class. Questions addressed should not be fake problems to motivate the use of statistical tools (decided beforehand), but real problems a statistical inquiry approach can address better.

5.2.2.2. Study and research paths for teacher education

To investigate the *ecology* of the transition of paradigms (i.e., the conditions needed to encourage the transition between the two pedagogical paradigms and the constraints that hinder it), Chevallard (2015) put forward a teaching proposal called *study and research path* (SRP). It can be described through the so-called *Herbartian schema*: $S(X; Y; Q_0) \hookrightarrow A^{\heartsuit}$, where a group of students X, helped by a group of teachers Y, form a didactic system S to address an initial question Q_0 and provide a final answer $A^{\heartsuit}$. In the process from Q_0 to the collective elaboration of $A^{\heartsuit}$, the *didactic system* $S(X; Y; Q_0)$ displays Q_0 into derived questions Q_i, searches already available "labeled" answers $A^{\Diamond}{}_j$, elaborates and adapts them to Q_i, finds new questions during the process which, in turn, call for new answers, and so on.

Bosch (2018) pointed out the importance of the *questions and answers (Q-A) dialectic* to ensure the dynamics of SRPs. The *Q-A* dialectic provides visible proof of the progress of the inquiry and contributes to the overall process management. To elaborate $A^{\heartsuit}$, the didactic system creates a didactic *milieu* M: $[S(X; Y; Q) \rightarrow M] \hookrightarrow A^{\heartsuit}$. This *milieu* is composed of derived questions Q_i, "ready-made" answers $A^{\Diamond}{}_j$ that seem helpful to answer Q_i, any kind of works W_k (knowledge or material), and the sets of data D_m gathered during the inquiry. The extended *Herbartian schema* is symbolised as:

$$[S(X; Y; Q) \rightarrow \{Q_1, Q_2, \ldots, Q_i, A^{\Diamond}{}_1, A^{\Diamond}{}_2, \ldots, A^{\Diamond}{}_j, W_1, W_2, \ldots, W_k, D_1, D_2, \ldots, D_m\}] \hookrightarrow A^{\heartsuit}$$

The *media-milieu* (*Me-Mi*) *dialectic* becomes crucial during the whole SRP. To analyse this dialectic, scrutiny is placed on where external information, data, and answers come from, and how access to data and information are managed (*media*). Explored also is how data and information are validated and transformed, and with what materials the final or intermediate answers are developed (*milieu*). Finally, an SRP is a collective inquiry process during which small groups X_i are formed, and individual work is also carried out. X_i and Y_j should organise themselves to work together. To analyse the responsibilities and roles assumed by X_i and Y_j during the development of an SRP, the notion of *didactic contract* explained above, is used.

Ruiz-Olarría (2015) extended the proposal of the SRP to the *study and research paths for teacher education* (SRPs-TE) to provide teachers with relevant (theoretical and practical) tools to undertake activities close to the paradigm of questioning the world. For this purpose, an SRP-TE consists of five modules (Figure 1). As in all SRP, the starting point is a generating question related to the teaching profession (Module 0). Partial answers to generating question will appear throughout the SRP-TE (and it is expected that afterwards, it will also continue to appear), making it a transversal module of the educational process. Then, teacher-students progress through an SRP to experience the inquiry process (Module 1) and analyse the joint experience from a didactic perspective (Module 2). The proposal is adapted to suit school conditions and implemented (Module 3), to be finally analysed (Module 4) to provide provisional answers to the initial teaching question (e.g., What is possible and how? What seems difficult and why? What are the consequences?).
The first online SRPs-TE were designed and implemented in Mexico and the findings in that investigation highlighted the practicality of the methodological adaptation, especially in the identification of institutional constraints that are typically hidden from instructors in their daily work (Barquero et al., 2018). In this study, we report on the analysis of the teachers' behaviour during the different modules of an SRP-TE, paying special attention to the difficulties found in using the ATD tools introduced to describe, design, and implement inquiry processes related to data treatment. These difficulties are likely to be constraints coming from the prevailing paradigm of visiting works and the related phenomenon of invisibility of data treatment in secondary school mathematics. The study also investigates how the teacher education proposal made it possible to promote inclusive statistics using digital resources in an inquiry process, based on the study of an environmental question close to the students' social context. The design and implementation of the online SRP-TE applied to the Brazilian teachers' education context; adaptations were made based on previous studies that will be detailed in the following section.

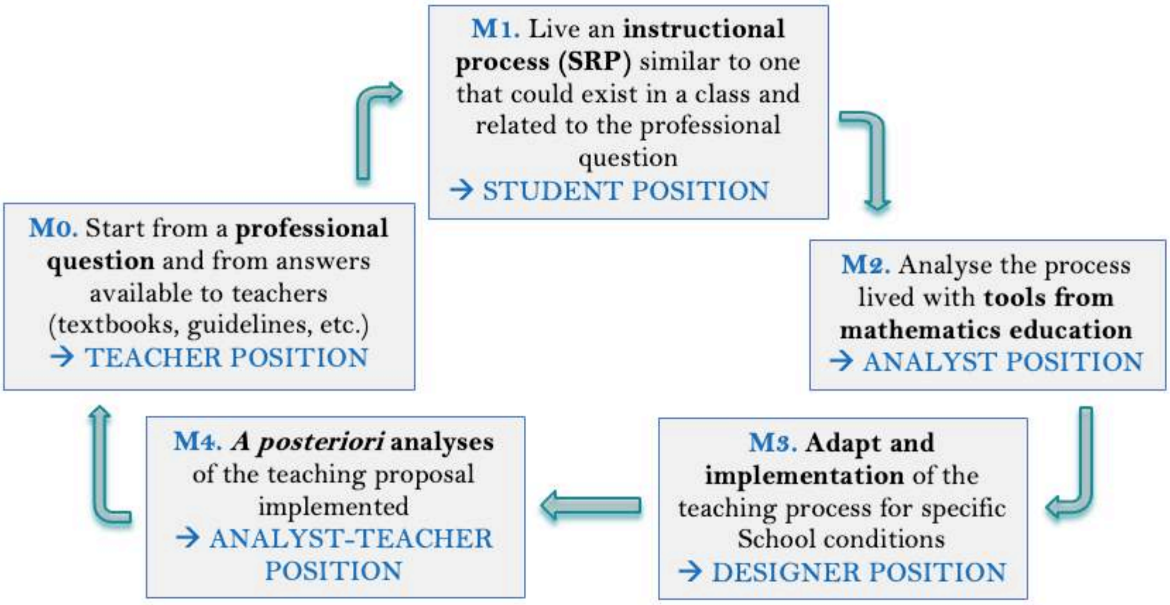


Figure 5.2.1. The general structure of an SRP-TE (Ruiz-Olarría, 2015; Barquero et al., 2018)

The methodology of our research is qualitative and exploratory. The design, implementation, and analysis of the SRP-TE follows the didactic engineering research principles (as described in Barquero and Bosch, 2015). They consist of four major steps. The first step is the *preliminary analysis* where we identify the didactic phenomena to be addressed. In our case, as mentioned before, it corresponds to the invisibility of knowledge related to data treatment at the secondary school level. The second step is the *a priori analysis* and entails designing an inquiry activity related to the phenomenon under consideration—in this case, an SRP-TE course—to make the phenomenon visible and test the potentialities of the proposed instructional activity. The third step is its implementation, observation, and *in vivo analysis* of the activities planned in each of the modules. Finally, the fourth step is the *a posteriori analysis* based on the analysis of the teachers' educational activity, as well as the development of knowledge about the initially identified didactic phenomenon. The data gathered include the educators' and teachers' productions during all the SRP-TE, the transcripts of their exchanges in the course sessions, and interviews with the participants who implemented an adapted SRP in their classes[9].

In section 3, we present the *a priori* analysis of the different modules of the SRP-TE. The *in vivo* and *a posteriori* analysis will be presented in sections 4 and 5, as results and to discuss them concerning the research questions.

[9] In Appendix D.1 and D.2 we provide the educators' and teachers' productions during all the SRP-TE. We maintained these productions in the original language: Portuguese.

5.2.3. The design of an online SRP-TE for in-service teachers in Brazil

5.2.3.1. Previous studies and results

In a previous investigation, when looking at Brazilian textbooks, we found a school exercise about a bar graph of "Distribution of Brazil's water resources, surface, and population." Although the context of the school exercise seemed highly relevant to be discussed in the classroom, it only presented questions about the kind of graph and the interpretation of some of its elements. Building upon this exercise, we developed a comprehensive set of a generating question and derived questions that could arise during an inquiry-based approach (Verbisck et al., 2022a). This *a priori* analysis serves as a potential SRP to be implemented in mathematics teacher education.

Subsequently, we conducted an online pilot study with preservice teachers from the Federal University of Sergipe (Brazil). Our goal was to turn the school exercise about "Distribution of Brazil's water resources, surface, and population" into an inquiry activity, moving it away from the paradigm of visiting works toward the paradigm of questioning the world (Verbisck et al., 2022b). The characterisation of the main phenomenon considered here, the *invisibility of data treatment*, resulted from this study. This experience inspired us to design and implement the online SRP-TE presented in this study. The discussion and analysis of the SRP-TE are explained from Section 4 onwards. Before that, we present a more detailed description of the design of this proposal and the digital infrastructure created for the development of the course.

5.2.3.2. The online SRP-TE structure and its main digital infrastructure

The SRP-TE was implemented as an online modality course from September to December 2022 for in-service secondary school teachers in Brazil (voluntary participation). It was planned and implemented in partnership with researchers from the Pontifical Catholic University of São Paulo, the Federal University of Mato Grosso do Sul, the Federal University of Bahia, and the University of Barcelona. The course took place in 14 sessions on Saturday mornings, each session lasting three hours, with a short break. Participants were required to attend the course sessions and complete some after-class work, implement a teaching proposal some weeks in November/December, and to write a final report about the different phases of the SRP-TE. We used the *Microsoft Teams* platform as part of the infrastructure for online synchronous sessions, some in small groups and others with the whole class. *Microsoft Teams* was also used as a document repository. All online *Microsoft Teams* sessions were recorded.

In M0, the generating question $Q_{0\text{-TE}}$ was *How should we teach statistics and probability in secondary school?* In this module, the educators presented $Q_{0\text{-TE}}$ as a cross-curricular issue in all the other modules. Our didactic system was composed of three groups working in parallel and carrying out different SRPs under the guidance of three educators. Reported in this study are the outcomes for one group of teachers, those addressing the question related to water scarcity. It was formulated as follows: "*Brazil has lost 15% of its water resources in 30 years, a loss of almost twice the water surface area of the entire Northeast region.*" *How to analyse the veracity of this news?*

The choice to divide the participants into three groups to carry out different SRPs in M1 was due to several factors. First, the initial group of registered participants comprised of over 100 people, and we considered that only groups of up to 30 participants were manageable. Second, we thought that proposing three different SRPs would illustrate different contexts for inquiry activities and broaden the type of statistical and probability tools expected to be implemented during the inquiries. Hence, the moments of general discussions become critical to share the progress of the SRPs, the common and specific difficulties encountered, and the ways used to overcome or interpret them. The author of this study coordinated the group included in the analysis.

In *Microsoft Teams,* we created four permanent rooms: a general room for discussions and sharing with the three groups together, and a room for each group to work on the SRP-TE modules based on the different SRPs. The sessions had a general structure: all sessions but two (8^{th} and 13^{th}) were divided into two parts, one with all the participants to introduce the work to do (including discussing issues related to the methodology of the SRP), provide new tools and explanations, and discuss the results obtained; and another one dedicated to group work. Between one session and the next, the team of researchers analysed the development of the online SRP-TE, adapted the initial schedule when necessary, and made decisions about the next steps to take in the course. Many discussions corresponded to the choice of digital resources that would enable the interaction of the work groups and the storage of their productions. The main digital tools employed during the SRP-TE and their main functions are listed in Table 5.2.1. More details of the sessions' structure are presented in Verbisck et al. (2024).

Table 5.2.1. Digital resources and their functions in the online SRP-TE

Digital resources	Mean functions of use during the course
Microsoft Teams	- Synchronous meetings, group work and collective discussions. - Document repository for suggestions of materials and readings for participants, as well as storage of teamwork. - Used to store the recordings of the sessions.
WhatsApp	- Keep in touch with participants from one session to the next (news, reports, etc.). - Follow the progress of the implementations of research activities during M4.
Padlet	- Online collaborative platform used for the collective production of questions-answer maps in M1.
Websites and digital publications	Webpages, e-books, online journals, and articles used to: - Search for "ready-made" answers $A^{\lozenge}_{j}$, - Search and download data D_m, - Search for objects O_k of different kinds.
Microsoft PowerPoint	Software used by educators to present the course structure, timetable, progress of sessions, and interventions on specific topics. For example: at each session, in the general *Microsoft Teams* room, PowerPoint was used to carry out recaps of the work developed so far in the three different groups.
Google Drive	Cloud-based file storage and synchronisation service used for teamwork productions: each team worked collectively in the same *Google Drive* document, which facilitated the storage and recording of all group productions.
Online Databases	Digital repositories of information used to search and download data about the subject being investigated, for example, water resources in Brazil.
Microsoft Excel	Software used to work with data: the data found in the database were downloaded in ".xls" format and for the organisation, analysis and visualisation of the data, the group members used the functions offered by the software to construct absolute and relative frequency tables, graphs of different types, comparison parameters, etc.

All these digital resources, and especially the connections between them, were crucial for the construction and evolution of the *media-milieu dialectic* in our online SRP-TE. In the result and discussion sections, we will see how some of these digital resources were used during the modules, especially by one of the teachers participating in the course (Paola's case) in adapting the SRP experienced in module M1 to her sixth-grade classroom.

5.2.4. Results

5.2.4.1. Modules 0-2: Equipping teachers with didactics tools and promoting inclusive statistics

In M0, the $Q_{0\text{-M0}}$ was, "*How should we teach statistics and probability in secondary school?*" In this module, the educators presented $Q_{0\text{-M0}}$ as a cross-curricular issue in all the other modules and gathered more specific questions raised by the participants. While grouping and commenting on these questions, the differences between the paradigm of visiting works (to which most of the questions refer) and the paradigm of questioning the world were explained.

In M1, the author of this study (represented by *y*) was the coordinator of Group 1. This group was comprised of six secondary in-service teachers. An inquiry activity was proposed starting with a Brazilian newspaper report related to water resources, about which the question of the veracity of the data presented was raised ($Q_{0\text{-SRP1}}$). We saw how considering this activity within the paradigm of questioning the world leads to its extension by incorporating dimensions of the statistical work that tend to be absent in secondary education, such as the search, collection, cleaning, and representation of data, as well as the critical reception of information based on data. We also saw how the generating question $Q_{0\text{-SRP1}}$ encouraged critical thinking and research skills to evaluate the credibility of news sources and claims related to environmental issues.

Once the generating question was presented by *y*, the participants had difficulties in raising questions about the theme exposed in the report. To address this issue, *y* assumed the role of motivator of the group and stimulated the participants to first carry out a careful reading of the report and then discuss possible questions that would derive from $Q_{0\text{-SRP1}}$ and the reading performed. Putting the participating teachers in the position of "inquirers" of a news report required a change in the didactic contract with which they were not familiar. The coordinator *y* tried not to interfere in this first moment of raising questions and observed the dialogues that emerged. She noticed that some participants asked, "But, what should we ask?" as if *y* expected certain specific questions. This type of behaviour is strongly linked to the dominant paradigm of visiting works, in which teachers usually know the answer to the assignments proposed in advance, and consequently, students believe that the teacher expects a certain answer to a requested task.

After reading the entire newspaper report, we realised that the participants took $Q_{0\text{-SRP1}}$ seriously and began to elaborate on questions derived from the newspaper report. The questions were grouped by themes (Table 5.2.2) and related to aspects of inclusive statistics (Kollosche et al., 2019; OECD, 2023; Skovsmose, 2019; Witmer, 2021), showing the focus of the questions on water resources, conservation, and their impact on various aspects of society and the environment.

Table 5.2.2. Questions grouped and their relationship with inclusive statistics

Questions raised	Relationship with Inclusive Statistics
Q_1*: What data are presented in the news?* $Q_{1.1}$*: Are there official databases that provide these data? Can we access them?* $Q_{1.2}$*: What information can we get from the databases?* $Q_{1.2.1}$*: How are the data from the databases produced?*	• Promoting awareness of available data sources, how to access and interpret this information for informed decision-making. • Promoting the exploration of the methodologies behind data collection, stimulating the study of the techniques used to gather accurate and reliable information.
$Q_{1.2.2}$*: What data can we use for the analysis?*	• Selecting relevant data for analysis, developing a deeper understanding of the factors contributing to water resource loss.
Q_2*: How can we corroborate the diagnoses mentioned in the news report?* $Q_{2.2}$*: What statistical knowledge could help get results and see if they match the news information?*	• Encouraging cross-referencing data and seeking expert opinions to verify claims. Incorporating statistical analysis skills to interpret and draw conclusions from data, and validate or question the claims made in the news.
$Q_{2.1}$*: What is the water situation of each Brazilian region?*	• Promoting learnings about the specific water situations faced by different regions in Brazil, emphasising the importance of understanding the local context

Overall, these questions reflect the application of critical thinking, data analysis, understanding of local contexts, and awareness of interdependencies within the environment. Due to the limited time, the group decided to investigate these questions, but they also raised eight more questions. In the corresponding answers to questions $Q_{1.1}$, $Q_{1.2}$, $Q_{1.2.1}$, and $Q_{1.2.2}$, the group downloaded the data from the *MapBiomas* database (the one presented in the newspaper report) and worked with *Excel*. To elaborate $A^{\heartsuit}$, they organised the data from 1985 to 2020 into a table and constructed bar graphs and line graphs of the five Brazilian regions to make comparisons among water losses. They were unsure about the main tools to create tables and graphs. They also had difficulties in choosing between a bar graph or a line graph to represent these data and raised some questions concerning these difficulties.

Four sessions were set aside for the development of this SRP. The research team was aware that it would not be enough time to investigate in depth all the issues raised. The aim of M1, however, was for in-service teachers to realise, albeit superficially, how an inquiry activity can work. We also intended to equip teachers with didactic tools, especially with the introduction of an inquiry activity within the paradigm of questioning the world, so that they could collectively design an inquiry activity to be implemented in the classroom. The educators also helped teachers to find some answers to the statistical questions raised by providing them with resources or directing them to information.

In M2, the collective analysis of the SRP was conducted using didactic tools that the educators introduced according to their pertinence. Three main discussions were promoted in this module. First, a description of the experienced SRP using the *Herbartian schema* and the two dialectics: the participants identified the derived questions raised (Q_i), the works (W_k) and data (D_m) used for the elaboration of some answers, and the questions that remained open. They did not incorporate any already available ready-made answer ($A^{\Diamond}{}_j$) because of the time shortage but highlighted some of the themes they believed were important to study and incorporate in their *milieu*, such as:

- *Water resources in Brazil: several types of analysis carried out by other agencies*
- *Construction of tables and main elements*
- *Types of statistical graphs, main elements, and relations to the data properties*
- *Loss comparisons in percentages, calculation of annual changes (absolute and in percentages)*
- *Functioning of hydroelectric plants, rainfall rates, and deforestation in Brazil*

The educators also promoted a discussion about the difficulties met in trying to develop the inquiry activity within the new paradigm of questioning the world, especially to notice the elements of the traditional didactic contract that were strongly modified. Finally, the participants were asked to identify the statistical knowledge used for the elaboration of $A^{\heartsuit}$. The group pointed out the construction of tables with data, the construction of graphs, comparisons of loss in percentages, and the calculation of the annual variation (absolute value and percentages). It is worth noting that much of the previous work—database search and access and its organisation—was not considered as part of the statistical knowledge.

5.2.4.2. Modules 3-4 and Paola's case: Transferring the SRP to a secondary classroom

M3 was devoted to the collective design of a proposal for an inquiry activity based on the SRP developed in M1. To do so, teachers were given the following instructions:

> *Work in your group to design an activity project adapted to the secondary school level. Think about the following questions:*
> - *What is the generating question?*
> - *How is the question posed, what is proposed, and what tools are available?*
> - *How is the study planned to be administered in class (approximate timetable)?*
> - *How is the activity expected to be completed?*
> - *What curriculum content (in mathematics and other subjects) can be addressed by the proposed activity?*

Paola volunteered to implement an SRP with sixth-grade students (11–12 years old). After all the discussions held in M1 and the collective analysis in M2, she became interested in working on the topic of rainfall levels in the region where she lived, Agreste, Northeast of Brazil. In relation to other regions, it suffers the most from lack of rainfall and, consequently, extreme drought for much of the year. This makes the residents of this region look for ways to store water from rain and rivers. Paola believed that it would be interesting and important to design and propose an inquiry connected to a social issue and the students' daily life. In addition, as she worked in a rural school with few resources and poor infrastructure, she sought to address these difficulties and promote inclusive education.

Paola took the initiative to propose a generating question based on a news report from a local newspaper entitled "Emergency decree in Pernambuco due to drought in 61 cities of Agreste". The news report mentioned the list of cities affected by drought and, among them, the city where the school is located (Pesqueira). Group 1 and *y* designed an SRP that took this news as a starting point: *What is a drought emergency? When is it declared?* With this in mind, we designed an SRP proposal based on the general guidelines: we raised questions that we believed would arise in the classroom, we planned the class time and the digital resources and other materials to be used in each lesson, we anticipated how Paola would lead and organise the students in nine lessons, we prepared how the inquiry activity would be closed, and decided on the curricular contents to be used.

Paola implemented this inquiry activity over three weeks, during a total of nine 50-minute lessons. In her class of 26 students, three of them had reading disabilities. These three students did not have any medical report attesting to a disability and had not been referred to specialists for any type of care. Table 5.2.3 presents how the SRP was implemented in Paola's lessons, highlighting the main activities developed by the students and how they were organised and guided by the teacher, also detailing the media used and the curricular contents involved.

Table 5.2.3. Paola's SRP in sixth-grade

Lessons	Description	Media	Curricular contents
1st: 17/11/22	In the regular classroom, Paola organised the students in pairs or groups of three. She carried out a collective reading of the newspaper. She proposed Q_0, and then the teams began to elaborate Q_i: • Q_0: What is a drought emergency? When is it declared? • Q_1: What are the water resources in our region? What is the name of the river that runs near the school (Pesqueira town)? • Q_2: Is the water from rivers and wells drinkable? • Q_3: Where is the town of Salgadinho? • Q_4: What are the rainfall levels in our region?	The report was *printed* for each student. Paola used the *blackboard* to write down the questions raised. The students prepared a report and wrote the list of questions about the text in their notebook.	- Identifying variables and their frequencies, and the elements (title, axes, legends, sources, and dates) of different types of graphs. - Interpreting and solving situations involving research data on environmental contexts, sustainability, traffic, responsible consumption, presented by the media in tables and different types of graphs and writing texts with the aim of synthesising conclusions. - Planning and collecting research data on social practices chosen by the students and using electronic spreadsheets to record, represent and interpret the information in tables, various types of graphs and text. - Interpreting and developing simple flowcharts, identifying the relationships between the objects represented (e.g., position of cities considering the roads that connect them, hierarchy of employees in a company, etc.) - Solving problems involving information presented in tables and/or graphs (Brasil, 2018).
2nd: 18/11/22	In the technology room, the teams performed some searches in different Internet media to find answers $A^{\lozenge}_j$ to Q_i. The students incorporated some $A^{\lozenge}_j$ into their *milieu* and wrote down the "source" in which they found the answer (as requested by Paola). In some moments, the students also used Paola as a media source and indicated in the notebook: "source: teacher Paola".	*Computers* with *Internet access* for the sources in various *Websites*. The students wrote the answers found in their *notebook*. Using *PowerPoint*, Paola presented a map to identify the water resources of their region and the cities surrounding them that were present in the list of the newspaper report.	
3rd-5th: 21/11/22 and 22/11/22	Q_4 prompted Paola to hand out a table with data on average monthly rainfall for the town where they live and for eight surrounding towns (Fig. 2, Part A). These data were collected by Paola from a database she found online. Hence, to seek answers to Q_4, the students began to construct bar graphs of the nine cities they had selected (Fig. 5.2.2, Parts B and C). First, Paola made two collective bar graphs representations, then, in the other lessons, she guided the students to create their own.	*PowerPoint* to present an example of a bar chart representation made in *Excel*. Paola used the *blackboard* to construct the bar graphs. The students made the graphs in their *notebooks* using *checkered sheets*, *rulers*, and *pens of different colours*.	
6th-7th: 29/11/22 and 1/12/22	Paola reviewed everything the students had done so far. She summarised the inquiry in four parts: "research and questions about the newspaper story", "presentation of data", "working with data and representations" and "presentation and analysis". As they were still in the third part, with the monthly rainfall levels table, the students continued with the bar graphs representations guided by Paola.	Paola used *PowerPoint* to present the summary of lessons 1-5. The students made bar graphs representations on checkered sheets, using rulers and pens of different colours.	
8th-9th: 6/12/22	In the final lessons, Paola proposed a final analysis of the bar graphs. She displayed all the graphs on the blackboard and led a collective discussion on the following guiding questions (Fig. 5.2.2, Part D): *What comparisons can we make between the graphs constructed? Which months of the year show less rainfall in the municipalities observed? What conclusions can we reach about the different levels of rainfall?*	Blackboard for displaying the graphs. The students wrote the answer for these final questions in their notebook.	

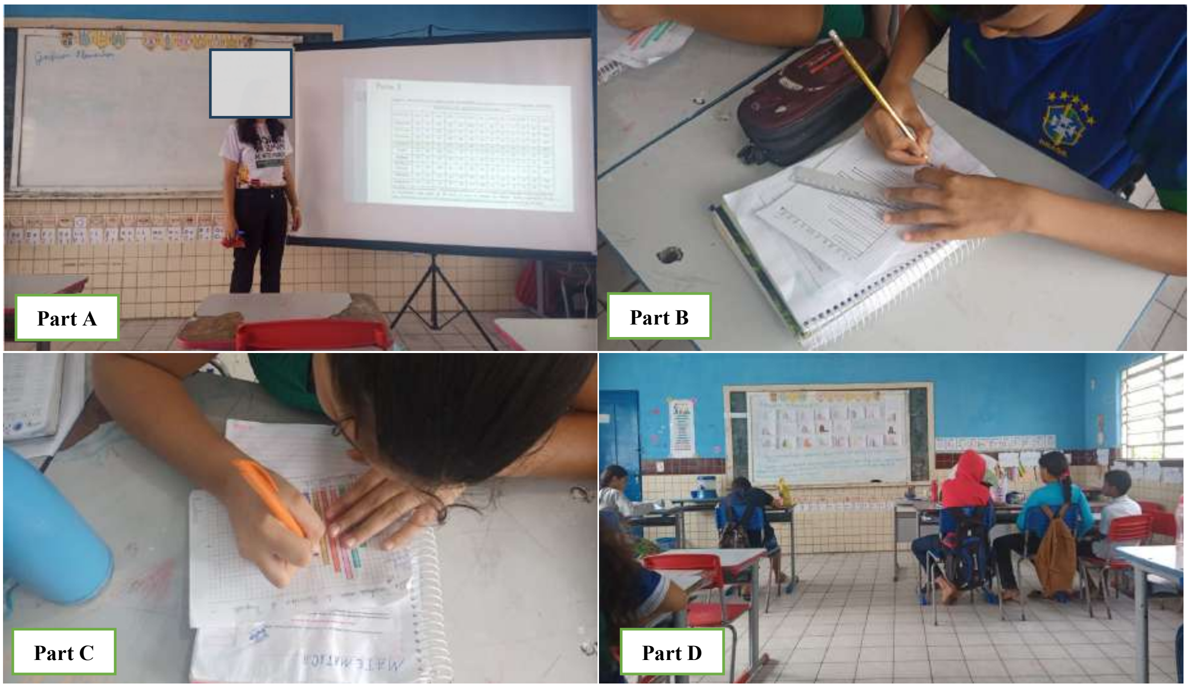


Figure 5.2.2. Illustration of Paola's class work using statistical graphs (lessons 3-9)

As seen during the SRP developed by the trainee teachers (M2) and in the case of Paola's implementation, the questions studied by her sixth-grade students incorporated aspects related to environmental issues. Q_1 was about the importance of understanding local water resources, and especially, identifying the river near Pesqueira town's school as an example of a valuable natural resource. Q_2 was a quest to understand water safety from various sources and the importance of knowing whether river and well water are fit to drink, and it relates to the broader goals of environmental awareness and public health. Q_3 was raised by the students only because they were interested in the singular name of the city. Answering the question, however, incorporated discussion about the geography and environmental issues to discuss the importance of locating the city of Salgadinho in its broader environment. It also showed how understanding the location of the city could contribute to a holistic understanding of its ecological challenges and potential strategies for sustainable development. Q_4 promoted analysis and discussion about monitoring and understanding regional precipitation patterns as a crucial aspect of environmental awareness and the subsequent implications for water availability.

During the implementation (M3), Paola received support mainly through conversations in a *WhatsApp* group with the other teachers. She commented she felt safer when implementing these lessons with the support of her group and always kept the other members informed (even if at a distance and virtually) about the progress of the activity. In addition to the *WhatsApp* group, the implementations were also shared and monitored during the Saturday sessions, in which discussions were held in the *Microsoft Teams* general room about how the participants' implementations went during the week (e.g., difficulties or unforeseen events and what created them; and adjustments and readjustments of the planning of activities, etc.). In summary, Paola's implementation of the SRP created an inclusive statistics education scenario by connecting the inquiry to students' realities, adapting methodologies to suit the school's context, fostering collaboration, and employing diverse teaching strategies. However, some unforeseen events arose throughout the implementation, such as class cancellations or unplanned school events, which led to adaptations to the nine lessons. While certain limitations were acknowledged, the project demonstrated a commitment to inclusivity by engaging students in a meaningful and interdisciplinary learning experience.

In M4, Participants and educators jointly analysed the SRPs implemented And took the opportunity to share comments about the experience. In the case of Paola, we identified how the teacher succeeded in managing the different dialectics of the inquiry process. For the *Q-A dialectic*, she encouraged her students to elaborate on other questions and search for answers in different media. For the *Media-Milieu dialectic*, she used different sources, such as the Internet, maps of the region, or a database she pre-selected. All these elements contributed to incorporating new objects into the students' *milieu*. The students were asked to include the source of information ("the Internet", "the teacher", etc.) in the partial answers proposed in their logbooks. It was observed that Paola tried to change the predominant *didactic contract* by working on a different classroom dynamic, including organising students in pairs and trios, proposing to go to the technology room, and preparing collective graphs. She also tried to adopt a new role and took on the responsibility of guiding the inquiry process without being the privileged media or providing answers to the students' questions.

Finally, we would like to highlight an episode showing that the SRP-TE was not fully able to remove some constraints related to the *invisibility of data treatment*. The episode refers to the comment by another participant x_3 regarding Paola's implementation:

> x_3: Paola, it seems interesting to me what you did, but statistics is different because it is data analysis, the calculation of measures of central tendency, even more so. But you [Paola] did not have the opportunity to do this part which, in my opinion, is the most important.

Although Paola' students created graphs from information about the rainfall in different months and cities—using them to analyse the data (creating, interpreting, comparing graphs, and summarising conclusions)—x_3 expressed the invisibility of this statistical knowledge yet again. She continued to identify it with the calculations of measures of centre.

5.2.5. Discussion and conclusions

5.2.5.1. Promoting inclusive statistics through the SRP-TE

With the design and implementation of the online SRP-TE, we identified conditions that promoted the development of inclusive statistics resorting to digital resources (RQ1). Because we offered an online course, we had the participation of in-service teachers from all the regions in Brazil. This course was only possible by using and integrating several digital resources in all the modules (M0–M4). Proposing an inquiry activity developed from *a priori* studies and adapted from a school exercise on "distribution of water resources in Brazil" helped promote the transition towards the paradigm of questioning the world by incorporating dimensions of statistical reasoning, literacy, and thinking that do not appear enough in school practices. Moreover, statistics turned out to be a valuable tool to address environmental issues that appeared naturally during the inquiry (M0–M4).

From M0–M2, the SRP-TE was inclusive because of its accessibility, ensuring all teachers interested could participate, regardless of their geographical location. Successful implementation was ensured by integrating real-world issues (Brazilian water resources) and making the use of statistics knowledge relevant to the broader societal and environmental contexts (Stevenson et al., 2013).

Once the inquiry was experienced and analysed by the participants and educators (M1–M2), the transfer to the classroom was possible, as illustrated by Paola's case (M3–M4). Paola sought to develop an inquiry activity with her sixth-grade students seeking to problematise a social issue that she and her students experience daily. This enabled an inclusive *milieu* to emerge. Given the school's limited infrastructure, Paola used most of the material and digital resources available. In addition, given the case of her three students with reading difficulties, Paola introduced class retakes and supplementary readings that better structured and nourished the inquiry process, thus benefiting all the students. Paola also found that all the students had more difficulties than expected in creating the graphs. The work required to create graphs, however, was strongly motivated by the need to respond to the questions raised. It was thus unavoidable for Paola to devote the time necessary for her students to create and make use of the graphs. It can be said that, by being driven by the logic of the inquiry, the activity naturally became more inclusive, environmental, and "statistical."

The experience with the complete module structure of the SRP-TE showed how “double inclusion” was made possible. On the one hand, the opportunity for teachers from different regions of Brazil to take part in continuing training through participation was only possible because it was offered remotely. On the other hand, facilitated, was the inclusion of secondary school students (in Paola’s case) through the experience of an SRP on water scarcity (adaptation of the one experienced in previous modules) approximating a paradigm of questioning the world. These findings corroborate research undertaken already (Kollosche et al., 2019; Monteiro & Carvalho, 2023; Ni Bhroin & King, 2020; Skovsmose, 2019; Witmer, 2021).

5.2.5.2. Breaking the invisibility of data treatment: consequences for the ecology of the teaching of inclusive statistics

The online SRP-TE on statistics for in-service teachers made conditions and constraints related to the phenomenon of data treatment invisibility visible (RQ2). First, we observed difficulties for the teachers in starting the inquiry of their respective SRP in M1, especially when searching for empirical data, and organising and analysing the data. The educators needed to encourage them in much of this work, many times providing more hints and suggestions than initially expected. The online modality did not help, but similar difficulties were found in face-to-face teacher education context (as shown by the pilot study previously implemented). Second, the invisibility of statistics knowledge related to data gathering, processing, and reporting emerged strongly. This was evidenced in M2 when the participants did not value some of these aspects when analysing their inquiry process, and in M4 as exposed in the teacher’s comment [x_3: Subsection 5.2.4.2].

These findings reinforce the prevalence of the invisibility of the knowledge at stake related to data treatment, which aligns with the results presented by Newton et al. (2011), who highlighted the little importance given by teachers when proposing data collection to their students in statistical processes. Our results aim to take the discussion further and focus more on the unclear *status* of statistical knowledge related to data treatment at school, despite its recent introduction in many countries’ curricula. We postulate that taking a broad vision of statistics and data treatment at school requires granting statistics education specific status as part of the “official school mathematical knowledge”. Although the SRP-TE implemented showed the difficult challenges ahead to be addressed, we entertain the hope of future developments. The experience of carrying out real inquiries seems necessary for teachers to acknowledge the importance and the elements of data treatment firsthand. It is, however, also important to give statistical ideas and concepts not only visibility but also a more dominant status in school mathematics. Future research should go in this direction to change the school culture and provide teachers with epistemological and educational tools to question and build a common understanding of what statistics or data analysis may encapsulate at the school level.

CHAPTER VI: A FACE-TO-FACE SRP-TE FOR PRESERVICE TEACHERS

6.1. INTRODUCTION

The previous study (presented in Chapter IV, section 4.1), started from the consideration of a school task found in a Brazilian textbook (Giovanni Júnior & Castrucci, 2018) about a bar graph of the "Distribution of Brazil's water resources, surface and population". The task proposed a highly relevant issue about the presence of water scarcity in some regions, despite Brazil being the fifth country with more water resources in the world. However, the task only presented questions about the kind of graph used and the interpretation of some of its elements. Building upon this exercise, we developed a comprehensive set of derived questions that could arise when inquiring about this issue (Verbisck et al., 2022b). The *a priori* analysis showed the generating power of this question and led to the design of an SRP-TE. It turned the school task into an inquiry activity, moving it away from the paradigm of vising works towards the paradigm of questioning the world (Verbisck et al., 2022a). Subsequently, we conducted an online pilot study with preservice teachers from the Federal University of Sergipe (Brazil) (presented in Chapter IV, section 4.2). The main hypothesis of the *invisibility of data treatment* resulted from this study (H2). The initial SRP of Module 1 was focused on a dataset and its analysis. In Module 3, students designed SRPs based on the inquiry of questions related to water consumption, rainwater storage, water degradation and water scarcity. Despite these choices and the fact that datasets were critical in the inquiry promoted by the proposed SRPs, students did not include "statistics" in the list of knowledge contents and disciplines engaged in their instructional proposal. This experience inspired us to design and implement an online SRP-TE from September to December 2022 with in-service teachers in Brazil, presented in Chapter V, sections 5.1 and 5.2 and presented in Verbisck et al. (2024). The phenomenon of the invisibility of data treatment reappeared in this second SRP-TE through the comments of a student during the *a posteriori* analysis of the instructional proposal implemented by one of the course participants.

A distinctive feature of the present study is the complete face-to-face implementation of an SRP-TE within a preservice teacher education programme at the university where the author completed her bachelor's and master's degrees. This was made possible through the *Pedagogical Residency Programme* (PRP) of Brazil's National Policy for Teacher Education aimed at integrating theory and practice by immersing preservice teachers in compulsory education schools, the same programme that hosted the pilot SRP-TE in the Federal University of Sergipe. The PRP's structure, which includes classroom management and pedagogical interventions, guided by experienced schoolteachers and university lecturers, presents an optimal setting for implementing SRP-TE (details about the PRP were presented in the subsection 4.2.3.1). By placing the programme within initial teacher education for future mathematics secondary school teachers, it was made visible the epistemological division that exists between school and university mathematics, fostering meaningful teacher professional development.

Let us recall the *scale of levels of didactic codeterminacy* (Chevallard, 2002) that is used to locate the different types of institutional conditions and constraints that affect instructional processes (Figure 6.1, see also Chapter 2, section 2.1.2).

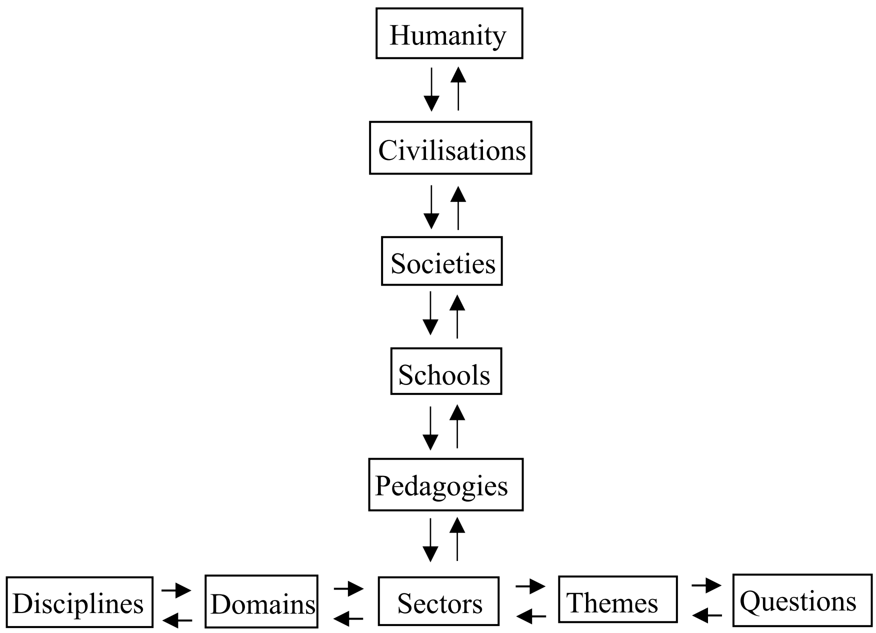


Figure 6.1. Scale of levels of didactic codeterminacy (adapted from Chevallard, 2002)

At the higher levels of the scale (*Humanity* and *Civilisation*), we find constraints that affect any kind of society organisation. These kinds of constraints are usually the most difficult to identify because they underlie our deepest beliefs and behaviours. Regarding statistics, Wozniak (2005, pp.188-197), considering the study of Hecht (1977) points to a constraint that can be located at the civilisational level. She mentions different passages of the Old Testament where David proceeds to count different sets of people, troops, herds, etc., sometimes under Yahveh's approval, but other times disobeying orders or being impelled by the Evil itself. The curse of counting will live long in the collective memory, affecting the study and "quantification" of state affairs. Constraints at the level of *Societies* can be visible when changing from one country to another and observing the differences in the corresponding educational systems. The level of the *Schools* includes all "the infrastructures provided by educational institutions to organise didactic systems and help them run: organisations of groups of teachers and students, grades structures, physical and virtual spaces, time schedules, final exam obligations, access to knowledge resources and experts, accreditations, etc." (Bosch, 2018, p. 4003). *Pedagogies* encompass all the tools employed by teachers and students during teaching and learning processes and that are not specific to the subject matter at stake. The promoted change from a "teacher-centred" to a "student-centred" pedagogy corresponds to this level, as is the focus on competencies and skills to complete study processes only focused on knowledge organisations.

The lower levels are the specific levels of the scale, linked to a given *discipline* or body of knowledge. They typically appear closer to the teachers' margin of action, even if they also correspond to long-term constructions of knowledge organisations following a complex process of didactic transposition (Chevallard, 1985). In this context, teachers usually elaborate at the level of the themes but may have little control or autonomy over more structured content blocks or discipline-level constraints. Additionally, they are subject to conditions and constraints imposed by higher levels of the scale, which affect how they organise disciplinary studies within different educational contexts.

Addressing the *ecological problem* through the study and research paths for teacher education (SRP-TE) involves addressing the institutional constraints with the teacher-students by experiencing with them teaching and learning processes that move forward the tradition of their schools. Hence, SRP-TE aim to help teachers identify institutional constraints at different levels of the scale by promoting the design of inquiry proposals, experience the limitations found when implementing them in real school contexts and providing tools to analyse the enacted instructional processes. This approach does not only enhance teachers' professional development but also contributes to a more dynamic and responsive educational system.

This chapter aims to examine and categorise the conditions and constraints encountered by preservice secondary school teachers in Brazil during an SRP-TE about the teaching and learning of statistics. The SRP-TE is analysed in terms of its *chronogenesis* (tracking the temporal evolution of the Modules, including the questions addressed and partial answers elaborated), its *mesogenesis* (examining how the learning environment and resources are structured), and *topogenesis* (studying the distribution of roles, determining how responsibilities shift between educators and future teachers).

The study is guided by the following research questions and specific objectives:

RQ_4 related to the implementation of an SRP-TE for preservice teachers: What conditions implemented by a face-to-face SRP-TE can help preservice teachers address the phenomenon of the invisibility of data treatment and what constraints appear to limit it?

SO_B: Study the SRP-TE ecology through the design, implementation, analysis, and development of different modalities of SRPs-TE for preservice secondary school teacher education in the field of probability and statistics.

RQ_5 related to the ecology of SRP-TE and the dissemination of didactic research tools in teacher education: What collaboration mechanisms between teachers, researchers and educational institutions? What didactic tools would facilitate the dissemination of SRP-TE and research in statistics preservice and in-service secondary teacher education?

SO_D: Identify the general conditions that favour and the constraints that hinder the implementation of SRP-TE, comparing preservice and in-service modalities.

As said before, in this study, we address these research questions and specific objectives through a case study based on a TE proposal for statistics we designed and implemented in a project developed with preservice teachers in the Pedagogical Residency Programme (PRP) in Brazil. The proposal pursues three main aims. First, getting preservice teachers involved in inquiring into a question that requires the collection and management of quantitative data. Second, providing teachers with design tools to adapt and implement a similar activity in secondary schools. Third, helping preservice teachers analyse the ecology of the teaching proposals and approach instructional resources more critically and productively, especially in what concerns the treatment of quantitative data.

6.2. *A PRIORI* ANALYSIS: AN SRP-TE IMPLEMENTED IN THE PEDAGOGICAL RESIDENCY PROGRAMME

The SRP-TE was implemented at the Federal University of Mato Grosso do Sul as a project linked to the PRP, involving sixteen pedagogical residents (secondary school preservice teachers) and one secondary in-service teacher. The project was entitled "Articulations between theoretical, pedagogical, and didactical aspects in the teaching of statistics from the perspective of questioning the world" and was conducted over sixteen 90-minute sessions under the guidance of the first author of this study (under the role of the educator/instructor). We used a session structure very similar to what was designed and implemented online for in-service teachers (Chapter V, Table 5.1.1). The main difference is that, in the case of the pedagogical residents, they were linked to four different schools, so we preferred to organise them into working groups from the same school. Consequently, they were organised into working groups according to their respective schools, which facilitated the implementation phase (M3) when they introduced the SRPs they had designed in their specific educational contexts.

Hence, the SRP-TE implemented for in-service teachers in the online modality served as an *a priori* analysis to the face-to-face SRP-TE implemented in the UFMS for pedagogical residents. The redesign and main adaptations of the SRP-TE organisation sessions are detailed in Table 6.1., which summariee information concerning the sessions, modules, tasks undertaken, and the specific roles assumed in each stage of the process by the participants and the educators.

Table 6.1. Face-to-face SRP-TE sessions organisation and SR

Sessions/ Modules	Description of the work	Role of participants	Role of educators
1st: 17/3/23 **M0**	General introduction: aims, chronogram, technological devices, group dynamics. Didactic concepts: change of paradigms and the modules of the SRP-TE.	Student	Lecturer
	Task 1: Word cloud in *Mentimeter*. "Write five words that you associate with statistics". Task 2: "Raise questions about statistics teaching in a shared *Padlet*".	Teacher	Guide
2nd: 24/3/23 **M1**	General discussion about tasks 1 and 2. Introduction of the Q_0 and the start of the SRP about water resources, territory, and Brazilian population. First derived questions and data searching. Didactic tools: questions-answers maps in a *Padlet* for each group. Proposal of report written by groups: questions raised, issues addressed, outstanding issues, new questions, answers found, sources consulted.	Student	Guide
3rd: 31/3/23 **M1**	Inquiry about Q_0: working with databases. New derived questions. Continuing the report.	Student	Guide
4th: 14/4/23 **M1**	Groups presentation of the final answer $A^{\heartsuit}$ to Q_0.	Student	"Reactor"
5th: 21/4/23 **M1–M2**	Finalisation of the written reports about M1.	Student	Guide
6th: 28/4/23 **M2**	Educator's intervention (first author): Presentation of statistics concepts by the educator: types of variables, mean elements of a bar graph, characteristics of a table, types of graphs, and measures of central tendency. Comments about errors found in the graphs constructed during the SRP (M1)	Student	Lecturer
7th: 5/5/23 **M2**	Groups' analysis about the learning process. Didactic tools: *Herbartian scheme* and dialectics: *questions-answers*, *media-milieu*, *individual-collective*.	Analyst	Guide

Sessions/ Modules	Description of the work	Role of participants	Role of educators
8th: 12/5/23 **M2–M3**	Educator's intervention (first author): Discussion about the transparency of data treatment in the analysis presented by the groups in the reports: addressing their description about "statistical knowledge" (M2) with the word cloud (M0), data treatment in the curriculum, statistical modelling.	Student	Lecturer
	Analysis if the SRP lived in M1	Analyst	Guide
	Participants started to think about the adaptations for secondary schools' conditions	Teacher designer	
9th: 19/5/23 **M3**	Design and adaptations for secondary schools' conditions	Teacher designer	Guide
10th: 26/5/23 **M3**	Design and adaptations for secondary schools' conditions	Teacher designer	Guide
11th: 2/6/23 **M3**	Implementations in secondary schools	Teacher	Lecturer and "Reactor"
12th: 9/6/23 **M3**	Implementations in secondary schools	Teacher	Lecturer and "Reactor"
13th: 16/6/23 **M3**	Implementations in secondary schools	Teacher	Lecturer and "Reactor"
14th: 23/6/23 **M3–M4**	Implementations in secondary schools Discussion about groups' *a posteriori* analysis	Teacher and Analyst	Lecturer and "Reactor"
15th: 30/6/23 **M4**	Discussion about groups' *a posteriori* analysis	Analyst	"Reactor"
16th: 7/7/23 **M4**	Final report on *a posteriori* analysis	Analyst	"Reactor"

The structure and duration of the SRP-TE sessions present some important differences in comparison to the online and face-to-face modalities. The online SRP-TE (which the sessions organisation was explained in the Table 5.1.1, in Chapter 5) consists of 14 sessions, each lasting three hours, following a structured progression from M0 to M4. This format allows participants to engage in group work, data exploration, and discussions, culminating in the individual implementation of SRPs and the submission of final reports in M4. In contrast, the face-to-face SRP-TE (described in the previous Table 6.1) extends over 16 sessions, offering a more immersive experience aligned with the pedagogical residency programme. The extended timeframe provides opportunities for deeper engagement with concepts, iterative discussions, and direct classroom implementations, allowing preservice teachers to gain hands-on experience in their teaching practice.

The implementation and reflective phases further highlight distinctions between the two models. In the online SRP-TE, implementation is primarily conducted by selected teachers who later share their experiences in M3 and M4, maintaining a focus on collaborative development and discussion. In contrast, the face-to-face SRP-TE requires all participants to implement their SRPs in real secondary school settings over multiple sessions, fostering richer data collection and stronger practical engagement. Additionally, the reflective phase in the online model is more collective, with educators guiding discussions on challenges and solutions in a general forum. Meanwhile, the face-to-face SRP-TE integrates a structured and iterative reflective process, linking discussions to prior tasks such as word clouds, curriculum analysis, and classroom implementation reviews. The cyclical transition from analyst to teacher and back to analyst strengthens professional development, providing a dynamic model of inquiry.

6.3. *IN VIVO* ANALYSIS

6.3.1. Modules 0: Professional issues about statistics teaching

In Module 0, two initial activities and a general discussion were proposed to address the question of *how to teach statistics at the secondary school level*. Our aim with this first approach to the initial question was to identify the participants' conceptions of statistics and their needs concerning teaching this subject in secondary education. First, the educator requested the participants to write five words associated with statistics on the *Mentimeter* platform, which led to the word cloud of Figure 6.2. The word cloud visually represents the frequency and prominence of the responses, with larger words indicating terms that were mentioned more frequently by the participants.

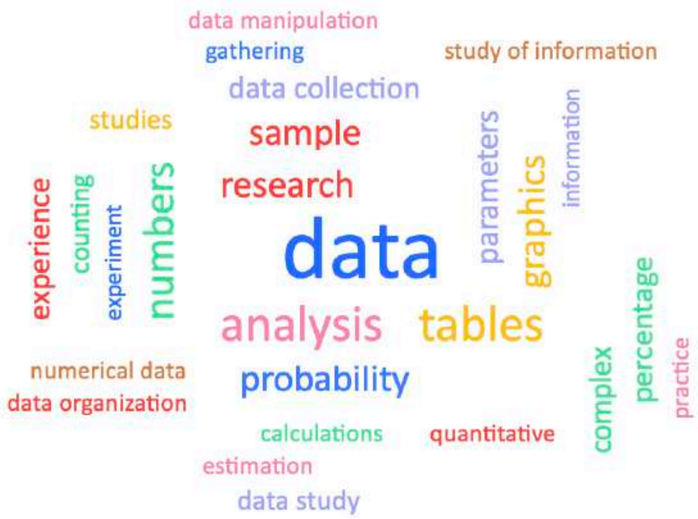


Figure 6.2: Word cloud about "statistics" made by the teacher-students in the first session

A notable observation from Figure 6.2 is the emphasis on terms such as *data*, *analysis*, *probability*, *research*, *sample*, and *numbers*, which suggests that participants primarily perceive statistics as a field centred on numerical data manipulation and analysis. The recurrence of words like *tables*, *graphics*, and *parameters* indicates an awareness of statistical representation tools, though the inclusion of *calculations* and *percentage* suggests a potential reduction of statistical work to arithmetic operations, aligning with concerns about the "arithmetisation" of statistics in school curricula (Chevallard & Wozniak, 2007). Furthermore, terms such as *study of information*, *data organisation*, and *quantitative* reflect an emerging recognition of statistics as an investigative discipline, though the limited presence of terms related to critical thinking or decision-making highlights areas for further conceptual development.

This initial activity highlights the influence of traditional approaches to statistics education, where procedural and computational aspects are often prioritised over inquiry-based and interpretative dimensions. The prevalence of terms related to data processing, yet the relative absence of expressions linked to statistical reasoning, variability, or context-driven analysis, suggests that participants may still hold a narrow view of statistics, rooted in formulaic procedures rather than exploratory inquiry.

The second activity, proposed by the educator, was to raise questions related to the teaching of statistics in secondary schools. Figure 6.3 presents a categorised collection of questions raised by teacher-students in response to the educator's prompt regarding the teaching of statistics in secondary schools. This activity aimed to elicit participants' concerns, challenges, and areas of interest related to statistics education, thereby revealing their perspectives on both pedagogical and didactic aspects of the subject. The Figure 6.3 depicts the columns of a Padlet activity, each column having been preconfigured by the educator into four principal question-categories: Statistics in lower secondary school, Statistics in upper secondary school, Pedagogical questions, Didactical questions, and General statistics questions Participants were then invited to contribute their own queries and to place each question in the column they judged most appropriate. Through this collaborative exercise, the Padlet not only visualises the breadth of participant concerns but also reveals points of intersection and divergence across these thematic dimensions.

Figure 6.3: Word cloud about "statistics" made by the teacher-students in the first session

A key observation from Figure 6.3 is the predominance of pedagogical questions, which reflect participants' concerns about making statistical content more engaging and accessible for students. Questions such as "*How can statistics be taught to bridge the knowledge gap among public school students?*" and "*How do we approach statistics in an interdisciplinary way?*" indicate an awareness of the difficulties students face in grasping statistical concepts and the need for cross-curricular connections. The recurrence of inquiries regarding teaching strategies and student motivation highlights the pressing need to develop methodologies that foster meaningful learning experiences in statistics education.

In contrast, didactic questions appeared less frequently, suggesting that participants may not yet have developed a critical perspective on how statistical knowledge is structured and transmitted within the educational system. The few didactic questions present, such as "*Why is there a greater focus on data treatment?*" and "*Are activities involving modelling easier for the construction of statistical knowledge than lessons involving 'only' calculations?*", indicate a nascent awareness of curricular design. The relative scarcity of didactic concerns aligns with findings from previous studies indicating that teacher education often emphasises instructional techniques over deep reflections on the nature and function of disciplinary knowledge.

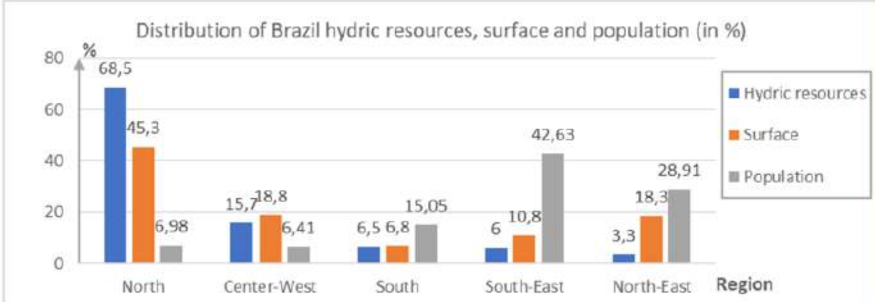
Additionally, the presence of general statistics questions, or questions related to the domain of statistics, such as "*What is a population?*" and "*What is dispersion?*", suggests that some participants may still be grappling with fundamental statistical concepts. This highlights a potential gap in their own statistical literacy, reinforcing the need for professional development initiatives such as the SRP-TE to provide educators with both conceptual and didactic tools.

The limited number of questions addressing disciplinary structures or statistical reasoning further suggests that the participants' focus remains on the procedural aspects of teaching statistics rather than on fostering inquiry-based learning approaches. The diverse range of questions collected in this activity highlights the complexities of teaching statistics at the secondary level and underscores the importance of integrating inquiry-based approaches to foster deeper engagement and understanding. Overall, this second activity provided valuable insights into the participants' conceptions of statistics and their perceived needs for teaching it at the secondary level. The predominance of pedagogical concerns over didactic ones suggests that initial teacher education programmes may need to place greater emphasis on developing preservice teachers' awareness of didactic questions and the necessary didactic knowledge to bridge the gap between pedagogical concerns and the epistemological nature of statistical knowledge.

6.3.2. Modules 1-2: The experience and analysis of an SRP on the distribution of water resources in Brazil

In Module 1, we proposed an SRP starting with the textbook exercise about water distribution that was used in the pilot study (Chapter IV, sections 4.1 and 4.2). The generating question was presented as follows in Figure 6.4.

Brazil has about 13.7% of the total fresh water supply in the world, being considered a territory rich in terms of water. However, the country is experiencing serious problems related to both the degradation of water quality, especially in the vicinity of urban areas, and the lack of control of excess and shortage of water, which affect several Brazilian locations. Not just floods affect Brazilian cities, water scarcity also imposes serious restrictions and high costs on economic and social development of large cities. Looking at the chart below, answer in your notebook:



Information obtained from: Ministério do Meio Ambiente <http://wwww.mma.gov.br/estruturas/sedr_proecotur/_publicaco/140_publicacao09062009025910.pdf>. Accessed: 1st July 2018.

Q_0: Considering the graph presented in the textbook exercise, what questions can we raise about the water problem in Brazil? Is the graph still valid? Will anything change if we update the data on population, territory/surface area, and water resources?

Figure 6.4: Generating question proposed in Module 1

The participants, working in four groups of 4-5 students, were asked to raise derived questions Q_i from the initial generating Q_0 and to use any tool they considered relevant to find or elaborate answers to Q_i and Q_0. One tool provided by the instructor was a *Padlet* so that each group could create a questions-answers map throughout the inquiry work. The Q-A map elaborated by each group can be seen in Figure 6.5, divided into four parts (the Q-A map of the four groups together can also be accessed here: https://padlet.com/janiellyverbisck1/groups-1-4-srp-in-m1-9s9q21rsz3k1kxve).

Group 1

Brazil's water basins

Graphic update

Is the resource distribution graph (textbook exercise) up-to-date after so many years, or has it not been updated since the last data was made available in 2018?

Population data for 2022

What is the Brazilian population by region?
Where can we find the most up-to-date database on this?

Source: https://www.ibge.gov.br/estatisticas/sociais/populacao/22827-censo-demografico-2022.html?=&t=resultados

Source: https://www.ibge.gov.br/estatisticas/sociais/populacao/22827-censo-demografico-2022.html?=&t=resultados

2022 surface data

What is Brazil's surface area by region?
Where can we find the most up-to-date database on this?

Source

ibge.gov.br

15761-areas-dos-municipios

And has the transposition of the São Francisco River changed this scenario?

Open question (not answered).

What was the distribution like in Brazil in the years before this survey?

Transposition Map

Water resources data for 2022

How much water is there in Brazil by region?
Where can we find the dataset with the most up-to-date data on this?

Distribution

Compared to the rest of the world, can Brazil present an unequal distribution of water in its regions?

Open question (not answered).

Generating Question

Brazil has about 13.7% of the total fresh water in the world, being considered a territory rich in water terms. However, the country is experiencing serious problems related to both the degradation of water quality, especially in the urban areas' proximity, and the lack of control of excess and lack of water, which affect several Brazilian locations. Not just the floods affect Brazilian cities: water scarcity also imposes serious restrictions and high costs on economic and social development of large cities. Looking at the chart below, answer in your notebook:

Information obtained in: Ministério do Meio Ambiente <http://www.mma.gov.br/estruturas/sedr_proecotur/_publicacao/140_publicacao09062009025910.pdf>. Accessed: 1st July 2018.

Q_0: Considering the graph presented in the textbook exercise, what questions can we raise about the water problem in Brazil? Is the graph still valid? Will anything change if we update the data on population, territory/surface area, and water resources?

Source

brasil.mapbiomas.org

MapBiomas Brasil

Comparison

What was the percentage of water resources, population, and surface area in the south before the deforestation of the Atlantic rainforest grew dramatically?

Open question (not answered).

"Brazil has around 13.7% of the world's fresh water, making it a water-rich country."
Is 13.7% a lot considering Brazil's territorial extension in relation to the world?

Do other countries have large amounts of fresh water?

Open question (not answered).

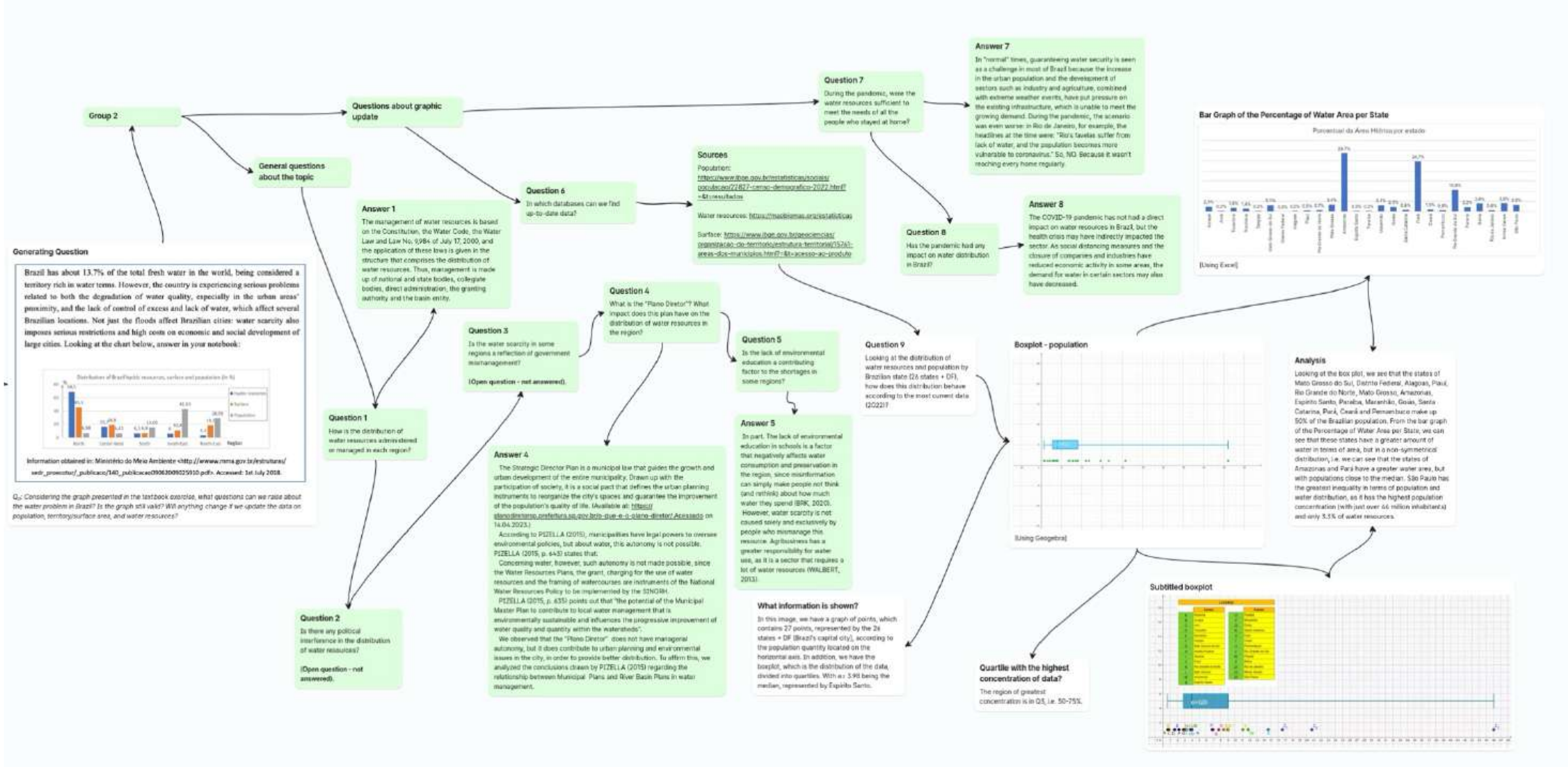
How do climatic conditions or the type of vegetation relate to the population? Example: The North has the Amazon, the largest area in Brazil, but the second smallest population. The South, on the other hand, has the smallest surface area, but more than twice the population of the North.

Open question (not answered).

The group met to gather information to search for the tables, then we organized these tables and made an updated graph according to the most up-to-date tables. [Using Excel]

Informações obtidas em: MINISTÉRIO DO MEIO AMBIENTE. Disponível em: <http://www.mma.gov.br/estruturas/sedr_proecotur/_publicacao/140_publicacao09062009025910.pdf>. Acesso em: 1º jul. 2018.

We can see from the updated graph that the Northeast grew its distribution of water resources by almost 7%, the North had no significant change in either population or distribution of water resources, while the Midwest had a 2% increase in population and a drop of almost half in water resources, from 15.7% to 8.65%. The South, on the other hand, more than doubled its distribution of water resources, from 65% to 13.2%, while its population did not undergo any significant adjustment. The Southeast remained stable across all three labels, as did the land area in all Brazilian regions.



Group 2
Questions about graphic update
General questions about the topic
Generating Question
Brazil has about 13.7% of the total fresh water in the world, being considered a territory rich in water terms. However, the country is experiencing serious problems related to both the degradation of water quality, especially in the urban areas' proximity, and the lack of control of excess and lack of water, which affect several Brazilian locations. Not just the floods affect Brazilian cities: water scarcity also imposes serious restrictions and high costs on economic and social development of large cities. Looking at the chart below, answer in your notebook:
Information obtained in: Ministério do Meio Ambiente <http://www.mma.gov.br/estruturas/sedr_proecotur/_publicacao/140_publicacao09062009025910.pdf>. Accessed: 1st July 2018.
Q₀: Considering the graph presented in the textbook exercise, what questions can we raise about the water problem in Brazil? Is the graph still valid? Will anything change if we update the data on population, territory/surface area, and water resources?
Answer 1
The management of water resources is based on the Constitution, the Water Code, the Water Law and Law No. 9,984 of July 17, 2000, and the application of these laws is given in the structure that comprises the distribution of water resources. Thus, management is made up of national and state bodies, collegiate bodies, direct administration, the granting authority and the basin entity.
Question 1
How is the distribution of water resources administered or managed in each region?
Question 2
Is there any political interference in the distribution of water resources?
(Open question - not answered).
Question 3
Is the water scarcity in some regions a reflection of government mismanagement?
(Open question - not answered).
Question 6
In which databases can we find up-to-date data?
Sources
Population: https://www.ibge.gov.br/estatisticas/sociais/populacao/22827-censo-demografico-2022.html?=&t=resultados
Water resources: https://mapbiomas.org/estatisticas
Surface: https://www.ibge.gov.br/geociencias/organizacao-do-territorio/estrutura-territorial/15761-areas-dos-municipios.html?=&t=acesso-ao-produto
Question 4
What is the "Plano Diretor"? What impact does this plan have on the distribution of water resources in the region?
Answer 4
The Strategic Director Plan is a municipal law that guides the growth and urban development of the entire municipality. Drawn up with the participation of society, it is a social pact that defines the urban planning instruments to reorganize the city's spaces and guarantee the improvement of the population's quality of life. (Available at: https://planodiretorsp.prefeitura.sp.gov.br/o-que-e-o-plano-diretor/ Acessado on 14.04.2023.)
According to PIZELLA (2015), municipalities have legal powers to oversee environmental policies, but about water, this autonomy is not possible. PIZELLA (2015, p. 643) states that:
Concerning water, however, such autonomy is not made possible, since the Water Resources Plans, the grant, charging for the use of water resources and the framing of watercourses are instruments of the National Water Resources Policy to be implemented by the SINGRH.
PIZELLA (2015, p. 635) points out that "the potential of the Municipal Master Plan to contribute to local water management that is environmentally sustainable and influences the progressive improvement of water quality and quantity within the watersheds".
We observed that the "Plano Diretor" does not have managerial autonomy, but it does contribute to urban planning and environmental issues in the city, in order to provide better distribution. To affirm this, we analyzed the conclusions drawn by PIZELLA (2015) regarding the relationship between Municipal Plans and River Basin Plans in water management.
Question 5
Is the lack of environmental education a contributing factor to the shortages in some regions?
Answer 5
In part. The lack of environmental education in schools is a factor that negatively affects water consumption and preservation in the region, since misinformation can simply make people not think (and rethink) about how much water they spend (BRK, 2020).
However, water scarcity is not caused solely and exclusively by people who mismanage this resource. Agribusiness has a greater responsibility for water use, as it is a sector that requires a lot of water resources (WALBERT, 2013).
Question 7
During the pandemic, were the water resources sufficient to meet the needs of all the people who stayed at home?
Answer 7
In "normal" times, guaranteeing water security is seen as a challenge in most of Brazil because the increase in the urban population and the development of sectors such as industry and agriculture, combined with extreme weather events, have put pressure on the existing infrastructure, which is unable to meet the growing demand. During the pandemic, the scenario was even worse: in Rio de Janeiro, for example, the headlines at the time were: "Rio's favelas suffer from lack of water, and the population becomes more vulnerable to coronavirus." So, NO. Because it wasn't reaching every home regularly.
Question 8
Has the pandemic had any impact on water distribution in Brazil?
Answer 8
The COVID-19 pandemic has not had a direct impact on water resources in Brazil, but the health crisis may have indirectly impacted the sector. As social distancing measures and the closure of companies and industries have reduced economic activity in some areas, the demand for water in certain sectors may also have decreased.
Question 9
Looking at the distribution of water resources and population by Brazilian state (26 states + DF), how does this distribution behave according to the most current data (2022)?
Bar Graph of the Percentage of Water Area per State
[Using Excel]
Boxplot - population
[Using Geogebra]
Analysis
Looking at the box plot, we see that the states of Mato Grosso do Sul, Distrito Federal, Alagoas, Piauí, Rio Grande do Norte, Mato Grosso, Amazonas, Espírito Santo, Paraíba, Maranhão, Goiás, Santa Catarina, Pará, Ceará and Pernambuco make up 50% of the Brazilian population. From the bar graph of the Percentage of Water Area per State, we can see that these states have a greater amount of water in terms of area, but in a non-symmetrical distribution, i.e. we can see that the states of Amazonas and Pará have a greater water area, but with populations close to the median. São Paulo has the greatest inequality in terms of population and water distribution, as it has the highest population concentration (with just over 46 million inhabitants) and only 3.3% of water resources.
What information is shown?
In this image, we have a graph of points, which contains 27 points, represented by the 26 states + DF (Brazil's capital city), according to the population quantity located on the horizontal axis. In addition, we have the boxplot, which is the distribution of the data, divided into quartiles. With a≈ 3.98 being the median, represented by Espírito Santo.
Quartile with the highest concentration of data?
The region of greatest concentration is in Q3, i.e. 50-75%.
Subtitled boxplot

Group 3

ibge.gov.br

Censo 2022 | IBGE

According to the Brazilian Institute of Geography and Statistics (IBGE), the Brazilian population in 2022 was approximately 207 million people.
Here is the Brazilian population by region, according to the IBGE in 2022:
North Region: 17,834,762 million people
Northeast Region: 55,389,382 million people
Center-West Region: 16,492,326 million people
Southeast: 87,348,223 million people
South Region: 30,685,598 million people

Question 9
What is Brazil's population (number of inhabitants)? What is the population (number of inhabitants) by region? Where can we find the most up-to-date database on this?

Question 8
Looking at the distribution of population by Brazilian state (26 states + DF (capital city)), how does this distribution behave according to the most current data (2022)?

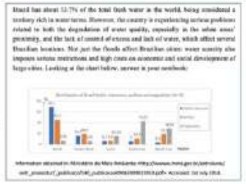
Question 7
Could the problem in the north and northeast be natural scarcity or poor distribution?

Open question (not answered).

Question 6
How is the distribution of water in the different regions of the country?

Open question (not answered).

Generating question



Q₃: Considering the graph presented in the textbook exercise, what questions can we raise about the water problem in Brazil? Is the graph still valid? Will anything change if we update the data on population, territory/ surface area, and water resources?

Question 5
How is the water quality in the different regions of the country?

Open question (not answered).

Question 4
Are there alternatives to improve water management and/or distribution?

Open question (not answered).

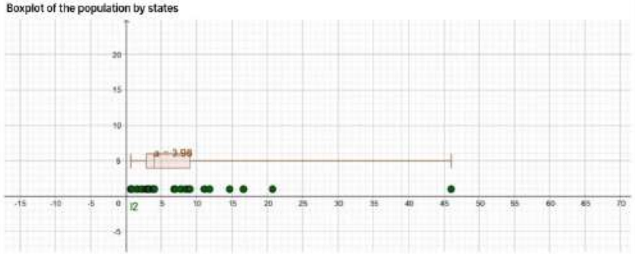
Question 3
How does water resource management work?

Open question (not answered).

Question 1
Is there a water quality control and/or monitoring system?

Question 2
How can climate change affect the availability and quality of water in Brazil?

Boxplot of the population by states



[Using Geogebra]

What information is shown?
We have a point graph with 27 points, which are the Brazilian states according to population, located on the horizontal axis. The boxplot also shows the distribution of the population in the Brazilian states. The red line inside the box represents the median, which is Espírito Santo. The data in the fourth quartile shows the states that have a much higher population than the states in the second and third quartiles, or the first quartile.

How to interpret the cash chart?
It's a graph that sorts the data into four parts (quartiles):
1st quartile: From the start of the data collected to the start of the box diagram.
2nd quartile: From the beginning of the box diagram to the median.
3rd quartile: From the median to the end of the box diagram.
4th quartile: From the end of the box diagram to the last data collected.

Answer 1

sisagua.saude.gov.br

página interna

The National Water Quality Surveillance Program for Human Consumption (VIGIAGUA) consists of a set of actions adopted continuously by public health authorities to guarantee the population access to water in sufficient quantity and of quality compatible with the potability standard established in current legislation as an integral part of actions to prevent waterborne diseases and promote health, as provided for in the Unified Health System (SUS). Vigiagua's actions are developed by the Municipal, State, and Federal District Health Departments and by the Ministry of Health through the General Coordination of Environmental Health Surveillance.

Answer 2

gov.br

Mudanças climáticas e recursos hídricos

The water cycle is directly linked to the climate. Thus, changes in the climate that alter the rainfall regime can lead to an increase in the occurrence of extreme hydrological events, such as floods and long periods of drought. These events affect the supply of water, threatening the supply of water resources for all.

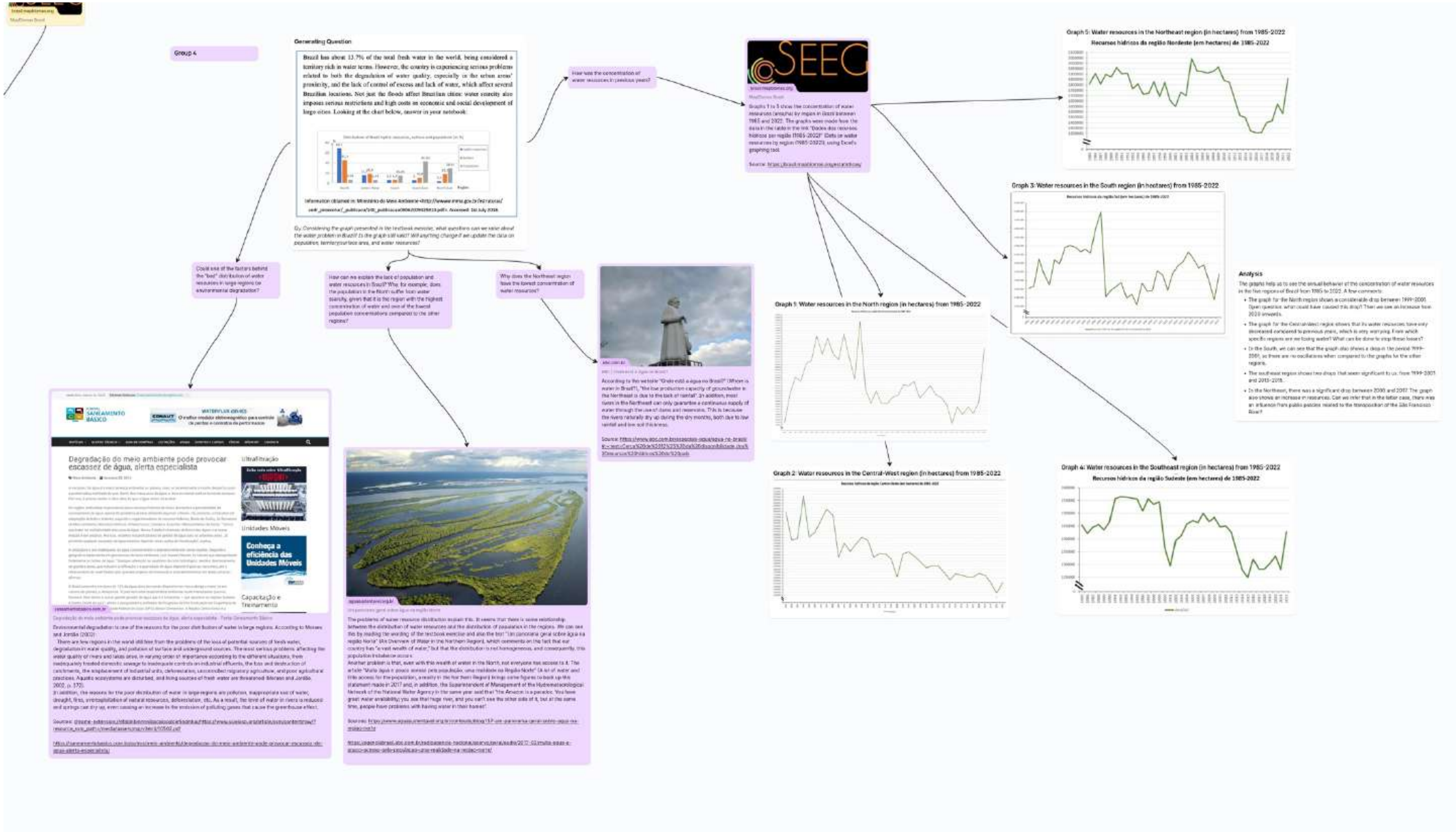


Figure 6.5: The Q-A maps produced by each working group in Module 1

The questions-answers maps generated in the Module 1 of the SRP-TE reveal the technological tools and resources that preservice teachers utilised to study the inquiry. These maps serve as structured visual representations of the inquiry process, illustrating how participants formulated their derived questions Q_i from Q_0, explored various sources for answers (A_j), and constructed their final responses ($A^{\heartsuit}$). One of the key observations from these maps is the extensive use of online databases and governmental sources for accessing statistical data. Participants frequently consulted platforms such as Brazilian Institute of Geography and Statistics (IBGE), MapBiomas (https://brasil.mapbiomas.org/estatisticas/), among others, to retrieve official datasets on water resources, population distributions, and environmental statistics. These sources provided real-world datasets that enabled the preservice teachers to explore statistical modelling in an applied context.

As evidenced by the Q-A maps from each group, not all derived questions generated during the sessions could be fully addressed. This outcome was influenced by the limited duration of Module 1, which spanned three and a half sessions. The decision regarding the number of sessions was determined by the educator. The primary objective of this module was to provide preservice teachers with an opportunity to engage with an inquiry-based approach, beginning with a generating question and subsequently identifying the need to explore statistical concepts to address the derived questions that emerged. Throughout these sessions, the educator guided the groups to prioritise specific questions for deeper study. Notably, the following questions were emphasised:

- Group 1 – $Q_{\text{Graphic update}}$: Is the resource distribution graph (textbook exercise) up-to-date after so many years, or has it not been updated since the last data was made available in 2018?
- Group 2 – Q_9: Looking at the distribution of water resources and population by Brazilian state (26 states + DF [capital city]), how does this distribution behave according to the most current data (2022)?
- Group 3 – Q_8: Looking at the distribution of population by Brazilian state (26 states + DF [capital city]), how does this distribution behave according to the most current data (2022)?
- Group 4 – How was the concentration of water resources in previous years?

The exploration of these questions was explicitly encouraged by the educator to enable participants to formulate answers grounded in statistical studies, when deciding on the questions that seemed more “studyable” than others, reinforcing the idea of statistical investigative questions proposed by (Arnold & Franklin, 2021). To support these investigations, the teacher also recommended the integration of various technological tools. A notable technological resource evident in the question-answer maps was the utilisation of data analysis software and digital spreadsheets. Groups 1, 2, and 4 employed Microsoft Excel and Google Sheets for organising, cleaning, and processing datasets. These tools facilitated the creation of statistical graphical representations, including bar graphs and line charts, thereby enhancing participants' ability to visualise and interpret data. Additionally, Groups 2 and 3 incorporated GeoGebra to generate scatterplots and boxplots, effectively illustrating the variability of the Brazilian population across different regions. The use of these digital tools not only aided in the representation of statistical data but also reinforced the importance of employing technological resources to support data-driven inquiry in statistical education. Such applications align with contemporary pedagogical approaches that emphasise the role of technology in enhancing conceptual understanding and analytical reasoning. To ensure that participants could effectively use these software, the educator provided guidance to facilitate their familiarity with its functionalities. In addition, in order to offering direct support, the educator recommended a selection of explanatory videos from YouTube to supplement their understanding. Furthermore, the educator occasionally assisted participants in the graphical representation of data, ensuring they could accurately visualise and interpret statistical information.

Beyond software applications, the question-answer maps further reveal that preservice teachers engaged with a range of educational websites, scientific articles, and multimedia resources to develop responses to their derived questions. Several groups referenced academic publications, instructional videos, and open-access learning platforms to contextualise their findings. This engagement with diverse sources reflects an inquiry-based approach to learning, in which preservice teachers actively sought out and synthesised information from multiple perspectives. Such an approach is instrumental in fostering critical thinking and equipping future educators with the skills necessary to guide students in conducting their own statistical investigations.

The last session was used to share and present the inquiries carried out by each group. A researcher (the supervisor, Marilena Bittar) was invited to attend the presentations and provide comments and discussions about the groups and the work done.

Following the participants' presentation of their final answers during the SRP, the educator deemed it essential to intervene before the beginning of Module 2, where participants were expected to analyse the SRP they had just experienced. This intervention, led by the educator, focused on consolidating key statistical concepts to provide participants with a stronger theoretical foundation for their subsequent analyses. The educator's intervention comprised a structured presentation of fundamental statistical concepts, including types of variables, the principal components of a bar graph, the characteristics of statistical tables, different types of graphical representations, and measures of central tendency. To contextualise these concepts, the educator referenced examples from secondary school textbooks, highlighting how statistical content is typically presented in formal educational settings.

Furthermore, particular attention was given to errors identified in the graphs constructed during the SRP (in Module 1). The educator provided analytical feedback on these inaccuracies, fostering a reflective discussion among participants regarding common misconceptions and best practices in statistical representation. This phase of the intervention was designed to enhance participants' critical engagement with statistical content, equipping them with the necessary analytical tools to evaluate their own and their peers' work more effectively.

In Module 2, as implemented in the SRP-TE for in-service teachers in the online modality (a study presented in Chapter V), the educator introduced a set of didactic tools to support the groups in analysing the SRP they had experienced in Module 1. These tools were designed to facilitate a structured and reflective examination of the learning process, enabling participants to critically assess their engagement with the inquiry-based approach. The main didactic tools introduced included the *Herbartian Schema* and three key dialectical frameworks: the *questions-answers dialectic*, the *media-milieu dialectic*, and the *individual-collective dialectic*. These analytical instruments provided participants with a theoretical foundation to explore the interactions between knowledge construction, pedagogical mediation, and collaborative inquiry within the experienced SRP.

To make explicit their reflections, participants were required to produce a written report addressing the following questions:

Questions-Answers dialectic: Comment on the usefulness of this tool during the SRP.

- Positive points:
- Negative points:
- Difficulties encountered?

Media-milieu dialectic:

- What media were used during the SRP?
- What objects were used during the SRP?
- What statistical knowledge was used during the SRP?
- What data was used during the SRP?
- What studies could or need to be further developed?
- Difficulties encountered?

Individual-Collective dialectic: Comment on the organisation of the group during the SRP.

- Collective discussions?
- Division of tasks?
- Difficulties encountered?
- Could group work be improved? In what aspects?

To facilitate the completion of these questions, the educator provided each group with a Word document on Google Drive, enabling simultaneous collaboration. This document served as a central repository for their responses and was subsequently used to compile the comprehensive project report (available in Portuguese in Appendix D.1). In Module 2, the phenomenon of invisibility of data treatment appeared, as teacher-students did not have the impression of doing statistics until they were calculating measures or drawing graphs, considering the search and organisation of data a “loss of time” that can be saved for school students. For example, when they were asked, in Module 1, to describe the statistical knowledge that emerged during their inquiry process in the SRP, they described aspects such as:

> Group 1: analysing the graph, analysing data, organising this information, it is all part of statistics.
> Group 2: Sample: population, territory, and water resources. Median: used to analyse the box plot. Percentage: when interpreting the graph in the initial question. Boxplot and dot-plot: when constructing graphs.
> Group 3: Median used in the box diagram, sample, population, mean, and variables.
> Group 4: Proportion, ratio, amplitude, percentage.

In analysing the inquiry process, the educator conducted a general discussion on the invisibility of data treatment. This discussion encompassed the introduction of new terminology to refer, explicitly, to key aspects, such as searching for datasets, downloading, cleaning, organising, and representing data. Additionally, the educator highlighted an intriguing observation: a significantly greater amount of terminology related to “data” emerged during the first session (Figure 6.2) compared to the subsequent analysis of the SRP. This discrepancy led to the identification of the phenomenon of the invisibility of data treatment, which was linked to the predominant conception of statistics in secondary education. The discussion explored how this invisibility reflects a narrow interpretation of statistical knowledge, often limited to procedural and computational aspects rather than a comprehensive understanding of data handling and interpretation. Furthermore, the educator contextualised this analysis by aligning the discussion with the guidelines set out in the Brazilian National Common Curricular Base (BNCC) (Brasil, 2018), examining how curriculum prescriptions influence the way statistics is approached in secondary schools.

6.3.3. Modules 3-4: Design, implementation, and analysis of SRPs at secondary schools

In Module 3, the pedagogical residents had the opportunity to design or adapt an SRP to be implemented in the secondary schools where they were doing the internship. As we explained earlier, the four groups were organised according to the schools assigned during the Pedagogical Residency Program. This choice was made to facilitate the design of the SRP proposals to be implemented in these schools. Here is a summary of the main characteristics of their proposals:

Group 1 – One group of Grade 8 and three groups of Grade 9 - Six 50' sessions

Generating question: Socio-economic and socio-emotional questionnaire on "bullying".

Description: The students (13-14 years old) were interested in discussing the topic of "bullying" and, to work on it, the pedagogical residents prepared a questionnaire with 24 questions. The questionnaire was printed out and distributed to each student. Most of the questions were categorical and some open. As the answers were on paper, the students first had to carry out the task of collecting the papers, gathering the answers, and organising them. With the data gathered, during the sessions a collective analysis was carried out on a selection of the questions. In each classroom, the students were divided into groups and each group was responsible for organising the answers from the paper into tables of relative and absolute frequencies, representing them graphically and analysing them.

Questions selected to be analysed:

Q_2: How old are you? Q_5: Do you have internet access at home? $Q_{5.1}$: If yes, where does your internet come from? Q_6: How do you access the internet? Q_7: How many people live with you? Q_9: What type of home do you live in? Q_{10}: What means of transportation do you use to get to school? Q_{11}: What is your main source of entertainment outside of school? Q_{12}: Have you ever been bullied? $Q_{12.1}$: What kind of bullying have you suffered? Q_{14}: In which environment have you been bullied? Q_{15}: Have you ever witnessed someone bullying? $Q_{15.1}$: Whoever witnessed the bullying, what did they do about it? Q_{16}: Have you ever witnessed someone being bullied? $Q_{16.1}$: If so, what was your attitude towards it? Q_{17}: Have you ever mutilated yourself? Q_{18}: Do you feel comfortable in class? *Group 2 – One group with students from grades 10, 11 and 12 (multi-grade) - Six 50' sessions*

Generating question: How much is "1 real" [the currency of the Brazilian monetary system]? Have you ever stopped to think how much the Brazilian real is worth concerning other world currencies?

Description: The pedagogical residents had to work on the topic of the "value of the Real" as a curricular demand in an elective subject called "curricular unit", that was offered to students of grades 10 to 12. The proposal aimed to study "the exchange rate and purchasing power parity using graphical and statistical resources". They organised the students by groups and proposed the generating question and, as case studies, the groups collected petrol prices from different countries and continents on the internet using their mobile phone, converted them to the "real", and made tabular and graphical representations to discuss the valuation or devaluation of petrol. To finish the inquiry, they organised a collective discussion.

Main questions that appear during the inquiry: What is inflation? What factors do you think influence the value of the Real? Name three currencies you know. How is the Real exchange rate calculated? What does this formula $E = e \cdot \frac{p1}{p2}$ mean? (E is the real exchange rate; e is the nominal exchange rate; p_1 is the product value in country 1; and p_2 is the product value in country 2) How can you compare the price of petrol if in some countries it is measured in "litters" and in others it is measured in "gallons"?

Group 3 – One group with students from grade 6 - Six 50' sessions

Generating question: Simulating a survey: draw up a question to be polled in the classroom.

Description: As the pedagogical residents would be implementing a proposal in a school with few technological resources, the preservice teachers, together with the schoolteacher, decided to create a survey within the classroom, with the students themselves collecting the answers from their classmates. So, the students (11-12 years old) experienced the elaboration of questions (one question by group), the collecting of answers from their classmates, and the organisation and representation

> of the data in tables and bar graphs as a "small opinion poll". The students prioritised tables as the main learning in statistics, and all the work of collecting and organising data did not appear in the discussions. Examples of questions of the survey are: Which of these modes of transport do you prefer? Which of these soccer players is your favourite? Which of these sports do you prefer? What month of the year is your birthday? How old are you?

While pedagogical residents were implementing their teaching proposal, four sessions of the teacher education course were allocated to discuss the carrying out of the classroom inquiries. They described the sessions with the students, identified some difficulties encountered in classroom management and discussed them with the educator and the other residents. Decisions were then made about the next steps to follow and the possible adaptations from the initial designs. The case of the implementation of Group 4 at the school appeared as an illustrative example of the curriculum constraint in the paradigm of visiting works. When the pedagogical residents had to implement their proposal, the planned topic at school was "functions" and the schoolteacher did not allow it to be changed for an inquiry activity in statistics.

In Module 4, the pedagogical residents presented their implementation to the educator and the mathematics education researcher (Marilena Bittar). They were asked to identify some of the constraints that hindered their teaching proposals and classify them according to their origin: school, pedagogical tradition, didactic contract, etc. To this end, the participants were introduced to the notion of *ecological analysis* and the idea that "Changing the *raison d'être* of knowledge clearly implies a change in the way it is conceived. But this change is not trivial, it is marked by: Conditions that can favour transition and constraints that prevent or hinder changes to happen". So that preservice teachers could situate the conditions and constraints they identified, the educator also introduced the *scale of levels of didactic codeterminacy* (Figure 6.1) as a final didactic tool for analysis. The orientation for this last analysis activity was explained in the following terms:

> One last homework...
>
> 1) Get together in your respective groups;
>
> 2) Discuss and answer the question: was the activity you implemented at school an SRP? Justify it.
>
> 3) Identify conditions and constraints that prevented the implementation of SRP in the lessons you taught. This could be any number of aspects: from the number of students per class, availability of technological resources, student motivation...
>
> 4) Try to categorise them and see at what level of the scale of levels of didactic codeterminacy they appear.
>
> 5) Prepare a presentation for the next session.

Group 1 (G1) worked with 8th and 9th graders and focused on the topic of “bullying”. The students (13-15 years old) were interested in discussing this topic and working on it. The pedagogical residents prepared a questionnaire with 24 questions. The questionnaire was printed out and distributed for each student to be answered individually. Most of the questions were categorical and some open. As the answers were on paper, the students first had to carry out the task of collecting the papers, gathering the answers, and organising them. With the data gathered, during the sessions a collective analysis was carried out on a selection of the questions. In each classroom, the students were divided into groups and each group was responsible for organising the data collected into tables of relative and absolute frequencies, representing them graphically and analysing them. The preservice teachers in Group 1 highlighted two main conditions that favoured the development of the SRP. The first related to the fact that the school, particularly the main teacher, allowed the pedagogical residents to implement their teaching proposals with complete autonomy. And the second is related to the fact that it was possible to carry out the survey activity with four classes of eighth and ninth graders, thus making it possible to collect 126 responses from all the classrooms. This group also identified some constraints in the development of the SRP and found it difficult to situate these restrictions in just one of the levels of didactic codeterminacy. To summarise, they indicated the following constraints and in brackets the levels they believed them to be situated:

- Access to the internet. (Societies / Schools)
- Access to computers or tablets (Schools)
- Time: The schoolteacher only gave us four lessons to finalise the project because he needed to revise for the bimonthly exam. (Schools / Societies)
- Content and knowledge required: The secondary students had difficulty calculating percentages and constructing the table. (Schools/Disciplines/Domains/Themes)

Group 2 (G2) worked with 10th to 12th grades (15-18 years old) and explored the value of Brazil’s currency, the Real, through the lens of exchange rates and purchasing power parity. The pedagogical residents designed an inquiry-based activity where students compared petrol prices across different countries, converting them to Brazilian Real and representing the data graphically. The session included discussions on inflation, currency exchange rates, and the complexities of comparing prices across different measurement systems. They organised the students into groups, proposed the generating question and, as case studies, the groups collected petrol prices from different countries and continents on the Internet using their mobile phone, converted them to “Reais”, and made tabular and graphical representations to discuss the valuation or devaluation of petrol. To finish the inquiry, they organised a collective discussion. Preservice teachers of G2 identified several conditions and constraints, and presented them in a table, which we reproduced in Table 6.2.

Table 6.2. Table produced by Group 2 about the conditions and constraints identified and their classification according to the scale of the levels of didactic codeterminacy

Favourable conditions	**Classification**
All the students had mobile phones and most f them had internet access.	Societies
The classroom had a data show	Schools
The schoolteacher was very flexible and extremely friendly, giving us the freedom and autonomy to do whatever we wanted if it was within the theme of the lesson.	Schools; Pedagogies; Disciplines
The school has a different standard of quality to other schools, as the students are very committed.	Schools
Constraints	**Classification**
Rigidity of subject lessons plans	Schools; Pedagogies; Disciplines
Technology room [where there are computers with internet access] was not available on the days we carried out the survey activities.	Schools
Students' lack of ability with basic mathematical calculations	Domains
Students' difficulty handling the calculator	Domains
Students' difficulty in differentiating when to use the dot or the comma when making calculations	Domains
Complexity of the content [petrol prices]	Themes
The product [petrol] students choose is sold in different units of measurement.	Civilisations; Societies
Unreliable information on the internet; invalid results	Civilisations; Sectors; Themes
Student engagement (unmotivated students, sleeping students)	Societies; Schools; Disciplines
Lesson time, we realised it was short	Schools
Holidays in the middle of the SRP	Societies
External interruptions during the lessons	Schools

This group also encountered challenges in classifying the specific conditions and constraints observed. In many instances, their classifications encompassed multiple levels of the scale, reflecting the complexity of the distinctions. Furthermore, during their oral presentations, they explicitly acknowledged these difficulties in classification, highlighting the nuanced and overlapping nature of the constraints analysed.

Group 3 (G3) worked with sixth graders (11-12 years old) and designed a survey to know more about the classmates (habits, family, etc.), focusing on basic statistical skills. Given the limited technological resources, the students designed their own survey questions, collected responses, and organised data into tables and bar graphs by hand. So, the students experienced the elaboration of questions (one question per group), the collecting of answers from their classmates, and the organisation and representation of data in tables and bar graphs as a "small opinion poll". The students prioritised tables as the main learning in statistics and the work of collecting and organising data did not appear in the discussions.

They identified the following conditions and constraints and the levels at which they considered them to be situated.

Conditions:

- Students know how to represent data in bar graphs and tables. (Domains)
- Number of students in classroom (less than third). (Schools)
- The school was open to the proposal. (Schools)

Constraints:

- Technology room [where there are computers with internet access] was not available. (Schools)
- Holidays and absence of students from one session to another. (Societies/Schools)

The case of Group 4 appeared as an illustrative example of the curriculum rigidity constraint in the paradigm of visiting works. When the pedagogical residents wanted to implement their proposal, the schoolteacher, responsible of the class, had planned to focus on the topic of "functions" and she did not allow to change the initial course schedule for an inquiry activity in statistics. They carried out six tasks with 11th grade students (15-16 years old) to study functions explicitly situated in the paradigm of visiting works. To illustrate, one of the tasks was:

A salesman's salary consists of a fixed portion of R$1320.00 and a variable portion of 8% of the value of his sales for the month.

a) Write the equation that represents this case.

b) Write the function that represents this case.

c) If he manages to sell R$380 000,00, calculate his salary.

d) If he manages to sell R$550 000,00, calculate the value of his salary.

e) In one month this employee earned R$2 850,00. How much did he sell?

f) Draw the graph of this function.

This task exemplifies the paradigm of visiting works as it prioritises procedural knowledge and the application of pre-defined mathematical techniques over inquiry-based exploration. The problem presents a pre-structured scenario in which students are expected to apply formulas, execute calculations, and produce standardised representations (e.g., function equations and graphs) without engaging in deeper inquiry or critical reflection on the context. It does not prompt students to investigate the economic implications of commission-based salaries, compare different salary models, or analyse real data on sales and wages. Instead, it reinforces a static, repetitive learning process, where mathematical concepts are treated as fixed content to be applied rather than tools for understanding and interrogating the world.

Therefore, Module 4 was the opportunity to discuss the *a posteriori* analyses of the SRP implementations carried out by each group. In this final discussion, the educator and the researcher assumed the role of "reactors", facilitating and stimulating a critical discussion of all the groups' oral reports. Particular attention was given to the challenges reported by the groups in accurately placing the identified conditions and constraints within the scale of the levels of didactic codeterminacy. A key insight that emerged from this discussion was the realisation that many of the constraints encountered during the implementation of the SRPs were not within the teachers' direct sphere of responsibility. Numerous factors influencing the process were beyond their control, highlighting the broader institutional and systemic influences on educational practice.

6.4. *A POSTERIORI* ANALYSIS

To facilitate a comprehensive *a posteriori* analysis of all the modules of the SRP-TE designed and implemented in this study involving pedagogical residents, an evaluation questionnaire was designed and made available to the sixteen participants at the end of the project. A total of thirteen responses were collected, which are presented in detail below. The questionnaire was carefully structured into four sections: (i) General Organisation of the Pedagogical Teaching Project, (ii) Group Work, (iii) Content and Structure of each Module, and (iv) Evaluation of each Module of the Project. The questions were formulated based on previously developed instruments used by the research group to which this investigation is inserted. Additionally, some adaptations and specific considerations were incorporated based on the author's reflections to enhance data collection, ensuring more robust empirical data for the ecological analysis undertaken at this stage of didactic engineering.

6.4.1. Students' evaluation of the SRP-TE general organisation

Tables 6.3 and 6.4 provide quantitative insights into the participants' perceptions concerning the sections (i), (ii), and (iii).

Table 6.3: Evaluation of participants' perceptions on the SRP-TE implementation

Items	Scale	Mean	Median	Standard deviation
Project duration	1 (very short) 10 (very extensive)	6,4	6	1,6
Practical content	1 (very little) 10 (too much)	7,4	7	2,1
Theoretical content	1 (very little) 10 (too much)	7,5	8	2,2
Complexity	1 (Very little difficulty) 10 (Too much difficulty)	**5**	**6**	2,4
Group workload	1 (Very little work) 10 (Too much work)	7,5	7	2,1
Individual workload	1 (Very little work) 10 (Too much work)	4,5	5	2,3
Global workload	1 (Very little work) 10 (Too much work)	7	7	1,8
"The work carried out throughout the project seems useful for my work as a teacher"	1 (Strongly disagree) 10 (Strongly agree)	**8,9**	**10**	2
Difficulty in sharing responsibilities between group members	1 (Very little) 10 (Too much)	6,77	7	3,32
Work environment	1 (Very bad) 10 (Too good)	**8,38**	**8**	1,61
Difficulties adapting to group work	1 (Very difficult) 10 (Very little difficulty)	6,92	7	2,69
"The result of the deliveries would have been better if we had worked individually"	1 (Strongly disagree) 10 (Strongly agree)	4	3	3,16
"The structure of the project, the different modules and the activities proposed in each of them were interesting"	1 (Strongly disagree) 10 (Strongly agree)	**8,15**	**8**	2,12
"The relationship between the different activities proposed and the project modules was clear"	1 (Strongly disagree) 10 (Strongly agree)	7,46	8	1,76
"The different roles I had to take on in each module became clear: teacher, student, teacher-analyst, teacher-designer of an SRP"	1 (Strongly disagree) 10 (Strongly agree)	**8,15**	**9**	1,95
"It was difficult to write the report as a group in the different modules"	1 (Strongly disagree) 10 (Strongly agree)	5,23	6	2,49
"The moments of sharing/presenting my group's work to the whole team and/or other groups were enriching"	1 (Strongly disagree) 10 (Strongly agree)	**9**	**10**	1,41

Table 6.4: Qualitative insights into participants' reflections on the overall SRP-TE organisation, group work dynamics, report writing, and experiences in presenting their work

Comments on the overall organisation of the SRP-TE modules
A different approach to teaching was presented. The learning experience has greatly enriched the professionals we are becoming. I gained a lot during the period we spent in the classroom.
Comments on group work
Group work depends significantly on its members. The lack of time and communication within the group made it difficult to distribute tasks equitably. Consequently, some members were overwhelmed with responsibilities, whereas others did not actively participate.
I do not find working in a group difficult. However, I often observed a lack of interest and proactiveness among members when carrying out tasks. It is disheartening to see preservice teachers criticising the lack of engagement among school pupils when they themselves exhibit the same behaviour at university.
My group consisted of four people, but two of them did not actively participate in activities or attend meetings regularly. We did not implement our project in the Residency school, and I was the only one able to visit another school. What was supposed to be a group discussion turned into an individual experience report.
Comments on report writing (strengths and weaknesses)
Perhaps due to a lack of attention from project participants, it was not initially understood that we needed to update the report throughout the project. As a result, the researcher assumed we had neglected it and had to reinforce the importance of maintaining regular updates.
One of the challenges was that we often struggled to understand what was being asked in some of the questions. However, a positive aspect was that the report allowed us to visualise all the work we had carried out within the project.
We encountered minor difficulties with the first part of the report. My group and I were uncertain whether we were completing it correctly.
We did not experience much difficulty when we worked collaboratively.
The report included questions followed by spaces for responses, which facilitated documenting our work. My group members were responsible for writing the report, while I contributed by highlighting our progress and incorporating notes from our implementation sessions.
Comments on sharing/presenting group work to other groups (strengths and weaknesses)
The enriching aspect of these sessions was the questioning that occurred during the presentations. Observations from Professors Marilena, Sônia, and Janielly, as well as comments from fellow students, contributed significantly to the discussions.
During the presentation of our implementation proposal, we were advised to allow students to complete the tasks independently and simply provide instructions on what we wanted them to do (e.g., creating a table and a bar chart), as the objective was not the construction itself.

From Table 6.3, the project duration received a mean rating of 6,4, suggesting that, while neither excessively long nor short, its length was perceived as appropriate. The balance between theoretical (mean = 7,5) and practical (mean = 7,4) content was well maintained, indicating that both aspects were adequately covered. However, the complexity of the project received a rating of 5, with a relatively high standard deviation (2.4), implying that perceptions varied considerably among participants. The overall workload (mean = 7,0) was rated as demanding, particularly in terms of group workload (mean = 7,5) compared to individual workload (mean = 4,5), suggesting that collaboration played a significant role in the SRP-TE. Participants generally agreed that the work carried out was useful for their future teaching careers (mean =

8,9), highlighting the practical relevance of the project. Furthermore, the structure of the project, including the relationship between different modules and activities, was well received (mean = 8,15 and 7,46, respectively). However, the responses indicate some difficulties in group coordination. The difficulty in sharing responsibilities was rated, in mean, at 6,77, suggesting that some participants encountered challenges in distributing tasks equitably. Likewise, writing the report as a group (mean = 5,23) posed moderate difficulties, although the moments of sharing and presenting work to peers were considered highly enriching (mean = 9,0). This suggests that, while collaboration had its challenges, peer interactions and discussions were beneficial to the learning process.

Chronogenesis concerns how the Q-A dialectic evolved over the SRP-TE and how the participants' engagement with the inquiry process developed across modules. Table 6.3 suggests that the project duration was perceived as appropriate (mean = 6,4), allowing for a progressive inquiry process. The balance between theoretical (mean = 7,5) and practical content (mean = 7,4) suggests that both aspects were adequately covered over time, ensuring an evolution of learning from conceptual understanding to applied teaching design. However, the complexity rating (mean = 5,0, SD = 2.4) indicates that perceptions varied significantly, suggesting that some participants found the learning curve manageable, while others struggled with the depth of inquiry expected.

One crucial aspect of *chronogenesis* in SRP-TE is how responsibilities evolved across modules. The responses indicate that participants were expected to take on different roles (student, teacher, teacher-analyst, teacher-designer of an SRP), and this transition was generally well understood (mean = 8,15, median = 9,0). The structure of the project and its modular progression were also well received (mean = 8,15), reflecting that the inquiry process was perceived as coherent and logically sequenced. However, qualitative feedback from Table 6.4 indicates some difficulties in adapting to the responsibilities of each module, particularly in report writing and implementation, suggesting that clearer temporal scaffolding might have been needed to ensure a smoother transition between roles.

Mesogenesis refers to how the media-milieu dialectic was staged throughout the SRP-TE, including the creation and structuring of the learning environment. Participants were required to navigate various didactic tools (e.g., Padlet, statistical software, inquiry-based frameworks), which strongly influenced their engagement with the inquiry process.

The group workload (mean = 7,5) was perceived as significantly higher than the individual workload (mean = 4,5), reflecting a strong emphasis on collaborative inquiry. However,

Table 6.4 reveals challenges in resource allocation and task distribution within groups, with some participants reporting a lack of engagement from their peers, which in some cases resulted in individual rather than collective implementation. These challenges indicate a misalignment in how responsibilities were negotiated within the didactic system, potentially hindering the effective use of resources. Moreover, the work environment was rated highly (mean = 8,38), suggesting that participants felt supported within the project framework. However, the difficulty in sharing responsibilities (mean = 6,77) and writing reports as a group (mean = 5,23) suggests that more structured guidelines on collaborative resource management could have improved group coordination and workload distribution.

Topogenesis concerns how roles and responsibilities were distributed between participants and educators and how this distribution evolved during the course. A key finding from Table 6.3 is that participants largely understood the roles they assumed throughout the modules (mean = 8,15, median = 9,0), suggesting that the didactic contract was well established. However, qualitative feedback from Table 6.4 highlights a discrepancy between the intended and actual distribution of responsibilities within groups. Several participants noted that certain group members did not fully engage, resulting in unequal participation and increased workload for others. This disparity in *topogenesis* indicates that while the formal structure of SRP-TE promoted shared responsibilities, the actual classroom dynamic did not always align with this expectation. The difficulty in coordinating group work (mean = 6,92) further supports this observation. Another *topos* aspect relates to the interaction between participants and educators. The high rating for sharing/presenting work (mean = 9,0) indicates that peer-to-peer and instructor feedback were valuable elements of the learning process. The qualitative responses reinforce this, highlighting the constructive role of questioning and discussions in refining teaching proposals.

6.4.2. Students' evaluation of Module 1

Concerning the participants' evaluation of each module of the SRP-TE (iv), Tables 6.5 and 6.6 present an evaluation of Module 1 of the SRP-TE, highlighting participants' perceptions of its development, group work, and use of digital tools, as well as the positive and negative aspects they encountered.

Table 6.5: Responses about Module 1

Statements	Totally agree	Partially agree	Partially disagree	Totally disagree
"The development of the SRP was very difficult for me"		1 (7,69%)	**8 (61,54%)**	4 (30,77%)
"I am used to developing open-inquiry mathematics activities like the SRP (without necessarily taking them into the classroom)"	1 (7,69%)		5 (38,46%)	**7 (53,85%)**
"The group work in this activity was enriching"	**9 (69,23%)**	2 (15,38%)	1 (7,69%)	1 (7,69%)
"Exchanging reports or interacting with the other working groups allowed us to improve ours"	**9 (69,23%)**	2 (15,38%)	1 (7,69%)	1 (7,69%)
"The schematic presentation on the 'Padllet' (questions-answers map) was a good tool for summarising the work done in the group"	**10 (76,92%)**	2 (15,38%)	1 (7,69%)	
"The schematic presentation in Google Drive (Module 1 report) was a good tool for summarising the work done in the group"	**13 (100%)**			
"The use of digital tools (in general: Padllet, Drive, Geogebra, Excel, Power Point) was very interesting for the development of the SRP"	**12 (92,31%)**	1 (7,69%)		
"The educators guided us a lot during the Module 1"	**10 (76,92%)**	2 (15,38%)	1 (7,69%)	
"Putting myself in the role of 'student-learner' during this module was a simple task"	**9 (69,23%)**	2 (15,38%)	1 (7,69%)	1 (7,69%)

Table 6.6: Responses about positive and negative aspects of Module 1

Comments on "Two positive aspects"	Comments on "Two negative aspects"
Gaining familiarity with new platforms, such as Padlet, and stepping out of our comfort zone by seeking information on a topic that was not previously discussed in depth by the group.	*Difficulty in finding reliable sources for research.*
Reflecting on derived questions from a textbook problem was a positive aspect. Adopting the role of a student was not an easy task, but it proved to be a highly engaging activity. Another positive aspect was working on this module collaboratively, as discussing with peers made it less challenging.	*A significant challenge was the group's lack of conceptual knowledge regarding certain aspects of statistics, particularly graphical elements. As a result, the graphs we constructed contained several conceptual errors. Another negative aspect was the large number of derived questions from* Q_0*, which required the researcher to select specific questions for the development of Module I.*
Exploring new digital tools and employing creativity and inquiry for the first activity.	*Difficulty in correctly carrying out the proposed activity, as well as challenges during the research process, since we sometimes struggled to answer our own questions.*
Enhancing teacher education through new perspectives.	*Difficulty in understanding the purpose of the task and in developing the required work due to a lack of clarity.*
Acquiring knowledge of new didactic tools and refining teaching approaches when dealing with new content.	*Challenges in data collection and difficulties in applying relevant tools.*

Engaging in an investigative activity and exchanging ideas during group meetings.	*Personally, I found it difficult to narrow down the topic due to its broad scope and the requirement to develop questions. Additionally, we sometimes struggled to find answers to the questions we formulated.*
Accessing digital tools on the first day and participating in initial discussions on the activity.	*The initial objective of the activity was not clearly defined, and the task did not feel particularly innovative ("Just another standard activity").*
Interacting and learning how to develop the assigned tasks.	*A lack of understanding over the content and insufficient interaction.*
Working on the SRP with a cross-disciplinary theme and presenting new digital tools.	*Time constraints and the challenge of formulating meaningful questions; uncertainty about the reliability of data.*
Using technology and engaging in group discussions.	
Benefiting from the tools introduced to us and the instructor's guidance.	*The early stages of group work, combined with encountering many new concepts for the first time, made it difficult for me to fully grasp the task.*
Effective group organisation and commitment.	*No comments come to mind.*
Inquiring and acquiring knowledge.	*Limited knowledge of the topic and difficulty in finding relevant information.*

Concerning the *chronogenesis* responses in Table 6.5 indicate that while a majority (61,54%) partially agreed that the development of the SRP was difficult, only a small fraction (7,69%) found it extremely challenging. This suggests that over time, participants became more comfortable with the open-inquiry process, despite initial struggles. The fact that 76,92% of participants agreed that the educators guided them well supports the idea that careful structuring of the inquiry process facilitated their engagement. However, Table 6.6 highlights some challenges in the progression of the inquiry, such as difficulties in formulating and narrowing down questions, which suggests that some participants struggled with defining and refining their inquiry focus over time. The need for researcher intervention in selecting specific questions further indicates that the self-directed inquiry process had some limitations. This aligns with challenges in the *chronogenesis*, where participants required additional support in structuring their investigative processes effectively.

Regarding the *mesogenesis* the use of digital tools (Padlet, Google Drive, Geogebra, Excel, and PowerPoint) was overwhelmingly well-received, with 92,31% of participants agreeing that these resources were valuable for developing the SRP. Additionally, 100% of participants found Google Drive an effective tool for summarising their group work, highlighting the importance of digital platforms in facilitating collaboration and knowledge organisation. However, Table 6.6 indicates that some participants struggled with finding reliable data sources, which suggests a gap in the effective use of external resources. This aligns with the *media-milieu* *dialectic*, which deals with how participants engage with external data sources and validate the information they find. The difficulty in applying data analysis tools also reflects challenges in the *mesogenesis*, as some participants may have lacked prior experience in using digital tools for statistical inquiry.

About how responsibilities and roles were distributed among participants and educators (*topogenesis*), Table 6.5 suggests that group work played a significant role in Module 1, with 69,23% of participants finding it enriching. However, Table 6.6 reveals concerns regarding group dynamics, with some participants noting that not all members contributed equally, leading to an uneven distribution of tasks. This issue relates to the individual-collective dialectic, as the intended collaborative learning process was hindered by disparities in participation. Additionally, the transition between roles (student, analyst, designer, and teacher) in Module 1 appears to have been a key challenge, as noted in Table 6.6. Some participants found it difficult to assume the role of student-learner, while others struggled to understand the purpose of the task. These responses highlight a *topos* challenge, where the shifting of responsibilities between different roles within the SRP-TE framework was not entirely seamless for all participants.

6.4.3. Students' evaluation of Module 2

Module 2 of the SRP-TE introduced participants to didactic analysis tools, such as the Herbartian scheme and the dialectics (questions-answers, media-milieu, individual-collective), to help them reflect on the learning process. The tables present both quantitative responses (Table 6.7) and qualitative reflections (Table 6.8) regarding the participants' experiences.

Table 6.7: Responses about Module 2

Statements	Totally agree	Partially agree	Partially disagree	Totally disagree
"The notion of 'Herbartian schema' introduced by the educators was very useful for analysing the SRP"	**7 (53,85%)**	**4 (30,77%)**	1 (7,69%)	1 (7,69%)
"The notion of 'milieu' introduced by the educators was very useful for analysing the SRP"	**7 (53,85%)**	**6 (46,15%)**		
"The notion of 'questions-answers dialectic' introduced by the educators was very useful for analysing the SRP"	**7 (53,85%)**	**6 (46,15%)**		
"The notion of 'media-milieu dialectic' introduced by the educators was very useful for analysing the SRP"	**8 (61,54%)**	**5 (38,46%)**		
"The notion of 'individual-collective dialectic' introduced by the educators was very useful for analysing the SRP"	**7 (53,85%)**	**6 (46,15%)**		
"In general, the didactic tools introduced by the educators will be very useful for my teaching profession"	**9 (69,23%)**	4 (30,77%)		

"It was easy to analyse the SRP I developed with my group"	2 (15,38%)	**6 (46,15%)**	**4 (30,77%)**	1 (7,69%)
"I am used to carrying out analyses of this kind"	1 (7,69%)	2 (15,38%)	**6 (46,15%)**	**4 (30,77%)**
"I have already carried out didactic analyses in subjects during my teacher education"	1 (7,69%)	**5 (38,46%)**	**5 (38,46%)**	2 (15,38%)
"Putting myself in the role of 'analyst' during this module was a simple task"	1 (7,69%)	**6 (46,15%)**	3 (23,08%)	3 (23,08%)

Table 6.8: Responses about positive and negative aspects of Module 2

Comments on "Two positive aspects"	**Comments on "Two negative aspects"**
The opportunity to share difficulties and experiences encountered during Module I.	*I did not identify any negative aspects.*
A positive aspect was the reflection we engaged in during the analysis, assuming the role of a teacher while considering the process from the perspective of a student. Another strength was the instructor's approach, consistently bringing forward and discussing aspects identified in the groups' work without causing any discomfort.	*The only negative aspect I can mention in this module is the complexity of the theoretical concepts involved, including the Herbartian scheme, the dialectics, and the statistical concepts themselves.*
Adopting the role of an analyst and acquiring new knowledge related to data analysis.	*One difficulty I experienced was in fully understanding the Herbartian scheme presented during the module.*
Observing tangible results from the application of concepts and effective group collaboration with other students.	*Scheduling difficulties and logistical challenges in attending school.*
Engaging in roundtable discussions and receiving constructive feedback.	*Difficulties in presenting ideas and in applying everything we intended to implement in the classroom.*
Gaining an analytical perspective that I had not previously encountered during my training, as well as the method we employed to develop this viewpoint.	*Challenges in fully comprehending the concepts introduced in Module 2 and the duration of this phase.*
Discussions held during meetings and the process of self-analysis regarding the work we developed.	*(1) I was not entirely clear on what we were developing; (2) The activities seemed relatively standard and not particularly enriching.*
Interaction and analytical discussions.	*Limited practicality in analysing the SRP and understanding dialectics.*
Understanding statistical content, particularly aspects of box plots, which enhanced the quality of our analysis.	*Constructing the box plot diagram and understanding its significance.*
I was absent during some of the sessions in this module.	*I was absent during some of the sessions in this module.*
No comments come to mind.	*No comments come to mind.*
Engaging in debates on the projects and receiving constructive criticism on presentations.	*The absence of group members.*
Knowledge construction and research.	*Lack of knowledge on the topic and difficulty in finding reliable information.*

The results in Table 6.7 indicate that most participants found the didactic tools useful for analysing their SRP, particularly the Herbartian scheme (53,85%), the *media-milieu dialectic*

(61,54%), and the *questions-answers dialectic* (53,85%). Additionally, 69,23% of the participants agreed that these tools would be valuable for their teaching careers, indicating that Module 2 successfully built on the inquiry initiated in Module 1. However, only 15.38% found it was easy to analyse their SRP, while 46,15% partially agreed and 30,77% disagreed, suggesting ongoing difficulties in structuring the analysis. The challenges in comprehending theoretical concepts, especially the Herbartian scheme.

The didactic tools introduced by educators played a significant role in guiding participants' analysis, as reflected in the strong agreement on their usefulness. Structured discussions, particularly in a roundtable format, were identified as highly beneficial (Table 6.8), as they helped participants refine their understanding through peer collaboration and instructor feedback. However, some participants struggled to apply these tools in practice, especially in relation to statistical concepts such as box plots. Additionally, logistical constraints—such as scheduling difficulties and school attendance issues—were noted as barriers to fully engaging in the analysis.

Half of the participants (53,85%) had no prior experience with didactic analysis, and only 7,69% reported being familiar with this kind of analysis. This suggests that the role of an 'analyst' was unfamiliar to most of them and required a significant shift from the implementation-focused approach in Module 1 to a more reflective and analytical role in Module 2. Additionally, while some participants found the collaborative aspect enriching, others struggled with role distribution and group engagement. Several groups reported difficulties in sharing responsibilities, which led to unequal participation, with some members taking on a greater workload while others contributed minimally. Furthermore, the absence of group members was frequently noted as a barrier, affecting the overall collaborative learning process.

Regarding the strengths and challenges of Module 2, participants highlighted several positive aspects of Module 2 (Table 6.8). Engaging in analytical discussions, reflecting on the learning process, and receiving instructor guidance were seen as significant strengths. The structured analysis methods, including the questions-answers dialectic and Herbartian scheme, helped participants develop a more in-depth understanding of the SRP process. Furthermore, collaboration and peer discussions contributed to knowledge-building and self-assessment, reinforcing the importance of group work in professional development.

Despite these strengths, several challenges emerged. Difficulty in understanding theoretical concepts, particularly the Herbartian scheme and dialectical analysis, was a major issue for many participants. Additionally, some struggled to translate the theoretical analysis into practical applications, especially when working with statistical tools such as box plots. Logistical constraints—such as scheduling difficulties and absenteeism—also limited full participation in the module. Moreover, some participants felt that the theoretical analysis was abstract and were uncertain about its practical relevance, suggesting a need for further contextualisation and application-based learning.

6.4.4. Students' evaluation of Module 3

Module 3 of the SRP-TE focused on adapting and implementing an SRP (or a related activity) in real classroom conditions. The responses in Tables 6.9 and 6.10 reflect participants' experiences in designing (Module 3.1) and implementing (Module 3.2) inquiry-based teaching activities.

Table 6.9: Responses about Module 3

Module 3.1 – Adapting an SRP (or an activity close to an SRP) to certain school conditions)				
Statements	**Totally agree**	**Partially agree**	**Partially disagree**	**Totally disagree**
"In my group, we had no difficulties planning an inquiry activity"	1 (7,69%)	3 (23,08%)	**7 (53,85%)**	2 (15,38%)
"Designing the first version of the teaching proposal was a simple task"	**5 (38,46%)**	3 (23,08%)	4 (30,77%)	1 (7,69%)
"The collective discussions during the SRP design helped a lot"	**9 (69,23%)**	2 (15,38%)	2 (15,38%)	
"The discussions promoted by the educators during the SRP design were important"	**9 (69,23%)**	2 (15,38%)	2 (15,38%)	
"Thinking of an activity closer to the paradigm of questioning the world was easy"	1 (7,69%)	3 (23,08%)	**7 (53,85%)**	2 (15,38%)
"I felt a heavy workload because in my group the teaching proposal was not design collectively"	5 (38,46%)	**6 (46,15%)**	2 (15,38%)	
"Putting myself in the role of 'teacher-designer' while designing the SRP was easy"	2 (15,38%)	**6 (46,15%)**	4 (30,77%)	1 (7,69%)
Module 3.2 – Adapting an SRP (or an activity close to an SRP) to certain school conditions)				
Statements	**Totally agree**	**Partially agree**	**Partially disagree**	**Totally disagree**
"Implementing the designed activity was easy"		**6 (46,15%)**	**7 (53,85%)**	
"The secondary students accepted the inquiry activity with interest"	1 (7,69%)	**6 (46,15%)**	4 (30,77%)	2 (15,38%)
"Constraints emerged that presented us from achieving certain goals that had been planned"	5 (38,46%)	**6 (46,15%)**	1 (7,69%)	1 (7,69%)
"The previous designed activity was a great help to my performance in this implementation phase"	**8 (61,54%)**	5 (38,46%)		
"The collective discussions on the progress of the implementation helped a lot in continuing and finalising it"	**7 (53,85%)**	4 (30,77%)	1 (7,69%)	1 (7,69%)

"I felt a heavy workload implementing the activity on my own"	1 (7,69%)	3 (23,08%)	**5 (38,46%)**	**4 (30,77%)**
"Implementing the inquiry activity with other colleagues in my group was very important"	**6 (46,15%)**	**5 (38,46%)**	1 (7,69%)	1 (7,69%)

Table 6.10: Responses about positive and negative aspects of Module 3

Module 3.1 – Adapting an SRP (or an activity close to an SRP) to certain school conditions)	
Comments on "Two positive aspects"	**Comments on "Two negative aspects"**
Once again, stepping out of our comfort zone, creating a different lesson plan from those we had previously developed, and challenging our ideas. Having the educator available to assist with any difficulties was essential, enabling us to clarify numerous doubts and finalise our planning.	*A group member did not participate in the planning process, and we faced difficulties in creating the first version of the lesson plan due to the unfamiliarity of the new lesson format we had not previously worked with.*
(1) Considering an inquiry activity, as this approach differs from the planning we typically undertake during our practices in the teacher education. (2) Being able to rely on the guidance of the educator. We were not left entirely on our own during the planning process; instead, we always took into account the considerations of both the educator and our peers.	*Unfortunately, we were unable to design an inquiry activity related to statistics due to the decision made by the schoolteacher regarding the content to be implemented. As a result, I personally felt that I had "lost" the previous modules, in which discussions were focused on statistics. Another negative aspect was that my specific group relied on the teacher from a school where one of our members worked. This dependency caused delays in our planning, as we had to wait for her to contact us regarding the topic we were expected to teach.*
Learning about and planning an activity different from our usual classroom routines, encouraging creativity while designing an SRP.	*The difficulty in developing the lesson plan was primarily due to the class we were assigned, as we had to adapt to the pre-determined subject matter (Theme: monetary systems).*
Despite initial difficulties, we successfully implemented our plan, which was a novel experience.	*At the beginning, it was challenging to develop the lesson, as we were unfamiliar with the students, making it difficult to engage them with the proposed content.*
Engaging in discussions about other SRPs and the execution of activities.	*The lack of school resources and the necessity to adapt lesson plans according to immediate needs.*
Assuming the role of teachers was straightforward, and planning the lesson was also manageable.	*Unforeseen challenges in the planning process were problematic, as was the constraint of the topic to a statistical inquiry.*
I believe that the discussions were again a positive aspect of this module, as were the moments of reflection prompted by our meetings.	*(1) I was not entirely clear on what we were developing; (2) The activities seemed relatively standard and not particularly enriching.*
Workload distribution and discussions.	*Once again, I felt a lack of innovation; we ended up producing and planning yet another standard activity. I was also not particularly excited about planning this activity for implementation in the school, meaning that this module did not provide me with much enthusiasm.*
Not being restricted to our original lesson plan.	*Limited knowledge of the subject matter and insufficient time for designing.*
The group was highly focused, and we were already familiar with the students we were going to work with.	*Not knowing the class dynamics and designing an SRP that could be effectively implemented for 6th grade students.*

The dedication of some group members who actively contributed to the SRP design process.	*The university timetable made it difficult to dedicate more time to the design process.*
The entire lesson plan was successfully implemented, with active participation from all group members.	*The coordination of certain group members was challenging, and we were somewhat lost at the beginning.*
Secondary students already had interest in doing a servey about a theme of their interest [bullying], which aligned well with the project requested by the mentor teacher.	*Some days, during the design, only one group member was present.*
Module 3.2 – Adapting an SRP (or an activity close to an SRP) to certain school conditions)	
Comments on "Two positive aspects"	**Comments on "Two negative aspects"**
I was unable to participate in the implementation due to my tight schedule. However, after discussing with colleagues who were present, I found that a positive aspect was the students' enthusiasm in engaging with the activity. Although I did not take part in the practical implementation, the process of developing the inquiry design was of great significance to my professional development.	*I was unable to participate in the practical implementation at the school. Unfortunately, only one group member was able to be present, and certain unforeseen events at the school slightly delayed the planning process. Nevertheless, the proposal was successfully completed.*
Even during the implementation phase, we held meetings to discuss the progress of the activities, presenting our findings to the instructors. Additionally, these meetings provided opportunities to convene with our own group and deliberate on the decisions to be made regarding the continuation of the implementation process in the subsequent weeks.	*The fact that only one group member was able to implement the activity was a negative aspect, as we were a group of three, meaning that only one-third of us truly experienced this phase. This was problematic because we could only receive feedback on the designing process through the accounts of the member who carried out the implementation. Another negative aspect was that some group members, both in my group and in others, missed some meetings during the implementation phase. These meetings were crucial as they involved presenting the progress of activities.*
The experience of designing an SRP and implementing it in the classroom enabled us to perceive the benefits of such an activity and reflect on ways to improve its implementation in future sessions.	*Due to the topic we had to address, it was challenging to maintain student interest in the activity.*
I was unable to carry out the implementation.	*I was unable to carry out the implementation.*
Discussion with secondary students regarding the proposed activity and the posters they elaborated.	*Lack of student interest, particularly during the mid-year festive period.*
The collective discussions were highly beneficial for further developing our SRP, as well as for actively engaging in our roles as teachers.	*The topic was restrictive, and the students did not show much interest.*
The use of digital tools and the students' engagement during the sessions.	*In my view, I felt that I was "disrupting" the schoolteachers' teaching planning process to introduce this inquiry activity. I also did not feel that this experience contributed much to my knowledge, as it did not provide a meaningful learning opportunity for the students.*
Interaction with students and engagement with the content.	*Workload and subject matter addressed.*
Student interest and participation.	*Classroom noise—since there were only a few students present, proper implementation was not possible.*
Freedom to choose the subject matter and flexibility in the number of lessons.	*Student difficulties and lack of engagement.*

The students who were committed to the activity and the various debates we had in classroom.	*Some students were not interested, and scheduling the implementation in the school was a challenge.*
The entire SRP designed was implemented, and all team members contributed to its development.	*The topic did not align well with the classes covered in the pedagogical residency, and on certain days, only one group member attended.*
A different dynamic with the students.	*Lack of student interest and insufficient class time for implementation.*

Concerning the *chronogenesis*, it was particularly evident in how participants transitioned from designing an SRP to implementing it in schools. In Module 3.1, participants encountered difficulties in designing an inquiry-based lesson, with only 7,69% reporting no difficulties, while 53,85% partially disagreed and 15,38% totally disagreed. The results indicate that many participants found it challenging to structure an SRP aligned with the paradigm of questioning the world (53,85%), emphasising how difficult it is to change the conditions that exists in paradigms closed to a monumentalistic teaching, which is still highly prevalent with respect to mathematics. In Module 3.2, the implementation phase introduced new challenges, particularly in adapting the designed SRP to the realities of school conditions. 53,85% of participants reported difficulties in implementing the inquiry activity, and 38,46% agreed that constraints emerged that prevented them from achieving their intended goals. However, 61,54% agreed that the prior planning phase helped them perform better during implementation, indicating that the preparation in Module 3.1 was valuable, despite its challenges. Additionally, collective discussions during the implementation phase were seen as beneficial by 53,85% of participants, suggesting that ongoing collaboration supported the implementation of their SRPs.

The creation and structuring of the learning environment were reflected in participants' engagement with collaborative discussions and digital tools. In Module 3.1, 69,23% of participants found collective discussions during the SRP design useful, demonstrating that peer collaboration played an essential role in refining their teaching proposals. The presence of the educators as "reactors", promoting the discussions, was also valued by 69,23% of respondents, highlighting the importance of expert guidance in supporting the development of inquiry-based teaching strategies. However, participants reported difficulties in adapting their lesson plans to school conditions, particularly when external factors dictated the subject matter. One participant expressed frustration at being unable to design an SRP related to statistics due to the schoolteacher's decision on content. This suggests that, while inquiry-based teaching methods were encouraged, curricular constraints (in this case) sometimes limited their application, highlighting the ecological challenges of implementing innovative teaching strategies in traditional school systems. Some participants reported difficulties engaging students, especially when the lesson topic was pre-determined by the school. Additionally, only one group member was able to implement the lesson in some cases, which limited the collaborative experience and undermined the intended group-based nature of the project.

Topogenesis, which relates to the roles assumed by participants and the distribution of responsibilities, was a key issue in Module 3. Many participants struggled with assuming the role of "teacher-designer" during the planning phase, with only 15,38% agreeing that this role was easy to take on, while 46,15% partially agreed and 30,77% disagreed. This suggests that transitioning from a student perspective to that of a curriculum designer required more support. Additionally, 38,46% of respondents reported a heavy workload because their group did not collaboratively design the teaching proposal, indicating that group work dynamics influenced participants' experiences significantly. During implementation (Module 3.2), the role of teacher-implementer introduced new challenges. 46,15% reported that implementing the activity with colleagues was important, yet, for some groups, only one member carried out the implementation, resulting in limited shared experience among the groups and group members. The lack of student engagement was another challenge, with several participants reporting that secondary students were disinterested in the inquiry activity, particularly when external factors (e.g., school constraints or festive periods) influenced their regular schedule. Additionally, one participant noted that introducing an inquiry activity felt like a disruption to the school's existing teaching structure.

Regarding the strengths and challenges of Module 3, Table 6.10 presents qualitative reflections on the strengths and weaknesses of this module. Positive aspects included stepping out of the comfort zone, engaging in collaborative planning, and benefiting from educator guidance. Several participants appreciated the creativity and innovation involved in designing an SRP, even if it differed from their usual lesson planning experiences. Discussions with the educators and their peers were repeatedly highlighted as valuable, indicating that structured dialogue played an essential role in refining participants' teaching strategies. However, several negative aspects emerged. One significant challenge was the difficulty in adapting inquiry-based activities to school constraints, particularly when pre-determined subject content limited their flexibility. Additionally, some participants reported that the module lacked innovation, as the activities felt repetitive rather than transformative. This suggests that, while the intention was to promote inquiry-based teaching, the actual experience sometimes mirrored traditional lesson planning. Another recurring issue was the unequal participation within groups, with some members failing to contribute effectively to the planning and implementation phases.

6.4.5. Students' evaluation of Module 4

Module 4 was the *a posteriori* analysis of the SRPs implemented by the pedagogical residents in the secondary schools' conditions and their final reflections and opinions about the entire SRP-TE. The ecological analysis of the implementations carried out by the four groups in the Pedagogical Residency Programme (PRP) highlights the various conditions that facilitated the SRPs and the constraints that hindered their development. By employing the scale of levels of didactic codeterminacy (Chevallard, 2002), it was made possible to place these conditions and constraints at different levels, such as societies, schools, pedagogies, disciplines, etc. This analysis provides insight into the systemic factors influencing the feasibility of implementing inquiry-based teaching in secondary schools.

However, participants encountered difficulties in accurately situating some of these conditions and constraints within a single level of the didactic codeterminacy. This challenge highlights the interconnected and multi-layered nature of institutional influences on teaching practices. Many constraints, such as curriculum rigidity, time limitations, and technological restrictions, operate simultaneously at different levels, making their classification inherently complex. Given this, Table 6.11 summarises the main constraints identified by the participants, the corresponding levels they placed them, within the scale of levels of didactic codeterminacy.

Table 6.11: Restrictions identified by the groups of preservice teachers about the implemented SRPs

Levels	Description of the constraints
Societies	Reliability of data found on the Internet: students and preservice teachers found unreliable Internet information and invalid data (G2). Holidays and absence of students: it was difficult to maintain the continuity of the SRP because of the holidays they had during the weeks of implementation, also affecting on the absence of many students (G2, G3).
Schools	Material Infrastructure: The classrooms do not have access to computers or the internet (G1, G3). This restricted to carry out the data study using software, as well as a broader search for information (G1). Difficult access to the computers room (G2). Time: Preservice teachers just had four lessons (each of 50 minutes) to carry out the SRP, but they had planned the inquiry activity for six lessons (G1, G2). Students' motivation: preservice teachers reported that many students lacked the motivation to do the inquiry activity (G2).
Pedagogies	Rigid structure of the contents: although preservice teachers had autonomy to implement the SRP, the responsible teacher tended to fix the subject to work with the students (G2). External interruptions during the lessons: preservice teachers reported that there have been times when other teachers have interrupted them. For example, a teacher comes into the classroom to remind the students about an outing, or to pass on a message about a project, etc. (G2).

Disciplines (and other specific levels)	Students' statistical knowledge: the majority of students had difficulty calculating percentages and relative frequencies, as well as constructing tables and graphs (G1, G2). Students' difficulties to use calculators: students had difficulty differentiating when to use the dot or the comma when making calculations (G2). Complexity of the subject: develop the inquiry activity about the "the value of Brazil's currency" and petrol prices was considered as a complex subject to work with the students. Students chose to compare the petrol prices in various countries, but they found difficulties in making the comparison because the petrol is sold in different units of measurement (G2).

Preservice teachers identified several constraints at different levels of the didactic codeterminacy scale. Most prominently, issues arose at the school and disciplinary levels, such as school infrastructural limitations (e.g., lack of internet access), pedagogical organisation (e.g. time and curriculum) and students' insufficient background in basic calculations. Residents also struggled to delineate where certain constraints originated, reflecting the interconnectedness of the codeterminacy scale. As expected, preservice teachers often internalised institutional limitations and considered them as personal teaching challenges, an aspect that highlights the need for interventions targeted at the institutional (including disciplinary) constraints in both preservice and in-service teacher education. Moreover, while they addressed students' content difficulties, residents pointed out the difficulties in calculations and making tables and graphs, with a less pronounced focus on data treatment, signalling an area for improvement in teacher preparation. This calls for collective efforts within the educational community to address institutional barriers, aligning with the GAISE II framework (ASA, 2020), but necessitating more concrete and actionable steps.

By recognising these constraints, and placing them within the levels of didactic codeterminacy, as provided by the ATD, this study highlights the need for teacher education programmes to integrate the ecological analysis as a core component for teacher education—whether through dedicated research projects, structured training, or embedded curricular components. This integration would equip preservice teachers with the critical awareness necessary to identify and address institutional constraints, thereby fostering their ability to implement pedagogical innovations within the transition from the paradigm of visiting works to the paradigm of questioning the world, being aware of the conditions that they are able to set up and the constraints that appear limiting the possible teaching and learning processes. The SRP device explicitly seeks to challenge and overcome these constraints by redefining the role of inquiry and knowledge construction in mathematics education. Such an approach would provide preservice teachers with analytical tools that enable them to critically navigate and address institutional constraints, fostering the ecology of innovative pedagogical practices in mathematics education.

Tables 6.12 and 6.13 also provide the final participants' reflections into the *a posteriori* analysis of the SRP implementations and the conditions and constraints that influenced its execution.

Table 6.12: Responses about Module 4

Statements	Totally agree	Partially agree	Partially disagree	Totally disagree
"In my group, the collective analysis of the implementation was easily carried out"	3 (23,08%)	**5 (38,46%)**	**4 (30,77%)**	1 (7,69%)
"The notion of 'scale of the levels of didactic codeterminacy' introduced by the educator was very useful for analysing the implementation"	5 (38,46%)	**7 (53,85%)**	1 (7,69%)	
"One session of discussion and analysis of the implementation was enough"	**4 (30,77%)**	**5 (38,46%)**	2 (15,38%)	2 (15,38%)
"It was easy to put myself in the role of 'teacher-analyst' in this module"	2 (15,38%)	**5 (38,46%)**	**4 (30,77%)**	2 (15,38%)

Table 6.13: Reflections and adaptations regarding the SRP implementations

Still in the position of 'teacher-analyst', if you could make adaptations to the inquiry activity (for future implementation), what would these adaptations be?
Comments
I believe our lesson plan was well-structured, and I would not make any alterations. However, depending on the class in which we were to implement it again, some adjustments might be necessary for better alignment. Nonetheless, I consider it to be a well-designed plan..
The adaptation would involve making the lesson plan more investigative, allowing students to make choices rather than merely presenting them with predetermined questions.
I would utilise the technology lab for the graphs representations.
I was unable to implement the activity. However, based on discussions, I would take more time to familiarise myself with the class in order to better understand its unique characteristics.
I would plan more sessions to develop the SRP.
I am uncertain about which changes I would make.
Instead of using traditional materials such as poster boards, I would opt for digitally generated graphs using Excel or another digital tool. In my view, knowledge production should align with technological advancements, as poster boards are somewhat outdated.
Becoming more familiar with the students would allow for a more tailored lesson plan.
I would incorporate digital tools in the teaching process.
I would aim to grant students greater autonomy, as in this lesson, we ended up adopting a more traditional teaching role.
I believe that finding more engaging and pedagogically effective strategies for planning is essential.
None
The responsibility for formalising the inquiry should lie with the students. In our case, we had already structured the inquiry and applied it accordingly.

Module 4 – *A posteriori* analysis of the SRPs implemented	
Comments on "Two positive aspects"	**Comments on "Two negative aspects"**
In this module, we shared our experiences and challenges, which was essential for analysing our processes and understanding our mistakes and successes.	*Some groups fell behind schedule, which prevented us from completing all analyses.*
I consider two aspects of Module 4 particularly positive: the fact that the analysis was conducted in a single session with all participants present, allowing for a comprehensive overview of each individual's experiences and final conclusions.	*I cannot identify any negative aspects of this module.*
It was valuable and enriching to observe the challenges and successes we encountered throughout the designing and implementation phases.	*I cannot think of any negative aspects.*
I did not participate in the implementation.	*I did not participate in the implementation.*
Engaging in discussions with my university peers and reviewing observations from Janielly.	*Insufficient time to complete the project.*
The debates with project colleagues and the analysis of feedback provided by students at the school.	*I struggled somewhat with understanding the new concepts introduced. However, I did not identify any additional negative aspects worth mentioning.*
Firstly, analysing the activity was insightful, as it enabled us to observe the factors that influenced its implementation in the school setting. Secondly, the discussions led by the educators, including the thought-provoking questions they posed, were also particularly valuable. I consider both aspects to be positive.	*I cannot think of any specific issues. However, perhaps the lack of in-depth discussion within my group or the limited engagement of some group members throughout each stage was a drawback.*
Assuming the role of a teacher and understanding about the scale of the levels of didactic codeterminacy.	*Collaborative analysis and the completion of the final report.*
Learning about the scale of the levels of didactic codeterminacy. and understanding that an unsuccessful outcome is still an outcome. The SRP fosters student autonomy.	*Understanding how to classify conditions and constraints, as well as recognising that some constraints can be modified.*
The group worked exceptionally well together, and we were highly supportive of one another.	*The process felt somewhat rushed because we shifted our focus too frequently and did not fully implement the project in the school as an authentic SRP.*
None come to mind.	*None come to mind.*
The project was successfully completed, and the farewell celebration was enjoyable.	*None.*
The experience contributed to more effective implementation in future applications, as I was able to observe students' mathematical difficulties related to the content.	*The intended conclusions were not fully reached.*

Freely express your opinion on the entire SRP-TE:
Comments
Overall, it was an excellent project and a truly innovative experience for me. However, I was disappointed with my group's performance, which led to feelings of discouragement regarding my own progress within the activities. On several occasions, we struggled to complete tasks collectively, and due to some members' lack of commitment, the division of tasks was inadequate. As a result, a few individuals were overburdened with the majority of the work, making me feel inclined to complete activities individually. Regarding the implementation phase, my experience in this project would have been more enriching had I been able to participate in it directly. However, this was not feasible, as the implementation sessions coincided with my master's degree classes. Lastly, I would like to express my gratitude to the instructors for allowing me to be part of this project, and I sincerely apologise for any shortcomings on my part.

It was a unique project that provided us with an opportunity to explore new concepts and didactic tools. From previous experiences, I had already participated in similar activities that successfully motivated students in the classroom. However, it was only in this project that I was introduced to the concept of SRP, among other concepts. I believe that all experiences contribute to learning, and it was a pleasure to gain new insights through this project.
It was a well-structured project that provided practical training for the classroom environment. I believe the main challenge was ensuring continuity. The students did not resist participation, and their engagement remained consistent from beginning to end. I did not observe any major student errors. Socially, I found the project highly valuable, as I believe that, given more time, we could achieve significant outcomes for society through its implementation.
I appreciated the project, as it allowed me to learn about SRP, how to approach statistics in different classroom contexts, and how to develop lesson plans that foster student engagement. The implementation phase sparked necessary discussions relevant to a teacher's professional journey, particularly regarding the persistent challenges that remain unchanged even at the university level. Additionally, there were valuable conversations about strategies to encourage greater student participation, especially when students display a lack of interest.

The results from Table 6.12 reveal that while some participants found the collective analysis of the implementation manageable (23,08% totally agreed and 38,46% partially agreed), a notable proportion (30,77%) partially disagreed, indicating that challenges persisted in conducting the analysis. A possible constraint here is the lack of structured proposals for didactic analysis within the teacher education programme, which may have left preservice teachers unfamiliar with systematic methods for evaluating their teaching practice. This gap in their education could have contributed to difficulties in analysing the implementation effectively, as some participants may not have been adequately prepared to critically reflect on their own instructional decisions and their impact on student learning. Additionally, variations in group collaboration and differing levels of prior experience with didactic analysis further compounded these challenges, highlighting the need for more explicit integration of didactic reflection and analytical frameworks within teacher education.

The scale of the levels of didactic codeterminacy was considered useful by most participants (38,46% totally agreed, 53,85% partially agreed), demonstrating its relevance in structuring the analysis of the implementation. However, the statement "One session of discussion and analysis was enough" received mixed responses, with only 30,77% totally agreeing and 38,46% partially agreeing, while 30,76% felt that one session was insufficient. This suggests that time constraints limited the depth of analysis, reflecting an ecological constraint—the need to balance comprehensive reflection with the available instructional time.

The first part of Table 6.13 highlights the participants' reflections on possible adaptations to the inquiry-based activity for future implementations. A recurrent theme in the responses is the need for greater student autonomy, suggesting that the initial design may have retained some traditional teacher-led elements. The responses also indicate a strong desire to integrate more digital tools, such as Excel for graph plotting, aligning with a broader technological shift in didactic practices. These reflections align with the *mesogenesis* of the teaching process, which concerns the design and adaptation of the learning environment, including the tools and resources used. Another key adaptation proposed was the necessity of better familiarising themselves with the students before implementation, indicating that a lack of prior knowledge about student backgrounds posed a constraint in effective lesson planning. This highlights a *topos* issue, as it concerns the role distribution between teachers and students and how responsibilities are shared in the inquiry process.

Regarding the second and third parts of Table 6.13, one of the key positive aspects identified is the opportunity for participants to engage in collective reflection and experience-sharing, which aligns with the role of *a posteriori* analysis the SRP-TE. The presence of all participants in a single session enabled a comprehensive and structured overview of individual experiences, fostering a collaborative learning environment. The discussions and feedback from both peers and educators were seen as valuable tools for refining teaching practices and understanding challenges in implementation. Additionally, several participants acknowledged that the analysis of the conditions and constraints encountered, through the scale of levels of didactic codeterminacy, helped them recognise both modifiable and unmodifiable aspects of different natures in the teaching environment. The emphasis on SRP fostering secondary students' autonomy also demonstrates a favourable epistemological condition, as it indicates an awareness of shifting pedagogical roles.

Despite these strengths, several constraints emerged that affected the effectiveness of the *a posteriori* analysis. One key limitation made explicit again was time management, with some groups falling behind schedule, preventing the completion of all analyses. This reflects an institutional constraint, as limited time for deep reflection may hinder the thorough application of inquiry-based proposals. Another recurring issue was the difficulty in understanding certain didactic concepts, particularly those related to the scale of levels of didactic codeterminacy and the condition classification among the levels. This suggests a lack of sufficient prior exposure to these analytical frameworks within the teacher education programme, constituting an epistemological-didactic constraint for preservice teachers themselves. Additionally, some participants noted that their analysis process felt rushed due to frequent shifts in focus, which may indicate a lack of continuity in their investigative process, potentially linked to insufficient prior training in didactic analysis. Moreover, there were challenges in maintaining group engagement, with some members contributing less actively, making the analysis process uneven. This reflects a *topos* constraint related to group collaboration dynamics, which can impact the depth of shared reflections.

The implementation of the SRP-TE in this study has provided significant insights into the conditions that facilitate the visibility of data treatment in mathematics education and the constraints that limit it. Through an ecological analysis, preservice teachers critically examined their experiences, highlighting both the strengths and limitations of the approach. The findings from this study align with the research question RQ_4, as they identify the specific conditions (structured role transitions, digital tools, inquiry-based discussions, and didactic analysis frameworks) that support preservice teachers in making data treatment more visible. Simultaneously, the constraints observed (institutional limitations, role tensions, and statistical knowledge gaps) illustrate the challenges that hinder this process.

Regarding the specific objective *SOc* (ecology through different modalities of SRP-TE), this study contributes to understanding the ecology of SRP-TE by analysing how the face-to-face SRP-TE interacts with institutional, pedagogical, and disciplinary conditions in secondary school teacher education. The ecological perspective highlights the complex interplay between teacher professional development, classroom realities, and systemic constraints, reinforcing the need for longitudinal interventions that extend beyond isolated pedagogical experiences.

6.5. COMPARATIVE ANALYSIS: PILOT SRP-TE FOR PRESERVICE TEACHERS, ONLINE SRP-TE FOR IN-SERVICE TEACHERS, AND FACE-TO-FACE SRP-TE FOR PRESERVICE TEACHERS

In this section, we present a comparative analysis of the three SRPs-TE implementations discussed in this dissertation. As highlighted in Chapter I, our research focus has progressively expanded with each implementation of the SRP-TE. Initially, our object of study was the ecological aspects of SRP-TE within the contexts of initial mathematics teacher education and statistics at the lower secondary school level (for students aged 11-14). This goal was pursued through the study of the pilot SRP-TE, as presented in Chapter IV. Subsequently, an opportunity emerged to collaborate with Brazilian researchers from the Pontifical Catholic University of São Paulo and the Federal University of Bahia, facilitating the design and implementation of a development programme with in-service mathematics teachers who work both in the lower and upper secondary school levels (for students aged 11-17). This research collaboration enabled the design and implementation of an SRP-TE for in-service teachers in an online modality, which was redesigned considering the findings from the pilot SRP-TE. Finally, these two implementations made it possible to design and implement the last study, the face-to-face SRP-TE for preservice teacher education, which is presented and analysed in this chapter. It is evident that each SRP-TE version implemented in advance was a reference for the next implementation, following the different rounds of the didactic engineering process, even if the proposals were implemented under different conditions. Table 6.14 summarises the general aspects of the three SRPs-TE discussed in this dissertation.

Table 6.14: Reflections and adaptations regarding the SRP implementations

Aspect	Pilot SRP-TE (Chapter IV)	Online SRP-TE (Chapter V)	Face-to-face SRP-TE (Chapter VI)
Modality	Online	Online	Face-to-face
Research collaboration	Federal University of Sergipe (UFS) Federal University of Mato Grosso do Sul (UFMS) Universitat de Barcelona (UB)	Pontifical Catholic University of São Paulo Federal University of Bahia UFMS UB	UFMS UB
Context and participants	Preservice secondary mathematics teachers in Pedagogical Residency Programme (PRP) of the UFS	In-service secondary school teachers from different regions of Brazil	Preservice secondary mathematics teachers in Pedagogical Residency Programme of the UFMS
Implementation period	Short-term, end of academic year November and December 2021	Fourteen weeks (on Saturdays) September to December 2022	Semester-long programme March to July 2023
Duration and structure	Four 90-minute sessions	Fourteen sessions (3 hours each)	Sixteen 90-minute sessions (aligned with pedagogical residency)
Digital resources	Google Meet Google Drive Online databases	Microsoft Teams WhatsApp Padlet Websites and digital publications Google Drive Online databases Microsoft Excel	WhatsApp Padlet Mentimeter platform Websites and digital publications Google Drive Online databases Microsoft Excel Geogebra
Generating question in Module 1	How can the contradiction between the abundance of water resources and the water problems (scarcity, quality degradation, lack of control, etc.) of Brazilian regions, as illustrated in the graph, be explained? (Considering the textbook exercise)	"Brazil has lost 15% of its water resources in 30 years, a loss of almost twice the water surface area of the entire Northeast region". How to analyse the veracity of this news? (Considering a news report)	Considering the graph presented in the textbook exercise, what questions can we raise about water problem in Brazil? Is the graph still valid? Will anything change if we update the data on population, territory/surface, and water resources? (Considering the same textbook exercise from the pilot SRP-TE)

Aspect	Pilot SRP-TE (Chapter IV)	Online SRP-TE (Chapter V)	Face-to-face SRP-TE (Chapter VI)
Didactic tools introduced in Module 1	Questions-answers dialectic (already familiar to participants)	Questions-answers dialectic and map	Questions-answers dialectic and map
Didactic tools introduced in Module 2	We did not carry out this Module 2 in this SRP-TE	Herbartian scheme Q-A dialectic Media-Milieu dialectic Individual-collective dialect	Herbartian scheme Q-A dialectic Media-Milieu dialectic Individual-collective dialect
Mean characteristics of the design and implementation of SRPs in Module 3	SRPs for lower secondary schools were designed but implemented in real classroom context	An SRP for grade 6 was designed and implemented (Paola's case): nine 50' sessions	Three SRPs were designed and implemented in three schools: (1) One group of Grade 8 and three groups of Grade 9 - Six 50' sessions (2) One group with students from grades 10, 11 and 12 (multi-grade) - Six 50' sessions (3) One group with students from grade 6 - Six 50' sessions
Didactic tools introduced in Module 4	-	Didactic contract	Scale of levels of didactic codeterminacy
Reflection and analysis in Module 4	Validation of the potential of the generating question of Module 1 SRP. First manifestation of the phenomenon of invisibility of data treatment.	Accessible to teachers across Brazil, integration of digital tools. Opportunity of introducing didactic tools for the analysis in Module 2. Second manifestation of the invisibility of data treatment phenomenon. In Module 3, only three teachers implemented SRPs in 'good conditions': in their classrooms. So, no such strong restrictions arose.	SRPs implemented in secondary schools aligned with the PRP. Opportunity of introducing didactic tools for the analysis in Module 2. In Module 3, the institutional constraints that appeared during the implementations of the SRPs prompted the ecological analysis carried out by the pedagogical residents in Module 4. Retake of the didactic tools in Module 4 for a deeper reflective process about the implemented SRPs.
Key strengths	Validation of the potential of the generating question of Module 1 SRP. First manifestation of the phenomenon of invisibility of data treatment.	Accessible to teachers across Brazil, integration of digital tools. Opportunity of introducing didactic tools for the analysis in Module 2. Second manifestation of the invisibility of data treatment phenomenon. In Module 3, only three teachers implemented SRPs in 'good conditions': in their classrooms. So, no such strong restrictions arose.	SRPs implemented in secondary schools aligned with the PRP. Opportunity of introducing didactic tools for the analysis in Module 2. In Module 3, the institutional constraints that appeared during the implementations of the SRPs prompted the ecological analysis carried out by the pedagogical residents in Module 4. Retake of the didactic tools in Module 4 for a deeper reflective process about the implemented SRPs.

Aspect	**Pilot SRP-TE (Chapter IV)**	**Online SRP-TE (Chapter V)**	**Face-to-face SRP-TE (Chapter VI)**
Key constraints	Limited time, online mode, restricted students' engagement, lack of deeper didactic analysis.	Difficulties for participants to assume the role of students in Module 1. Teachers struggled with using digital tools effectively, some limitations in data collection and reflection. More effort should have been put in the study of elementary tools for data analysis.	The institutional constraints found by students when implementing their SRPs in Module 3 (curriculum rigidity, limited school resources, etc.) had a positive role in facilitating the ecological analysis. However, they also limited the effective implementation of SRPs. Difficulties in introducing totally new didactic tools for the didactic analysis in Modules 2 and 4.

The implementation of SRPs-TE across different contexts and modalities have revealed significant variations in the roles of the participants and educators, the nature of professional development, and the challenges encountered. The *Pilot SRP-TE* served as an initial experiment to explore the potentiality of the generator question designed for the SRP-TE within preservice teacher education. It primarily focused on structuring inquiry-based learning through a textbook exercise in which we proposed a change of paradigm, highlighting the invisibility of statistical work often present in conventional teacher preparation. The *topogenesis* of this implementation was predominantly teacher-centred, as educators played a dominant role in guiding the inquiry process at each step. Although participants were involved in the didactic analysis, their autonomy in designing inquiry-based activities remained minimal. Consequently, preservice teachers were positioned primarily as students, with limited opportunities to transition into the roles of independent researchers or teacher-designers.

The *online SRP-TE for in-service teachers* marked a shift towards shared responsibility, as it scaled the SRP-TE approach to a national audience through an online format. Unlike the Pilot SRP-TE, where educators maintained the guidance, the online format encouraged more autonomous practice among in-service teachers. Participants were required to implement SRPs within their own classrooms, taking on the dual roles of teacher-designers and analysts. Educators acted more as facilitators, supporting teachers while granting them autonomy in making contextual adaptations to the SRPs. Despite this flexibility, teachers faced challenges related to institutional constraints, such as curriculum, and a lack of didactic tool exposure.

These constraints hindered their ability to fully integrate statistical inquiry into their teaching practice.

The *face-to-face SRP-TE for preservice teachers* provided the most immersive and structured experience compared to the previous implementations. It aligned with the pedagogical residency programme, offering direct classroom engagement and more comprehensive reflective practices. The *topogenesis* in this implementation was the most progressive, as preservice teachers progressively assumed roles of students, designers, teachers and analysts. Educators acted as mediators rather than instructors, guiding participants through iterative cycles of planning, implementation, and reflection. This dynamic enabled preservice teacher to internalise the role of a teacher-researcher, which was less evident in the previous models. By directly confronting institutional and disciplinary constraints within the school context, the face-to-face SRP-TE offered a reference for integrating statistical inquiry into mathematics teacher education.

Throughout these SRP-TE implementations, the researcher's role (the author of this dissertation) evolved significantly. In the Pilot SRP-TE, the observer position dominated, even if combined with the role of educator, also shared with the main responsible of the course. During the online SRP-TE, the researcher more clearly assumed the role of main responsible and leader of the team of educators during the sessions. However, the decisions made during the *in vivo* analysis were clearly shared with the supervisors who had previous experience in online SRP-TE. Their role was also critical in the introduction of didactic tools. It was in the face-to-face SRP-TE where the researcher fully assumed the role of educator. She conducted the *in vivo* analysis alone, drawing on insights from the online SRP-TE experience to anticipate potential challenges and intervene as necessary to support participants' reflections and adaptations. The researcher played a pivotal role, particularly during Module 1, by assigning specific questions to each group to address emerging statistical knowledge gaps and by emphasising moments when the phenomenon of invisibility of data treatment became evident (Modules 2, 3, and 4). This reflective and adaptive approach strengthened the connection between theory and practice, allowing the SRP-TE framework to evolve in response to real-world challenges and the needs of preservice teachers.

In summary, the comparative analysis of these three SRP-TE implementations demonstrates a progressive shift from theoretical to practical engagement, reflecting an evolution of the participants didactic contract and *topogenesis*. The dynamic interplay between educators and participants shaped the didactic contract in each context, with varying levels of autonomy and support. The evolution also clearly appeared regarding the importance of the ecological perspective about the teaching of statistics inquiry at secondary school level. It underscores the potential of SRPs-TE to enhance secondary school teacher education in statistics, particularly by promoting critical reflection and didactic analysis. To fully realise this potential, sustained teacher training that combines theoretical foundations with practical didactic designs and their implementations is essential, fostering a pedagogical approach that challenges traditional paradigms and nurtures inquiry-based statistics education.

6.6. COLLABORATION MECHANISMS BETWEEN TEACHERS, EDUCATORS, RESEARCHERS AND EDUCATIONAL INSTITUTIONS

Finally, after presenting reflections on the main results of each study presented in this dissertation, with similarities and differences in each case, we discuss the collaboration mechanisms between teachers, educators, researchers and educational institutions, and what didactic tools would facilitate the dissemination of SRP-TE and research in statistics secondary teacher education (RQ5). With the findings of the three SRPs-TE we consider that to facilitate the dissemination of SRP-TE and research in statistics secondary teacher education, the following strategies would be essential.

Collaboration mechanisms:

- *Collaborative and interdisciplinary teams:* The constitution of interdisciplinary teams comprising teachers, educators, researchers, and educational institutions is crucial. These teams bring together diverse expertise, allowing for a comprehensive approach to teacher education. For example, the collaboration between statisticians, didacticians and lecturers from other fields (e.g., social sciences, environmental sciences, experimental sciences, etc.) could bring different perspectives to the training proposals and enrich the design and implementation of SRPs-TE. Collaborative teams ensure that multiple viewpoints are considered, thereby enriching the training process and addressing the complexities of teaching statistics in secondary education.
- *Institutional partnerships:* Developing strong partnerships between educational institutions, such as schools and universities, is necessary to create environments where SRP-TE can be effectively implemented. These partnerships enable the

integration of scholarly and practical knowledge, allowing teachers to bridge the gap between theory and practice. The findings in the case studies suggest that the types of institutions involved influenced the dynamics of the SRP-TE implementations, indicating the importance of institutional collaboration in the success of these programmes.

- *Opportunities for practical implementation in school conditions:* Ensuring that teachers can implement didactic proposals in real school contexts is essential. This practical application allows teachers to test and refine the strategies learned through SRP-TE, making the training more relevant and impactful. The findings highlight that the ability to implement these proposals was a critical factor in determining the effectiveness of the SRP-TE, suggesting that collaboration with schools to facilitate such implementations is necessary.

- *Continuous feedback:* Establishing systematic and structured feedback mechanisms among teachers, educators, researchers, and institutional actors—such as school leadership, curriculum coordinators, and university departments—enables the ongoing refinement of SRP-TE modules. Institutions can contribute feedback by providing insights from school policies, curriculum implementation, logistical constraints, and student performance data, all of which influence the feasibility and relevance of didactic proposals. This multi-level feedback loop ensures that the training remains responsive to the evolving needs of both teachers and learners, allowing for timely adjustments to didactic tools and strategies. The three case studies' findings underscore such collaborative feedback's crucial role in shaping effective and contextually grounded teacher education.

Didactic tools:

- *Didactic transposition processes:* The findings identify the need to study the didactic transposition processes that contribute to the phenomenon of the invisibility of data treatment in both school and scholarly mathematics. Addressing these processes through SRP-TE can help teachers understand and critically engage with the knowledge they are expected to teach. Didactic transposition involves comprehending and adapting complex academic knowledge into forms that are accessible and teachable at the school level, which is central to effective statistics education.

- *Questions and answers maps:* Incorporating Q-A maps as a didactic tool can help in addressing the challenges posed by Klein's double discontinuity, which refers to the gap between school knowledge and scholarly knowledge. By using Q-A maps, teachers can navigate and bridge these discontinuities, by the construction of intermediate reference epistemological models (expressed through the Q-A maps) in between university and secondary school mathematics. They allow teachers to make explicit and accessible the knowledge involve in an SRP, and the inquiry and study dynamics, which is in fact a first step to make it visible and accessible for the students. The SRP-TE framework aims to empower teachers to examine both the knowledge to be taught and its scholarly foundations, thereby overcoming traditional educational limitations.

- *Applicationist conception of teacher education:* The findings highlight the "applicationist" conception of teacher education, which tends to focus on the direct application of theoretical knowledge without encouraging critical examination. SRP-TE offers an alternative by empowering teachers to explore and question the foundations of the knowledge they teach. This shift from a purely application-based model to one that encourages critical engagement is facilitated through didactic tools that promote inquiry, reflection, and adaptation.

- *Media-Milieu (Me-Mi) dialectic:* The incorporation of media-milieu dialectic in SRP-TE helps teachers understand the interaction between different educational media, made accessible for students and teachers, and the learning environment. This tool enables teachers to design more effective learning experiences by considering how different media (e.g., textbooks, digital resources) interact with students' learning milieus. This tool was emphasised in the evolution of SRP-TE modules and is crucial for adapting teaching strategies to diverse classroom settings.

- *Scales of levels of didactic codeterminacy:* The use of the scale of levels of didactic codeterminacy helps in evaluating and adapting teaching practices based on the conditions and constraints of specific educational contexts. This didactic tool allows for a nuanced understanding of how various factors, such as institutional policies, student demographics, etc., influence the effectiveness of teaching strategies. By using these scales, teachers can better tailor their approaches to meet the needs of their students and overcome institutional constraints.

Overall, these collaborative mechanisms and didactic tools create a robust foundation for disseminating SRP-TE and advancing secondary statistics education. By establishing interdisciplinary teams and institutional partnerships, providing practical opportunities for implementation, and maintaining continuous feedback loops, participants can more effectively address the complexities of teaching statistics. Complementary to these mechanisms, the didactic tools offer concrete pathways for adapting scholarly knowledge to classroom realities. Through this synergy of collaboration and targeted pedagogical resources, SRP-TE emerges as a scalable and context-sensitive approach, capable of enriching teachers' professional development and ultimately strengthening statistical learning in secondary education.

6.7. CONCLUSIONS

One of the central goals of the study presented in this chapter was to create conditions to overcome "applicationism" within the context of teacher education. The design and implementation of a face-to-face SRP-TE aimed to foster critical reflection on statistical concepts rather than merely applying predefined procedures. Through iterative cycles of planning, implementation, and didactical analysis, preservice teachers were encouraged to assume roles as students-teachers-designers-analysts, critically analysing their instructional choices and reflecting on their impact on student learning.

6.7.1. Teacher professional development and the teaching of statistics inquiry

As with the previous online SRP-TE, an important aspect of this face-to-face SRP-TE was the phenomenon of the invisibility of data analysis, especially when dealing with real-world data sets. The preservice teachers were able to engage students in collecting, organising, and analysing data, promoting critical thinking and fostering statistical literacy. For example, the implementation of Group 1 (in the face-to-face SRP-TE) shows the practical use of data collection and analysis in addressing a real-world issue like school bullying. The pedagogical residents tried to show the importance of data organisation and representation through tables and graphs, engaging students in collaborative analysis, fostering critical thinking (Ben-Zvi & Garfield, 2004), and discussion based on the collected data. The implementation of Group 2 (detailed in section 6.3.3 of Chapter 6), illustrates the integration of data literacy following a curricular demand (Shreiner, 2018), focused on understanding economic concepts like exchange rates and purchasing power parity. The preservice teachers encouraged students to apply mathematical and statistical skills in analysing and interpreting data and, using real-world data (petrol prices from different countries). However, they managed the inquiry in a highly directive way, always presenting students with ready-made questions to answer. In the implementation of Group 3, as in the case of Group 1, students were empowered to actively participate in the data collection process emphasising the importance of data representation (tables and bar graphs) for conveying information effectively and facilitating comparison and analysis. In contrast, the survey in this case was not directed to answer any important questions. In all three cases, the data treatment appeared as a key element of the inquiry process, as highlighted during Module 2 and introduced in the design proposals during Module 3. In summary, all these inquiry processes highlighted the role of statistics in structuring, organising, and analysing the data collected.

6.7.2. Comparison with the previous SRP-TE

The results obtained show some commonalities and differences from the online SRP-TE previously implemented (Verbisck et al., 2024). First, in Module 1, compared to in-service teachers, preservice teachers assume the role of students very easily, certainly because they are younger and continue to be university students. They are not afraid to ask questions and admit to ignoring something. They accepted better the new didactic contract of inquiring into an open professional question, worked autonomously, searched data and were able to represent them and obtain conclusions about the situation of water in Brazil. Comparing the online SRP-TE for in-service teachers with the face-to-face SRP-TE for preservice teachers, the implementation of online SRP-TE for in-service teachers revealed significant challenges in achieving a paradigm shift from traditional teaching practices to inquiry-based approaches. In-service teachers often exhibited a conservative stance towards changing their established practices, partly due to institutional constraints and a lack of didactic tool familiarity. The didactic contract in this setting remained teacher-centred, with educators primarily guiding the inquiry process rather than encouraging autonomous investigation. Conversely, the face-to-face SRP-TE for preservice teachers demonstrated a greater receptiveness to new didactic contracts, as preservice teachers readily embraced open inquiry and the exploration of generating questions. Their dual role as learners and teacher-designers allowed for a more dynamic interaction with the teaching content, fostering a more flexible didactic contract that accommodated reflective practice.

6.7.3. Addressing the ecological problem in the teaching of statistics inquiries

Two major characteristics of SRP-TE must be highlighted. First, modules 1 and 2 offered a situation where teacher-students could carry out an inquiry where statistics appears as a useful tool. Second, modules 3 and 4 constituted a way to confront the teaching proposals with the schools' realities and helped residents identify some constraints derived from the paradigm of visiting works that affect directly to data treatment using the analytical tools introduced in Module 2.

The fact that the phenomenon of data invisibility appeared in Module 2 confirms its prevalence in school and university mathematics education. It was also an expected fact that helped the educator make it visible to the students and support the design of their teaching proposals in the schools. This is an interesting case of the role played by SRP-TE in the dissemination of results from research to the teaching profession. A didactic phenomenon is identified in research—the invisibility of data treatment—and evidenced in the first activity carried out by the teacher-students. The analytical tools proposed in the course included a description of this phenomenon and its relationship to the prevailing conception of statistics in secondary school education. The implementation and analysis by the preservice teachers of specific teaching proposals provide further evidence of this phenomenon and allow educatots to highlight other school constraints related to the prevailing paradigm of visiting works.

An essential aspect of this study has been the integration of the ecological analysis within the SRP-TE, as it allows for a comprehensive examination of the conditions and constraints that influence the implementation of inquiry-based teaching in statistical education. By introducing and using the scale of levels of didactic codeterminacy, the study has systematically identified institutional, pedagogical, and disciplinary barriers that preservice teachers encounter when adopting innovative didactic approaches. This ecological perspective not only facilitates a deeper understanding of the multifaceted challenges inherent in educational contexts but also informs more targeted interventions to address them. Consequently, incorporating ecological analysis as a core component of teacher education enhances preservice teachers' ability to reflect on their practices critically, adapt to diverse teaching environments, and develop robust strategies to overcome contextual limitations. This result goes in line with the study of Barquero and Ferrando (2024), who also highlight the critical role of the ecological analysis in a two different teacher education courses for secondary level in the case of mathematical modelling.

Last but not least, a more continued training concerning the ecological analysis within the SRP-TE, with the introduction of epistemological and didactic tools that facilitate dealing with this ecological dimension, seems indispensable for fostering sustainable and reflective professional development in mathematics and statistics education.

CHAPTER VII: GENERAL CONCLUSIONS

This chapter aims to revisit the main aspects of the doctoral research by developing a set of final reflections, which will cover the central elements of the investigation: the research questions, the objectives and hypotheses formulated, and the theoretical and methodological framework adopted. Additionally, in these conclusions, it is important to highlight the scientific validation of the research, the variables that are difficult to control, the implications and limitations of the research, the perspectives for future research, and the contribution of this research to the field of mathematics education. Despite the division into two main sections—*a posteriori* analysis and open questions—, we emphasise that the relevant aspects are globally interconnected throughout this final chapter.

7.1 *A POSTERIORI ANALYSIS* OF THE ENTIRE PROCESS

7.1.1. Hypotheses, research questions, objectives and methodology

In this research, we focused on the existing gap between the statistics in secondary school and the statistics in teacher education. In terms of the Klein's double discontinuity, we looked at the second discontinuity, which refers to the distance between the knowledge studied at university by preservice teachers and the knowledge they need when they get into the teaching profession. We place this research within the anthropological theory of the didactic (ATD, Bosch & Gascón, 2006; Chevallard, 1985; Chevallard, 2015; Chevallard & Bosch, 2020) and seek to provide elements of answer to the main research question:

RQ: *What conditions are needed to help preservice and in-service secondary school teachers design, analyse and implement new didactic processes for teaching statistics that overcome the phenomenon of data treatment invisibility?*

Based on this RQ, we have presented a general objective for the development of the research:

General objective: *Analyse the necessary conditions to implement SRP-TE in preservice and in-service secondary school teacher education, as well as the institutional constraints that hinder the transfer of tools to teachers for the design, analysis, and implementation of didactic processes of statistics in the transition to the paradigm of questioning the world.*

To this end, three general hypotheses have guided the research:

- Hypothesis 1. An important didactic phenomenon concerning statistics education at secondary level (and beyond) is the invisibility of data treatment in both school and scholarly mathematics. This phenomenon is the consequence of specific didactic transposition processes that are to be studied more in-depth. In any case, we postulate that it should also be approached as a teacher education problem as it is central to the Klein's double discontinuity problem mentioned before.
- Hypothesis 2. Concerning statistics teacher education, two mean types of constraints appear, one linked to a limited view of statistics (dominant epistemological model of statistics) and the other linked to an "applicationist" conception of teacher education (dominant didactic model of teacher education).
- Hypothesis 3 (the main statement defended in this dissertation). From the ATD, it comes the assumption that study and research paths for teacher education (SRP-TE) have the capacity to lead teachers to question the knowledge to be taught and even, sometimes, the related scholarly knowledge (Barquero et al., 2022). SRP-TE can also be a potential device to address the didactic phenomena of the "invisibility of data treatment" and the presence of "applicationism in teacher education". They are appropriate educational proposals to approach the limitations of the paradigm of visiting works and set the bases for a transition towards the paradigm of questioning the world.

Five research questions (RQ1-RQ5) were raised, leading to four specific objectives (presented schematically in Figure 2.5, Chapter II). They were addressed throughout four empirical studies that, taken all together, correspond to various cycles of didactic engineering processes. This dissertation was structured into distinct chapters, each contributing to the overarching research framework. Notably, some of these chapters have been published as journal papers, book chapters, or proceedings. The studies carried out and presented in each chapter address the specific research questions and objectives raised in section 2.2 and the didactic engineering phases (subsection 2.3.1), which is assumed as the general research methodology.

To summarise the main results of our research, we are considering the research questions one by one.

7.1.2. Research question 1: Dominant conceptions of statistics for teacher education

The study presented in Chapter III addressed the first research question and the specific objective A:

RQ1. *What are the dominant conceptions of statistics and statistical teacher education in institutions related to the knowledge to be taught at the secondary school level and teacher education? How might these conceptions hinder the establishment of favourable conditions for statistical education that emphasise the study of data and their variability?*

SO_A. *Analyse the conception or model of statistics as data science that prevails through curricular documents (textbooks, national and international curricula, teaching resources, etc.) in the noosphere and in scholarly institutions. Study the consequences concerning the phenomenon of "applicationism" and that of the "invisibility of data treatment" in teacher education.*

From our findings, two main conceptions of statistics are identified in the scholarly institution: "conceptualist statistics" and "data science". In the process of didactic transposition, "conceptualist statistics" dominates both scholarly knowledge and the traditional organisation of the statistical knowledge to be taught. This approach focuses on descriptive statistics, probability, and inference, with probability being the foundation for inference. This traditional view gives little importance to data treatment, which is not given a specific status within the mathematical work. In contrast, the conception of statistics as data science emphasises the importance of considering data and their variability, thus offering a more comprehensive framework for statistical education. In the dominant conception, the notion of measure in statistics is typically confined to the calculation of standard summaries such as the mean, median, and standard deviation. This aligns with the observations made by Ben-Zvi and Garfield (2004), who highlight that statistical reasoning often remains confined to procedural techniques, neglecting the broader contextual interpretation of data. The conception of statistics as data science underscores the importance of addressing data variability and engaging in the modelling of datasets through comprehensive measures and calculations of distance between these measures and the dataset itself (Cleveland, 2001). What is notably absent in "conceptualist statistics" is the concept of modelling a dataset through measures and calculating the distance between these measures and the dataset itself. This gap signifies a critical aspect that needs to be further developed.

In what concerns secondary school, three dominant epistemological models are outlined: one limiting statistics to the elaboration and interpretation of summaries, tables and graphs from given (and cleaned) small datasets; another based on the idea of starting with descriptive analysis of datasets and include some elements of informal inference; finally a vision of statistics closer to data science and the work with different types of datasets through project-, inquiry-, or problem-based proposals. Our research does not propose any characterisation of these school conceptions but only uses them to distinguish the scholars' and educators' vision of school statistics depending on their previous academic and professional backgrounds.

When considering the profiles of the teacher educators (researchers in theoretical or applied statistics, and researchers in statistics education), we could not find a clear relationship between the research profile and their vision of the statistics to be taught in teacher education or at secondary schools. However, the exploratory study we carried out did not provide a concrete delimitation of the different epistemological models prevailing in scholarly and teacher education institutions, nor a set of indicators that could help better identify the phenomenon of invisibility of data treatment.

Despite this weakness, our study emphasises the relevance of *applicationism indicators* as tools for researchers to detach from the various institutions involved in didactic transposition processes, a need previously overlooked in teacher professional development. The phenomenon of *applicationism* leads to proposing a unique organisation of the statistical knowledge independently of the specific professional domains where it will be applied. This narrows the focus on applying ready-made statistical techniques without fostering a deeper understanding of data treatment and interpretation. Consequently, teacher education often fails to address the complexities of data variability and contextual interpretation within school contexts. Addressing this gap requires the development of analytical tools that facilitate the identification of transposition processes of statistical knowledge. Such tools would help uncover how institutional constraints influence the way statistics is taught and how teachers develop their professional practices.

Our study acknowledges its limitations and suggests future development areas, such as diversifying the sample of interviewees to include a broader range of university and professional profiles, both in statistics and statistics education. The lack of a specialised mathematical infrastructure tailored to the teaching of statistics further exacerbates this issue, limiting the capacity of teachers to navigate the complexities of data treatment. Systematic dialogues based on PCK (Pedagogical Content Knowledge) and MCKT (Mathematical

Knowledge for Teaching) could be useful to create shared tools for connecting the analysis of transposition processes with the proposal of teacher education processes. The lack of distinction between dominant models of PCK and the absence of a specialised mathematical infrastructure for teachers, particularly in statistics, are highlighted. Integrating existing PCK and MCKT work with a vision that includes various institutions involved in didactic transposition is a significant opportunity.

We also acknowledge the limitations of our study and suggest several areas for future research and development. One essential direction is to expand the diversity of participants, incorporating a broader range of academic and professional profiles to capture diverse perspectives on statistical teacher education. Additionally, fostering interdisciplinary collaboration between mathematics educators and statisticians would help develop a richer understanding of the complexities of teaching statistical concepts. Further empirical investigations are necessary to validate these findings and examine other transposition processes that shape the construction of statistical knowledge within teacher education. This would enhance the contextualisation of statistics as a dynamic and evolving field, rather than a static collection of methods and formulas, as proposed by authors like Burrill and Ben-Zvi (2019) or Ponte and Noll (2018).

Further fieldwork is needed to better understand the different types of institutional constraints acting on secondary teacher education and explore transformation possibilities. Investigating other transposition processes of statistical knowledge is also crucial. We emphasise considering the processes of didactic transposition, particularly in statistics, as they shape and update the knowledge to be taught. Teacher education is crucial in these processes, as teachers play a critical role in implementing and disseminating curricular reforms and serve as intermediaries between scholarly institutions and the teaching profession.

7.1.3. A pilot study and research path for teacher education

Chapter IV presents a study based on the design, implementation and analysis of a pilot SRP-TE that takes as generating question a problem of water distribution in Brazil. The role of the pilot SRP-TE is attached to the design process of the future online and face-to-face SRPs-TE.

Therefore, the pilot study addressed the second research question:

RQ2. What educational proposal based on SRP-TE can be designed and implemented with a group of preservice mathematics teachers in Brazil? How can this proposal contribute to *providing future teachers with tools to design, analyse and implement new didactic processes for the teaching of statistics in lower secondary school?*

It partially contributes to the specific objective SO_B: *Study the SRP-TE ecology through the design, implementation, analysis, and development of different modalities of SRPs-TE for pre-service secondary school teacher education in the field of probability and statistics*.

The hypothesis H3 posits that designing and implementing an SRP-TE in statistics with preservice mathematics teachers will equip them with tools to develop, analyse, and implement statistics teaching proposals towards the paradigm of questioning the world. This paradigm shift might also encourage teachers and researchers to adopt a critical perspective on the potential social, educational, political, and epistemological role of statistics.

The first *a priori* analysis was based on the transformation of a textbook exercise on the distribution of water resources, population and territory into an SRP generating question. The proposal incorporated aspects of statistical work that are not currently covered in secondary school and that emphasises working with data. It also incorporated open questions that could arise in the inquiry process (Verbisck et al., 2023). As part of the *a priori* analysis, pilot SRP-TE was implemented to test the generating power of this question. The pilot study was conducted under restricted conditions, including a limited number of sessions, the end of the academic year, and the necessity for online interactions due to the COVID-19 pandemic. Despite these constraints, valuable insights were gained regarding the teacher education proposal and its potential future implementations.

The proposed generating question showed a clear questioning power. Preservice teachers spontaneously raised several types of derived questions, thus enriching our initial *a priori* map of questions and answers. They expanded their inquiries to encompass geographical, social, political, and environmental issues, showing the interdisciplinary strength of the proposal, and aligning with Chevallard's concept of a "co-disciplinary symphony" (Chevallard, 2004b).

However, a surprising result needs to be mentioned. Compared to the *a priori* design, statistical and data processing aspects appeared only tangentially in the SRP. The teacher-students did not prioritised searching for and using data as anticipated. Maybe limited time for the activity influenced their focus on easily accessible topics. Additionally, the statistical work underpinning many studies was often invisible and difficult to access, highlighting the challenge of establishing an effective media-milieu dialect and showing the phenomenon of the invisibility of data treatment. The phenomenon particularly appeared when students were asked to associate curricular knowledge with the inquiry they just lived and the adapted teaching proposal they were asked to design.

Although the design aimed to prioritise data treatment by the choice of the generating question, the practical implementation revealed that teacher-students did not place sufficient emphasis on data collection and analysis as anticipated. Integrating statistical literacy into teacher education requires explicit and systematic attention to data analysis practices (Batanero et al., 2011; Gould et al., 2018). The limited time available for the activity influenced the focus on easily accessible topics, and the prevailing invisibility of statistical work within educational practices became evident. To this respect, we can consider that the pilot study was a successful experience given the way it highlighted the phenomenon of the invisibility of data treatment and provided us with orientations to better approach it in the design and implementation of the subsequent SRPs-TE.

7.1.4. An online study and research path for in-service teacher education

Chapter V focuses on the third research question related to the implementation of an SRP-TE for in-service teachers:

RQ3. *What conditions implemented in an online SRP-TE can help teachers address and detach themselves from the phenomenon of the invisibility of data treatment in a context of online in-service teacher education? What constraints limit it?*

It contributes to the specific objective *SO_C*: *Study the SRP-TE ecology through the design, implementation, analysis, and development of different modalities of SRPs-TE for in-service secondary school teacher education in the field of probability and statistics*.

This study regards an online SRP-TE was implemented to in-service secondary teachers across Brazil, designed from the pilot study. Fourteen three-hour Saturday sessions—each comprising plenary and small-group work—were hosted synchronously via Microsoft Teams, which also functioned as a repository for readings, data and collaborative artefacts; all sessions were recorded. Co-developed by researchers from the Pontifical Catholic University of São Paulo, the Federal Universities of Mato Grosso do Sul and Bahia, and the University of Barcelona, the course guided participants through the five SRP-TE modules.

The ecological analysis of this implementation was presented in two publications: Verbisck et al. (2023) and (2024). The analysis highlights several conditions and constraints related to the invisibility of data treatment in an online SRP-TE for in-service teachers. Initially, groups faced significant challenges in starting their inquiries, particularly in the aspects of data collection, organisation, and analysis during the SRP lived in Module 1. Educators had to provide more guidance than anticipated, a problem not unique to online settings but also present in face-to-face environments (Ben-Zvi & Garfield, 2004; Batanero et al., 2011). Moreover, strong invisibility of statistical knowledge was observed, where participants undervalued essential aspects of data gathering and organisation in Modules 2 and 4. However, the successful experience of one teacher (Paola's case) highlights the potential for progress and transformation within the SRP-TE framework, demonstrating that targeted support can foster positive outcomes (Barquero & Bosch, 2015).

When the proposal succeeded in overcoming the invisibility of data treatment, it facilitated the development of inclusive statistics using digital resources. The inclusion of real-world issues, like the distribution of water resources, helped integrate statistical reasoning into broader social and environmental contexts. This integration fostered a meaningful connection between statistical inquiry and broader social and environmental contexts, aligning with recent literature on inclusive mathematics education (Kollosche et al., 2019; OECD, 2023). The structure of the SRP-TE supported teachers in transferring their inquiry experiences to their classrooms, promoting inclusivity despite infrastructural limitations, as seen in Paola's case. This double inclusion of diverse teachers and their students through the SRP methodology highlights the potential for broader applicability and relevance of statistical education. Despite infrastructural challenges, the SRP-TE managed to establish conditions for teachers to adapt their inquiry experiences to their classrooms, highlighting the potential of inclusive education through digital means (Alquati Bisol et al., 2015).

The study also underscores the need for addressing the enduring issue of data treatment invisibility. Participants often struggled with empirical data tasks and underestimated the importance of data processing, indicating a necessity for a shift in how statistical knowledge is conceptualised and taught. This finding aligns with the ongoing debate in mathematics education about the transition from traditional statistical teaching to more comprehensive and data-driven approaches (Ben-Zvi, 2020; Loy et al., 2019), emphasising the importance of experiential inquiries and the need for clearer recognition of data treatment within school mathematics.

Contrasting with the pilot implementation, this online SRP-TE for in-service teachers was convenient to introduce the participating teachers to some ATD analytical tools, especially the questions and answers, the media-milieu, and individual-collective dialectics, which were essential for the collective analysis (Module 2) of the SRP experienced in Module 1. In addressing the research question about the conditions and constraints of implementing an online SRP-TE for in-service teachers, the study reveals that while there are significant challenges, such as difficulties in initiating inquiries and undervaluing key aspects of data treatment, there are also promising conditions for success. These include the accessibility of online formats and the integration of real-world issues to make statistics education more relevant and engaging (Monteiro & Carvalho, 2023; Newton et al., 2011). The constraints mainly revolve around the invisibility of statistical processes and the need for greater support and clearer status of data treatment in school curricula. Future research is needed to regularly raise the question of the epistemological and educational tools relevant for the teaching profession, ultimately fostering a culture that values comprehensive data treatment in educational contexts.

7.1.5. A face-to-face study and research path for preservice teacher education

In Chapter VI, the study aimed to address the fourth research question related to the implementation of an SRP-TE for preservice teachers:

RQ4. *What conditions implemented by a face-to-face SRP-TE can help preservice teachers address the phenomenon of the invisibility of data treatment and what constraints appear to limit it?*

And the fifth research question related to the ecology of SRP-TE and the dissemination of didactic research tools in teacher education:

RQ5. *What collaboration mechanisms between teachers, researchers and educational institutions? What didactic tools would facilitate the dissemination of SRP-TE and research in statistics preservice and in-service secondary teacher education?*

The study contributes to the specifics objectives SO_B**:** *Study the SRP-TE ecology through the design, implementation, analysis, and development of different modalities of SRPs-TE for preservice secondary school teacher education in the field of probability and statistics*, and SO_D**:** I*dentify the general conditions that favour and the constraints that hinder the implementation of SRP-TE, comparing preservice and in-service modalities*.

Two key characteristics of the face-to-face SRP-TE emerged: First, Modules 1 and 2 provided a scenario where teacher-students could conduct inquiries using statistics as a useful tool. Second, Modules 3 and 4 helped align teaching proposals with school realities, allowing residents to identify constraints related to the paradigm of visiting works and data treatment using analytical tools introduced in Module 2. The invisibility of data treatment, evident in Module 2, confirmed its prevalence in school and university mathematics education. This phenomenon was made visible to students and supported the design of their teaching proposals. The SRP-TE played a crucial role in disseminating research findings to the teaching profession, highlighting the invisibility of data treatment and related school constraints.

The results highlighted some similarities and differences compared to the previous study, the SRP for in-service teachers (Chapter V). In Module 1, preservice teachers, assuming the position of university students, adapted to their student roles more easily than in-service teachers. They were open to asking questions and admitting gaps in their statistics education, working autonomously to search for and represent data and draw conclusions. However, one negative aspect observed was that preservice teachers expressed a desire to learn more statistics, indicating a gap in their statistical literacy, reasoning, and thinking. This gap aligns with challenges previously identified by Batanero and Díaz (2010) in teacher education to develop statistical knowledge. Consequently, this highlights the need for future investigations to design an SRP-TE that integrates more robust elements of statistical literacy, reasoning, and thinking, as advocated by Garfield and Everson (2009), to foster a deeper and more comprehensive understanding of statistics.

One important result appears in relation to the conditions and constraints of the implementations of SRPs in secondary schools by the four teams of the preservice teachers. The fact that the participants were pedagogical residents was an excellent condition to implement different teaching proposals in four secondary schools. Group 1 implemented practical data collection and analysis to tackle real-world issues like school bullying. The preservice teachers emphasised the importance of organising and representing data through tables and graphs, engaging students in collaborative analysis, critical thinking, and discussion based on the data collected. Group 2 focused on integrating data literacy in the teaching proposal, particularly around economic concepts like exchange rates and purchasing power parity. The preservice teachers encouraged students to apply mathematical and statistical skills to analyse and interpret real-world data, although their inquiry was very directive, providing students with predetermined questions. Group 3 involved students in data collection and emphasised data representation for effective information comparison and communication. However, their survey lacked direction towards answering significant questions. Unlike Group 1, which collected data to answer questions related to a social problem, the implementation of Group 3 was characterised as a small simulation of a statistical survey and more distant from the paradigm of questioning the world. In the case of the implementation of Group 4, it was able to observe the strong curricular constraint that prevented them from carrying out a teaching proposal involving statistics. In all cases, except the last one, data treatment emerged as a crucial element of the inquiry process, as highlighted in Module 2 and incorporated into design proposals in Module 3. These processes underscored the role of statistics in structuring, organising, and analysing collected data.

Another important aspect of this face-to-face SRP-TE is the opportunity presented in Module 4 to hold an extensive discussion about the conditions and constraints observed by the groups of teacher-students during their school implementations. The groups contextualised these conditions and constraints using the scale of levels of didactic codeterminacy, which served as a valuable analytical tool. Notably, a strong point of this dissertation lies in the ecological analysis carried out with preservice teachers, as the ecological dimension is seldom examined in teacher education courses. This perspective is crucial, as it addresses the interconnectedness between educational practices and their broader institutional, social, and cultural environments, as discussed by Bosch and Gascón (2006) and Chevallard (2004b).

The face-to-face SRP-TE also saw a significant increase in secondary school implementations, which demonstrated a greater alignment of teaching proposals with school realities and adaptive approaches to data treatment. Nevertheless, these implementations faced institutional constraints, highlighting the persistent challenge of data treatment invisibility. The ecological analysis provided insights into how preservice teachers navigated these challenges, making it a particularly innovative and essential contribution to the field. These findings underscore the importance of equipping preservice teachers with analytical tools and experiences that allow them to critically examine the institutional and ecological aspects of their practices.

Moreover, the comparative analysis between different SRP-TE modalities revealed that the face-to-face approach yielded deeper engagement and practical implementation opportunities compared to online models. While the online SRP-TE for in-service teachers faced challenges in promoting inquiry-based practices, the face-to-face model allowed for direct classroom implementation and practical reflection. Despite promoting innovative practices, systemic constraints remain, necessitating continued efforts to integrate critical reflection and practical strategies within teacher education processes. Furthermore, fostering collaboration among educators, researchers, and institutions is essential to ensure the sustainable dissemination of innovative practices in statistical education.

7.1.6. The three SRP-TE as a complete process of didactic engineering

In the preliminary analysis, we identified the didactic phenomenon related to statistics as knowledge to be taught and to statistics teacher education in terms of two hypotheses: the phenomenon of the *invisibility of data treatment* in both school and scholarly mathematics. This phenomenon is the consequence of specific didactic transposition processes that are to be studied more in-depth. In any case, we postulate that it should also be approached as a teacher education problem as it is central to the Klein's double discontinuity problem mentioned before. Assuming that SRP-TE can empower teachers to critically examine both the knowledge to be taught and its scholarly foundations, addressing didactic phenomena and overcoming the limitations of traditional educational paradigms, we aimed to illustrate the first phenomenon and overcome the second one based on evidence gathered from three SRPs-TE implemented and analysed: pilot SRP-TE, SRP for in-service teachers, and SRP for preservice teachers.

The transition from the paradigm of visiting works to the paradigm of questioning the world within the three SRPs-TE (pilot SRP-TE, online SRP-TE for in-service teachers, and face-to-face SRP-TE for preservice teachers) represents a significant rupture in how statistical education is conceptualised and practised. This transition, conceptualised within the framework of the Anthropological Theory of the Didactic (ATD), challenges the traditional approach of presenting knowledge as a static, fixed body of facts to be acquired (visiting works) and moves towards promoting an active inquiry where knowledge is constructed through questioning and investigation (questioning the world).

Although each version implemented in advance was a reference epistemological model for the following implementation (pilot SRP-TE → SRP for in-service teachers→ SRP for preservice teachers), each SRP-TE experimented was uniquely shaped by various conditions that influenced the process and the results. These conditions included the specific group of participants involved, ranging from preservice teachers to experienced in-service teachers, each bringing their own backgrounds, experiences, and expectations. The types of institutions involved and the modalities of the training proposals, whether online or face-to-face, played a significant role in determining the dynamics of interaction, collaboration, and engagement during the sessions. The number of sessions conducted also varied, impacting the depth of the material that could be covered, as well as the opportunities for participants to reflect, apply, and iterate on their learning. The composition and experts of the team of researchers-educators were crucial as well, with each team bringing different perspectives and emphases to the different modules of the training proposals. Moreover, a critical factor was whether the participants had the opportunity to implement the didactic proposals within a real school context. This aspect often determined the practical relevance and adaptability of the SRPs to school conditions, allowing participants to bridge the gap between theory and practice. However, the condition to implement these proposals was not always guaranteed, and this was due to significant constraints.

These various factors did not merely serve as background conditions; they actively influenced the trajectory of each implementation. In some cases, these factors posed constraints that could limit the effectiveness of the training. For instance, online modalities might have restricted the level of interactive engagement possible, or a limited number of sessions might have curtailed in-depth exploration of certain topics. Similarly, the inability to implement proposals in a real classroom might have prevented participants from fully experiencing the practical challenges and advantages of the SRP-TE approach. Understanding these constraints and their impacts is crucial for refining and improving future implementations. By acknowledging and addressing these conditions and constraints, it is possible to design more flexible and adaptive education programmes that better meet the needs of participants and achieve the desired educational outcomes.

Regarding the ATD approach, a comprehensive examination of these three case studies reveals a clear evolutionary process within each implementation. This progression is marked by the strategic introduction of didactic tools, thoughtful redirections, and a heightened emphasis on the modules proposed within the SRP-TE. Each case demonstrates how these elements collectively contribute to refining and enhancing the educational approach, allowing for a more effective and targeted teacher education experience. Over time, the iterative adjustments made in response to observed challenges and outcomes further solidify the robustness of the SRP-TE modules, aligning them more closely with the goals of improving both teaching practices and student outcomes in the context of statistics education. The evolution is illustrated in the Figure 7.1:

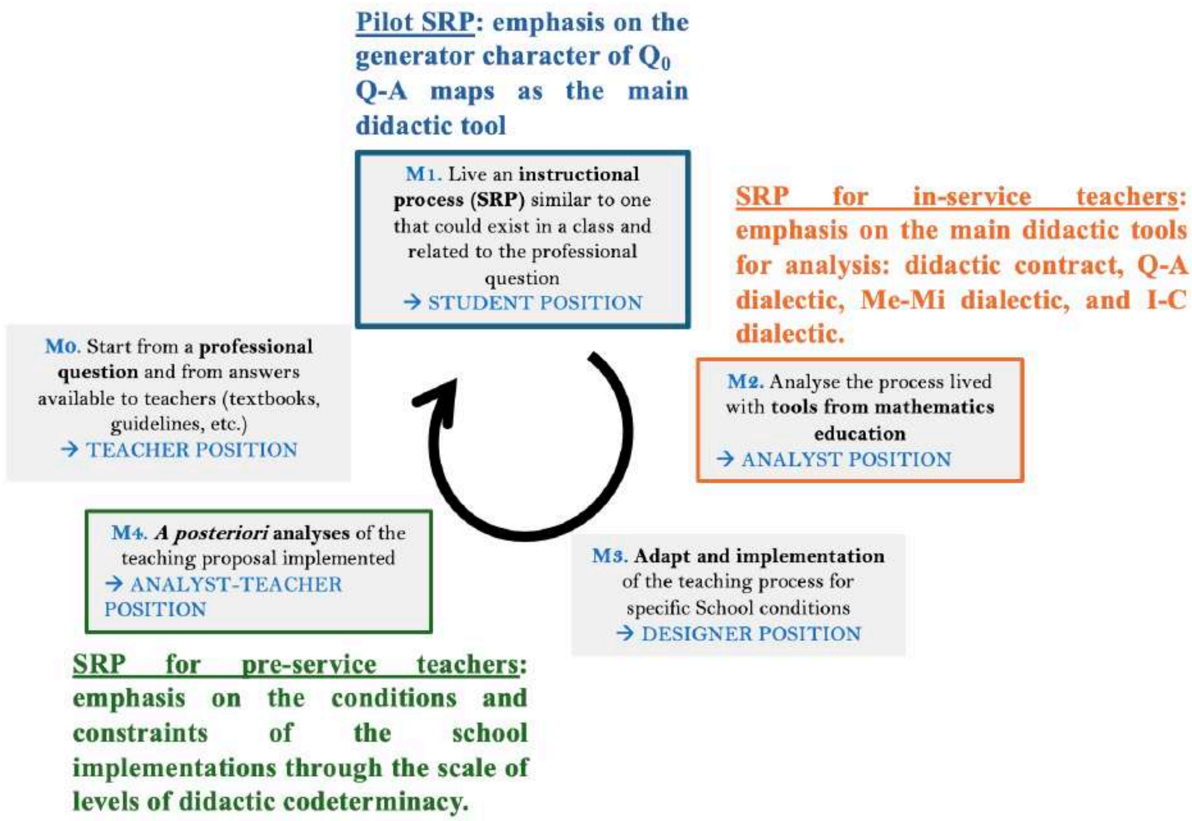


Figure 7.1: Evolution of didactic tools introduced and emphasised in each SRT-TE

A critical analytical perspective highlights the cyclical and unfinished nature of transitioning from one SRP-TE to another. The cyclical nature is rooted in the didactic engineering approach, where each implementation informs subsequent adaptations and refinements. As each SRP-TE attempts to break away from the paradigm of visiting works, "applicationism" and the invisibility of data treatment, it simultaneously uncovers new conditions and constraints that must be addressed in future implementations. The pilot SRP-TE served as a starting point, revealing the difficulties in fostering autonomous inquiry from a textbook exercise in the paradigm of visiting works to the paradigm of questioning the world. This experience informed the online SRP-TE, which sought to scale the approach while granting more teacher autonomy. However, constraints related to institutional inertia and lack of tool familiarity persisted, informing the face-to-face SRP-TE that followed. The latter represented a more progressive rupture but faced challenges related to didactic analysis and data treatment invisibility.

Each SRP-TE builds on the reflections and results from its predecessor, embodying the unfinished, iterative nature of inquiry-based teacher education. This process aligns with the ATD, which posits that knowledge practices are not static but dynamically evolve through continual questioning and reflection. The transition from visiting works to questioning the world is not a linear progression but a complex, iterative journey, shaped by evolving didactic contracts and contextual realities. In essence, the rupture of paradigms within the SRPs-TE is a gradual and cyclic process marked by continuous reinterpretation and adaptation. While substantial progress has been made, the path towards fully realising the paradigm of questioning the world remains an ongoing challenge, necessitating further exploration and critical reflection within both preservice and in-service teacher education.

Regarding the change of paradigm in statistics, innovative instructional proposals such as project-based learning (PBL) seem to share certain pedagogical intentions with SRPs, particularly in their mutual rejection of transmissive, lecture-based instruction and their commitment to student-centred inquiry. Both approaches foreground real-world phenomena and encourage learners to formulate questions, collect and analyse data, and draw conclusions, thus promoting active engagement with knowledge production. This convergence is evident in Freixanet et al. (2022), where a secondary-level project on noise pollution exemplifies an inquiry-based approach that mirrors the core features of both SRP and PBL. However, beyond these surface similarities, significant epistemological and methodological divergences emerge.

While PBL is frequently valued for its instructional, motivational and collaborative potential, it tends to operate without a robust theoretical apparatus. In contrast, SRPs are designed as didactic devices that are explicitly anchored in a structured theory of knowledge and learning. Within the ATD, the SRP constitutes not only a pedagogical strategy but also a didactic tool for analysing and reshaping the relations between knowledge, institutions, and learners (Chevallard, 2002; Bosch, 2018). As Markulin et al. (2021) argue, many implementations of PBL lack this analytical depth, often neglecting the systemic constraints and conditions under which such learning is to take place.

A further point of distinction concerns the orientation towards curriculum. PBL typically functions as an activity within an existing curricular framework, often without challenging the structure or purpose of that framework. The way of formulating its educational goals remains often inscribed in the paradigm of visiting the world, particularly when focusing of preestablished statistical content organisations, instead of the statistical inquiry itself. SRPs, conversely, are intended as instruments of curricular transformation. They aim to rupture the paradigm of visiting works by repositioning learners as co-investigators in the production of knowledge, an approach encapsulated in the shift towards questioning the world (Barquero & Romo, 2022; Chevallard, 2015). This paradigm shift demands a reconceptualisation of the educational process that goes beyond the scope of conventional PBL. Moreover, SRPs introduce analytical tools such as question–answer maps and the dialectic of theory and practice, which offer a refined approach to managing didactic constraints and fostering sustained inquiry (Florensa et al. 2021). These tools are generally absent in PBL methodologies, where the institutional, ecological, and epistemological dimensions of learning are often under-theorised. As such, SRPs-TE provide a more nuanced account of the conditions under which statistical literacy, reasoning, and thinking can be cultivated in authentic learning environments.

7.2. OPEN QUESTIONS AND PERSPECTIVES FOR FUTURE RESEARCH

Despite the significant contributions of our research to the secondary statistics teacher education field, several critical areas warrant further investigation. These open questions present valuable opportunities for future studies to deepen our understanding and enhance educational practices in these domains. Below, we outline these areas, accompanied by perspectives on how future research could address them.

- *Bridging the gap between teacher education and secondary school teaching:* How can the discontinuity between the knowledge acquired in university-level teacher education programmes and the knowledge required for teaching statistics in secondary schools be effectively bridged?

Talking in terms of Klein's double discontinuity seems to leave the type of statistics (or mathematics, in general) that is taught at the university as unquestioned and point at what should be done, as a complement, to prepare future teachers for the exercise of their profession. On the contrary, future studies might explore the development of university curricula to better integrate the necessities of secondary school teachers (and other professionals), instead of proposing a common knowledge organisation based on the conceptual construction of knowledge instead of promoting its use to formulate and address questions. The vicinity between educators that are researchers in statistics and those who are researchers in statistics education might help to develop new epistemological models to approach and support school statistics and new didactic infrastructure to support the didactic transposition processes.

- *Conceptualising statistics education beyond traditional pedagogical models:* How can we redefine and expand the conceptions of statistics education to include broader, more contemporary approaches such as data science, while ensuring that these are accessible and teachable at secondary school level?

Our study is a modest contribution to this big and crucial area of research, which has been developed at an impressive rate. Collective efforts are used to investigate how the "data science" perspective can be systematically introduced into secondary school curricula, emphasising real-world data gathering, processing, analysis and interpretation (Burrill & Pfannkuch, 2024). Additionally, there is a need to explore methods for modelling datasets and integrating these into existing educational paradigms. From the perspective of the ATD, such investigation would gain in introducing the ecological analysis about the institutional and epistemological constraints that hinder today the evolution of statistical education in secondary education.

- *Addressing the phenomenon of the invisibility of data treatment:* How can teacher education programmes be designed to better address the phenomenon of the invisibility of data treatment and ensure that preservice and in-service teachers develop a deeper understanding and appreciation of data processes?

The phenomenon of the invisibility of data treatment has appeared in our research with an unforeseen strength. It corresponds to a typical didactic transposition phenomenon in an evolutionary knowledge area such as data science. The phenomenon of the invisibility of data processing deserves to be studied in greater depth, for example, by proposing indicators (as in the case of "applicationism") to characterise this invisibility, which might support the analysis or their persistence.

How to address it in practice is a completely different challenge. Future investigations are needed to explore strategies for making statistical processes more visible and comprehensible to teachers, together creating the necessary conditions to give them an official mathematical or statistical status. Additionally, future lines of development can investigate how to foster a critical approach in teacher education, enabling them to question the prevalent traditional paradigms of data handling in education.

- *Expanding the implementation and impact of SRPs-TE:* What are the conditions and constraints that influence the implementation of SRPs-TE in different educational contexts? How can these be mitigated or leveraged to maximise the impact on teacher education?

The present study limits its empirical scope to Brazilian teacher education institutions. Even if we have not identified any specific feature related to the Brazilian society, future investigations could systematically analyse these factors across changes of contexts to develop adaptable and scalable SRP-TE. These include investigating how these models can be adjusted to accommodate different educational environments, from online to face-to-face settings, other countries, etc., and how they can be made more resilient to constraints like limited resources or time.

- *Institutional infrastructures for statistics teacher education:* What institutional infrastructures are necessary to sustain effective teacher education programmes in statistics, particularly those that emphasise innovative approaches like SRPs-TE?

Throughout this dissertation, we have undertaken three rounds of SRP-TE that have been, at each step, adapted to the teacher education context and refined considering the findings from its preceding implementation. This has led the researchers and educators to analyse what conditions were created in each implementation, thanks to which teacher education activities facilitated the work in each module of the SRP-TE. Future investigations might focus on analysing what conditions have been able to be created in each successive redesign of the SRP-TE, identifying best educational practices for institutionalising these conditions. Moreover, this analysis could contribute to detecting the necessary infrastructure to succeed in placing statistics teacher education closer to the paradigm of questioning the world. These include examining the conditions that can facilitate educational policies for teacher education and the statistics teaching at university and secondary school levels, to be detached from the dominant conceptualist and applicationist approach to statistics education.

- *Adapting and transferring SRPs-TE to diverse educational contexts:* What adaptation would be necessary to the SRP-TE model to respond better to specific needs and contexts of preservice and in-service teacher education?

In this dissertation, both preservice and in-service SRPs-TE were implemented under varying conditions —differing time allocations, opportunities for secondary classroom implementations, institutional supports, etc. These implementations revealed distinct needs and enabling conditions for each group. In-service teachers, on the one hand, benefit from direct access to school contexts, but are more resistant to changing many entrenched curricular and organisational conditions. Pre-service teachers, on the other hand, may be more willing to innovate but lack direct classroom intervention. In the case of the participants of the pedagogical residency programme, it was possible to work in the school. Even with these favourable conditions, some groups found difficulties in implementing their didactic designs due to deep-rooted institutional and curricular restrictions (lack of time, preference towards some mathematical domains, such as algebra, functions, etc.), which were mainly manifested by the school teachers accompanying teachers in training. Future research should, therefore, be developed to refine the modular structure of the SRPs-TE to better address the professional needs and conditions existing in the distinct contexts for teacher education.

In conclusion, while our research has made meaningful strides in contributing to statistics teacher education and the teaching of statistics, it has also opened the door to a host of additional questions. Addressing these in future studies will be crucial for advancing the field and ensuring that the educational strategies we advocate for continue to meet the needs of both teachers and students in an ever-changing educational landscape.

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APPENDIX A: GAISE II framework

Process Component	Level A	Level B	Level C
I. Formulate Statistical **Investigative Questions**	Understand when a statistical investigation is appropriate Pose statistical investigative questions of interest to students where the context is such that students can collect or have access to all required data Pose summary (or descriptive) statistical investigative questions about one variable regarding small, well-defined groups (e.g., subset of a classroom, classroom, school, town) and extend these to include comparison and association statistical investigative questions between variables Experience different types of questions in statistics: those used to frame an investigation, those used to collect data, and those used to guide analysis and interpretation	Recognize that statistical investigative questions can be used to articulate research topics and that multiple statistical investigative questions can be asked about any research topic Understand that statistical investigative questions take into account context as well as variability present in data Pose summary, comparative, and association statistical investigative questions about a broader population using samples taken from the population Pose statistical investigative questions that require looking at a variable over time Understand that there are different types of questions in statistics: those used to frame an investigation, those used to collect data, and those used to guide analysis and interpretation Pose statistical investigative questions for data collected from online sources and websites, smartphones, fitness devices, sensors, and other modern devices	Formulate multivariable statistical investigative questions and determine how data can be collected and analyzed to provide an answer Pose summary, comparative, and association statistical investigative questions for surveys, observational studies, and experiments using primary or secondary data Pose inferential statistical investigative questions regarding causality and prediction

Process Component	Level A	Level B	Level C
II. Collect Data/ **Consider Data**	Understand that data are information; recognize that to answer a statistical investigative question, a person may collect data themselves specifically for that purpose, or a person may use data that have been collected by other people for another purpose Understand how to collect and record information from the group of interest using surveys and measurements collected from observations and simple experiments Understand that a variable measures the same characteristic on several individuals or objects and results in data values that may fluctuate Understand that within a data set there can be different types of variables (e.g., categorical or quantitative) Interrogate the data set to understand the context of the variables as they may relate to statistical investigative questions Understand that data are not always pristine but may contain errors, have missing values, etc., and that decisions have to be made about how to account for these issues	Understand that data are information collected and recorded with a purpose and can be organized and stored in a variety of structures (e.g., spreadsheets) Understand that a sample can be used to answer statistical investigative questions about a population. Recognize the limitations and scope of the data collected by describing the group or population from which the data are collected Understand that data can be used to make comparisons between different groups at one point in time and the same group over time Recognize that data can be collected using surveys and measurements, and develop a critical attitude in analyzing data collection methods Understand that quantitative variables may be either discrete or continuous Understand how to interrogate the data to determine how the data were collected, from whom they were collected, what types of variables are in the data, how the variables were measured (including units used), and the possible outcomes for the variables Understand that data can be collected (primary data) or existing data can be obtained from other sources (secondary data) Understand how random assignment in comparative experiments is used to control for characteristics that might affect responses	Word as: Apply an appropriate data collection plan when collecting primary data or selecting secondary data for the statistical investigative question of interest. Distinguish between surveys, observational studies, and experiments Understand what constitutes good practice in designing a sample survey, an experiment, and an observational study Understand the role of random selection in sample surveys and the effect of sample size on the variability of estimates Understand the role of random assignment in experiments and its implications for cause-and-effect interpretations Understand the issues of bias and confounding variables in observational studies and their implications for interpretation Understand practices for handling data that enhance reproducibility and ensure ethical use, including descriptions of alterations, and an understanding of when data may contain sensitive information Understand how concerns about privacy and human subjects may affect the collection and distribution of data Understand that in some circumstances, the data collected or considered may not generalize to the desired population, or

Process Component	Level A	Level B	Level C
III. Analyze the Data	Understand that the distribution of a categorical variable or quantitative variable describes the number of times a particular outcome occurs Represent the variability of categorical variables or quantitative variables using appropriate displays (e.g., tables, picture graphs, dotplots, bar graphs) Describe key features of distributions for quantitative variables, such as: °center: mean as the equal share, and median as the middle-ordered value of the data ° variability: range as the difference between the greatest and least value, and dispersion as how many units from the equal share value ° shape: number of clusters, symmetric or not, and gaps Recognize distributions can be used to compare two groups Observe whether there appears to be an association between two variables	Represent the variability of quantitative variables using appropriate displays (e.g., dotplots, boxplots) Learn to use the key features of distributions for quantitative variables, such as: ° center: mean as a balance point, and median as the middle-ordered value ° variability: interquartile range and mean absolute deviation (MAD) ° shape: symmetric or asymmetric and number of modes Use reasoning about distributions to compare two groups based on quantitative variables Explore patterns of association between two quantitative variables or two categorical variables: ° measures of correlation: quadrant count ratio (QCR) ° comparison of conditional proportions across categorical variables	Use technology to subset and filter data sets and transform variables, including smoothing for time series data Identify appropriate ways to summarize quantitative or categorical data using tables, graphical displays, and numerical summary statistics, which includes using standard deviation as a measure of variability and a modified boxplot for identifying outliers Summarize and describe relationships among multiple variables Understand how sampling distributions (developed through simulation) are used to describe the sample-to-sample variability of sample statistics Develop simulations to determine approximate sampling distributions and compute p-values from those distributions Describe associations between two categorical variables using measures such as difference in proportions and relative risk Describe the relationship between two quantitative variables by interpreting Pearson's correlation coefficient and a least-squares regression line Use simulations to investigate associations between two categorical variables and to compare groups Construct prediction intervals and confidence intervals to determine plausible values of a predicted observation or a population characteristic

Process Component	Level A	Level B	Level C
IV. Interpret Results	Use statistical evidence from analyses to answer the statistical investigative questions and communicate results through structured answers with teacher guidance Make statements about the group or population from which the data were collected, recognizing that conclusions are limited to these groups and cannot be generalized to other groups Describe the difference between two groups with different conditions	Use statistical evidence from analyses to answer the statistical investigative questions and communicate results with comprehensive answers and some teacher guidance Acknowledge that looking beyond the data is feasible Generalize beyond the sample providing statistical evidence for the generalization and including a statement of uncertainty and plausibility when needed Recognize the uncertainty caused by sample to sample variability State the limitations of sample information (e.g., a sample may or may not be representative of the larger population, measurement variability) Compare results for different conditions in an experiment	Use statistical evidence from analyses to answer the statistical investigative questions and communicate results through more formal reports and presentations Evaluate and interpret the impact of outliers on the results Understand what it means for an outcome or an estimate of a population characteristic to be plausible or not plausible compared to chance variation Interpret the margin of error associated with an estimate of a population characteristic Acknowledge the presence of missing values and understand how missing values may add bias to an analysis Use multivariate thinking to understand how variables impact one another Communicate statistical reasoning and results to others in a variety of formats (verbal, written, visual) Understand how to interpret simulated p-values appropriately

Source: ASA, 2020, pp. 16 – 19

APPENDIX B: Interview analysis tables

Descriptions of abbreviations	
Positions	**Knowledge**
RS → Researcher in statistics	S_{SK} = Statistics as scholarly knowledge
RSE → Researcher in statistics education	$S_{KT\text{-}Sec}$ = Statistics as knowledge to be taught in secondary school
ULS → University lecturer in statistics	$S_{KT\text{-}Uni}$ = Statistics as knowledge to be taught in university
SETE → Statistics educator in teacher education	$S_{KT\text{-}TE}$ = Statistics as knowledge to be taught in teacher education

Temas de discusión y preguntas a las entrevistados	**Indicadores relacionados**					
	I_1	I_2	I_3	I_{1FP}	I_{2FP}	I_{3FP}
(1) Describe *what statistics is? How would you define statistics?*	**X**	**X**	**X**			
(2) Gather opinions on what statistics should be taught in secondary school and its *raison d'être*: *What do you think statistics should be taught in secondary school? What do these students need to learn statistics for?*	**X**	**X**	**X**			
(3) Gather opinions on what statistics should be taught in a mathematics teacher education course: *What do you think should be included in the statistics syllabus in the mathematics education programme? What do you consider essential in the initial training of mathematics teachers in statistics?*	**(X)**	**(X)**	**(X)**	**X**	**X**	**X**
(4) Discuss differences between the way probability and statistics are taught in bachelor mathematics and mathematics education (only for those teachers who teach both university courses): *Are there any differences between what is taught in the bachelor mathematics course and the mathematics education course with respect to the subject Probest?* If so, *what are they?*	**(X)**	**(X)**	**(X)**	**X**	**X**	**X**
(5) Gather opinions on variations or new proposals for teacher education: *Do you think that teacher education in statistics could be different from what exists today?* If so, *what kind of education could there be, and what factors do you think restrict this kind of education?*				**X**	**X**	**X**

Interviewee X_1 - comments	**Relations**	**Manifestations**
[*Topic 1*] I always tell my students on the first day of class that statistics is that science that will help how to collect and, before collecting, how to plan a collection; how to collect, how to organise, analyse and, mainly, interpret any kind of data set. I like to summarise **"statistics" as: how to plan, how to collect, how to organise, how to analyse and interpret any kind of data set, whether quantitative or qualitative**.	R_I(RS, S_{SK})	Not I_3. Non-invisibility of data treatment
[*Topic 2*] As I am a mathematician and I have not studied the teacher education, I did not have that experience from secondary school. So my experience is only with my first year students who have just left secondary school and come to study the degree. I think some of the **most important basic statistical skills in secondary school** are **how to work with the mean, the median and how to read a graph or a table**. **[...]** **Regardless of the field in which they are going to work, knowing how to correctly interpret a table or a graph and having at least some notions of the main statistics,** simple ones such as **mean, median, what is a standard deviation, or how to work mean and standard deviation together,** can be important for secondary school students**. Because they are going to use it, regardless of the field in which they work.** So having a basic knowledge of at least these points, I think, is fundamental.	R_I(ULS, $S_{KT\text{-}Sec}$)	I_3. Invisibility of data treatment I_1. Independence
[*Topic 4*] **In a pure mathematics course it is normal to focus on theory.** What I always like to say to students is: **when you graduate and work in the most diverse areas, you will have a notebook and a pen to do any kind of analysis.** So I always try to **show a little bit of theory,** because the person has to know what they are doing. [...] In pure mathematics**, because it is a degree for mathematicians, we try to** go a little bit more into statistical theory**.** We go a little deeper. That does not mean that in the teacher education we only look at the basics. I am not saying that. **In pure mathematics, we try to go a little bit deeper into the theory.**	R_I(ULS, $S_{KT\text{-}Uni}$)	I_1. I_3.
[*Topic 4*] On the other hand, **at the teacher education,** although the syllabus of these courses is basically the same**,** the focus is a bit different. The focus is on trying to **show what the main points are and finding that way to transmit concepts to the students** [future teachers] **so that they can then transmit them to other people.**	R_I(SETE, $S_{KT\text{-}TE}$)	$I_{2.TE}$. Formalisation and application
[*Topics 3 and 5*] I think that, if the aim of the mathematics teacher education is for primary and secondary school, it is not necessary to go a little deeper into statistics because it will not be of great interest for these school levels. I think that **today the syllabus that we have in the course is sufficient for the mathematics teacher education to** be able to work at secondary level**. So, I think that today, when it comes only to statistics,** I do not say anything about other areas of the exact sciences, **what I have experienced today in teacher education is enough to work in basic education.**	R_I(SETE, $S_{KT\text{-}TE}$)	$I_{1.TE}$. Uniformity

Interviewee X_2 - comments	Relations	Manifestations
[*Topic 1*] Basically, I **see statistics as a subfield of mathematics**. Some statisticians do not like that definition very much. But I see it as a subfield. A mathematician, for example, does statistics very well, he just has to adapt. **To me, the definition of statistics is just a sub-area of mathematics that allows you to draw conclusions from information, from data**. And you will draw those conclusions based on mathematical criteria. That is my definition of statistics [...] **Statistics is a mathematical process, with criteria and following the laws of mathematics that allow us, in the future, to draw conclusions**.	$R_I(RS, S_{SK})$	I_2. Conceptualist statistics
[*Topic 2*] Well, I do not have any experience with secondary education, but looking at my students who arrive in the first year of the course, I think that what the secondary school student has to know is that: there is information and that you have to summarise that information. How do you summarise that information? Through the so-called "central measures". **I think it is enough to know the mean; they do not even need to talk about what the median is, what the mode is**. And, above all, how to **interpret graphs**. In my opinion, that would be all. [...] If you can get a student to leave the secondary school knowing how to look at a graph and get information from that graph, I think that would be a great evolution, because here there are many students who do not know how to look at a graph and get some information... simple information, for example, looking at the variation... what that means in a graph. I think if I had to teach in secondary school today, I would focus on **constructing graphs and interpreting what it means**. Of the different types of graphs: pie chart, bar chart, histogram, those cumulative graphs. I think it is interesting.	$R_I(ULS, S_{KT\text{-}Sec})$	I_3. Invisibility of data treatment
[*Topics 3 and 4*] In the **teacher education**, it is as if the preservice teachers are users. So, they have to know that the result is valid, they do not need to prove it. **Because it is a course for engineers... so that is the vision, they see it as a user. I am talking about engineers because a lot of times the preservice teachers or the engineering students take the course together**. So I think they have to know that it is valid and know how to interpret the data. I focus on that approach. For the pure mathematics degree, no. I focus on the construction itself, on the mathematical concepts that allow it to be valid.	$R_I(SETE, S_{KT\text{-}TE})$	$I_{1.TE}$. Uniformity (among engineers and mathematics preservice teachers)
[*Topic 4*] I see **Probability and Statistics** [subject] as a house: the **bottom part** of the house **is probability**, the **middle part** of the house is the **descriptive statistics** part, and the **top part** (the final part) of the house is the **inference** part with which you make decisions and then you need a little bit deeper mathematical knowledge.	$R_I(ULS, S_{KT\text{-}Uni})$	I_2.
[*Topic 4*] In **teacher education**, I start with **probability** and move on to **statistics**. I have the following view of Probability and Statistics: as I said, I see the subject as if I were building a house. So I have to start with probability. Some other lecturers start with descriptive statistics first, then move on to probability and then to interval construction and hypothesis testing. In my opinion, it is a bit wrong, because you do not link probability with statistics for the student. [...] For the teacher education, I start by talking about the whole **concept of probability**; definition from the **classical** point of **view**. Then I present the **frequentist view of probability** and, in particular, I present the **axiomatic definition of probability**. Then comes **geometric probability**. Then I talk about **random variables** and, based on random variables, I **define that bunch of probability distributions**. I tell them: "It is like if you start studying calculus. In calculus you see constant function, linear, quadratic... and then it is the same in probability. Why do you see it? Because each of those probability distributions models a type of problem. Right? So what generates those variables are those models". And from that, I start doing **statistics**. First the **descriptive statistics of the data**, then the whole **process of**	$R_I(SETE, S_{KT\text{-}TE})$	I_2. $I_{3\text{-}TE}$. Invisibility of data treatment

estimation, **interval construction**, and **hypothesis testing**. That is the idea behind the Probability and Statistics (Probest) subject course.		
[*Topic 4*] For the **pure mathematics degree** it is separate: I have a course on Probability and I have a course only on Inference. So, when I talk about **probability, I talk a little bit more in depth**. I start the same way by defining what a random experiment is. But I tell them what a *sigma-algebra* is. So, I define it for them a bit more **from a mathematical, theoretical point of view**. Then I talk about random variables, but I do demonstrations. I also talk about probability models. I talk about the *weibull* model, the *gamma* model, for example, which are not discussed in Probest. I talk about transformations of random variables, which are not discussed in the Probest course. I talk about the moment generating function, characteristic function, probability generating function, which are not mentioned in the Probest course. And then the course ends with the Central Limit Theorem. Which is the theorem that guarantees that, if you take any distribution and you make the "n" tends to infinity, it goes to a normal distribution. Well, that is the end of the probability course. And then it goes on to the Inference course. So, they see inference as a decision-making process. How do you make a decision? First, they see what the estimation methods are. I tell them about the methods of moments, the method of mass likelihood, which is basically Fisher's Theory, where he talks about statistics in two points: having a sufficient sample and a complete sample. So, I define what sufficiency is, what completeness is. And **then I demonstrate a lot of things there**. And I finish the course with the construction of confidence intervals and hypothesis testing as well. The difference between the two courses is: **for the pure mathematics degree, I focus more on the demonstration part of things, on why things are valid**.	R_I(ULS, $S_{KT\text{-}Uni}$)	I_2.
[*Topic 5*] For the **teacher education**, I think the content in the **syllabus** is a lot. It is like you see a lot of very superficial things. So, you end up having to pick some things out of there to be taught. I would change some things, yes. **I think I would make the course less theoretical and more practical**. I think that, for preservice teachers, if they come out of the course knowing very well **how to interpret the results**, it is already very valid. I think the big difference between statistics and mathematics is that in statistics the idea is very simple. Then you take that idea and you mathematise it. And then you end up complicating that idea that was simple. So, you have to know how to read it and interpret it.	R_I(SETE, $S_{KT\text{-}TE}$)	
[*Topic 5*] As I said, I see statistics as a sub-area of mathematics. I think **the problem is that most statisticians here in Brazil, at least the most respected ones, are mathematicians. And we, as mathematicians, try to mathematise everything, everything is a mathematical view**. So, in statistics, what happens today is that people try to mathematise all statistics. **Forgetting the principles of statistics, the basic concepts, the basic ideas of statistics, which came out of practical problems**. So, it is much easier for a mathematician to go and start talking about functions. Because they have been trained in it. That is what I was trained to do. To turn everything into functions and draw conclusions based on that. Without thinking about the student, from the point of view of his day-to-day life... what will he work on? What does that number mean in his day-to-day life? For example, **let's take a Fisher work, that, … let's put it this way "he is the father of statistics", you take his ideas, they are all based on practical problems.** Then they got mathematised, and what happened? They ended up forgetting his basic ideas and focusing more on the mathematical part. I am not condemning the mathematical part; I think it is valid. But it is valid like this: for my research work, when I write a scientific paper, in this case I must rely on the mathematical part. Do you understand? A student doing a master's degree or a PhD must base everything on mathematical theory. **But for preservice teachers I see that the most important thing is the ideas and the interpretations**.	R_I(SETE, $S_{KT\text{-}TE}$)	Criticising I_2. Criticising I_1.

Interviewee X_3 - comments	**Relations**	**Manifestations**
[*Topic 1*] I acquired the concept that **statistics is a science of collecting, analysing and interpreting data** and is very important when we think about contemporary society and when we think, especially, about the use of technological resources, the speed and the amount of information that circulates in society.	$R_I(RSE, S_{SK})$	Not I_3.
[*Topic 2*] When we think of defending **statistical literacy** and **probabilistic literacy from early childhood education**, it is because they allow us to develop forms of reasoning, both probabilistic and statistical, that contribute to the criticality of people and allow a speed in **the process of analysis and decision making**. So, as we have a lot of information and you want to make a selection, a sifting of that information, whether it is the one you are interested in, or the one that is real, or the one you want to use for a certain argument. These forms of statistical and probabilistic reasoning are essential for this kind of sifting. [...] But this **literacy does not occur at a certain point in time as it is presented in curricula**, for example, divided by school levels and what concepts to teach. **That is, when it is studied from a more technical perspective: of rules, formulas, predefined concepts**.	$R_I(RSE, S_{KT\text{-}Sec})$	Criticising I_2.
[*Topic 2*] However, the **teaching of statistics** that we advocate (**that I advocate and that most researchers in the field advocate**) **is one in which it begins to be developed through activities related to real situations**.	$R_I(RSE, S_{KT\text{-}Sec})$	Criticising I_1.
The BNCC curriculum guidelines are a horror! **It does not take into account what research in statistics education has presented**. At no point does it point to statistical and probability literacy. There is no relationship between what is proposed for the child from one year to the next. There are no didactic orientations, no indications of theoretical or methodological references. So, they want to make the teacher responsible.	$R_I(RSE, S_{KT\text{-}Sec})$	Criticising the curriculum of secondary education
[*Topic 3*] I argue that this mathematics **teacher education should have the conceptual and procedural study part articulated to the study that has been evidenced in the research**. For example, **the classes could be divided into two phases**. The **first stage is conceptual and procedural study, with exercises to correct, solve and discuss**. The **second moment is to think about how these concepts will be worked on at school**: How do children think about statistics?, How do they think about probability?, What can be done to develop probabilistic reasoning, combinatorial reasoning and statistical reasoning? This has to be present and interconnected.	$R_I(RSE, S_{KT\text{-}TE})$	$I_{2.TE}$.
[*Topic 5*] The discussion that there is **a didactic approach to statistics and probability**, that **we can work with probability by analysing the conduct of experiments, analysing real context issues**, whether in the financial area or in the environment, **or even in statistics working in various contexts**, **only those who are into teaching statistics have these readings**. [...] I think **we have problems not only in the training of teachers who teach mathematics in compulsory education, but also in the training of educators** [those who teach future teachers]. In many cases, **secondary teachers do not work with statistics education because they were trained in statistics in their teacher education**. I see many students who come to the Institute to work in the teacher education course and, as they do not come from a statistical education background, they **do not have any accumulated reading or reflection on the subject**. So, they are going to reproduce it in a technical way as they learnt it. The same thing happens if we think about who teaches calculus, who teaches analytic geometry, trigonometry or any other subject. In other words, any subject that we teach in a training course can have this bias.	$R_I(RSE, S_{KT\text{-}TE})$	Constraint Criticising $I_{1\text{-}TE}$.

Interviewee X_4 - comments	**Relations**	**Manifestations**
[*Topic 1*] For me, **statistics is the science of data, of number in context**. Which is Moore's definition. With that, I try to work with, I think it is important that you understand all that data in the context in which it was extracted.	R_I(RSE, S_{SK})	Not I_3.
A few years ago, **the teaching of statistics was almost centred on mathematics**. **It was all about calculations and formulas**. **There was not much concern about doing analysis of the results of this calculation**. Today, I can teach a course that is **almost solely** focused **on analysis. I give the results to the students and tell them to write a paragraph saying what that means**. Let them interpret that result.	R_I(RSE, $S_{KT\text{-}Uni}$)	Criticising I_2.
[*Topic 2*] From secondary school I think it is fundamental for the student to have **descriptive statistics**, **probability**. Do they need inferential statistics? No. But they can **do some informal inference while doing descriptive statistics**. To develop the way of looking, right? The way of looking. But this part of informal inference is not really done, you see?	R_I(RSE, $S_{KT\text{-}Sec}$)	I_2.
For me, **what is in the BNCC is fine**. Now it even talks about frequentist probability, it does not limit equiprobability any more, it talks about non-equivariant spaces. So, for me it is fine. There are a lot of things about probability. There is no need to go into distributions, binomial distribution. Mainly because students do not study the random variable. So, you find probability without talking about the random variable. How do you talk about distribution without talking about what is a random variable? I want to die with this when I analyse the textbooks, I get indignant. So that is it, statistics is in the curriculum from the first years.	R_I(RSE, $S_{KT\text{-}Sec}$)	Commentary on the curriculum
[*Topic 3*] At secondary level, teachers need to be better prepared. **Teachers are not sufficiently prepared**. In surveys they tell us that they do not feel confident enough to deal with the subject. Even though it is in the national curriculum. If you think about teacher education, even a teacher who is going to teach statistics in higher education. So that it is not just about formulas but also about teaching students to work with the meaning of statistics. [...] I think, at the very least, **they need to know descriptive statistics, probability and informal inference**. **But they need to know it in a deeper way**. To go a little bit deeper into aspects of **statistical thinking and reasoning**. We have to remember that this teacher will make his pupil literate. He/she is the one who is going to develop statistical literacy, so he/she needs to have it very well developed. **Because just knowing the content does not help him/her to have the knowledge**. For example, to do the changes of representation, to do the *transnumerations*, as Pfannkuch preaches. That kind of thinking, playing with *transnumeration*... he/she [the teacher] will not have that vision. **The teacher who only knows the content but has not developed the reasoning or statistical thinking does not do it**.	R_I(RSE, $S_{KT\text{-}TE}$)	I_2.
[*Topic 5*] The first problem **is the limited number of teaching hours**. But if you look at it, **it is not possible to increase the number of hours of undergraduate courses, because they already have too many hours**. **But it is a problem that can be solved: redistribution, joint work with Teaching Practice, for example**. **Transforming a subject into "projects" to work on various aspects of mathematics in an articulated way**, for example.	R_I(RSE, $S_{KT\text{-}TE}$)	Constraint Conditions that could be implemented

[*Topic 5*] It would be different in this sense: it **would go deeper into aspects that today we still call "statistics education".** Which includes this part of developing **thinking, reasoning and statistical literacy**. [...] But there is another very important problem: **who is going to teach statistics at the teacher education?** It is not enough to be a teacher with a degree in mathematics or statistics. If you do not have this mentality, this culture of the "statistics education", you will not do it. **Because the trainer also needs to know other contexts, he cannot just do classical things**.	$R_I(RSE, S_{KT\text{-}TE})$	Criticising $I_{1\text{-}TE}$. Constraint

Interviewee X_5 - comments	**Relations**	**Manifestations**
[*Topic 1*] Statistics (with the help of probability) will work with **risk and variability**, which are constant elements in all natural phenomena.	$R_I(RSE, S_{SK})$	"Science of risk and variability"
[*Topic 2*] Some time ago **we began to see that it is important to develop statistics education**, right from compulsory education, because **it would be useless just to theorise everything if there was no pedagogical and didactic way of doing it, for the student who goes to any area of knowledge**. They need to feel what statistics is, to know that it is useful not only for their future research, but also **to know the world around them and not to have an aversion to it**. [...] Both the introductory courses and the "brush strokes" in compulsory school were given (when they were given) in a very instrumental way. Speaking of compulsory school, **if we look at the textbooks of years gone by** (now it is a bit better), **there was only calculus and no reflection. From calculus to calculus, we changed chapters and statistics was over**. This was a problem with statistics education when it became part of compulsory school.	$R_I(RSE, S_{KT\text{-}Sec})$	Criticising I_1. I_2.
[*Topic 2*] What is happening is that we are collecting data, and more data, and more data, by the millions, by the billions, but people do not have much idea that you can work with data and that every answer you are going to give is always **a partial answer**, it **is always an answer that is going to involve risk, that is going to involve uncertainty, that is going to have variability that has to be discussed**.	$R_I(RSE, S_{SK})$	"Science of risk and variability"
[*Topic 2*] If the student does not start doing statistics from primary school and only begins to study it at university, he or she **will have developed deterministic reasoning**. It is important to bear in mind that **science is constantly being updated**, with new evidence. So, from an early age, **children must be confronted with the improbable, the probable, the possible, the not possible because, otherwise, only a small part of the pupils will develop a critical spirit in this area and the others will be afraid to give their opinion and will think that the world is deterministic**, when in fact **phenomena are random**. You have to start young, **i.e. first analyse situations of uncertainty and variability and then you can go more directly into modelling, inference and more sophisticated things**. But for the moment, you have to be able to analyse data. **What does the data tell you? How do you collect it? What are the ethical aspects of data collection? What is the procedure you should use when you are dealing with research?** [...] What needs to be done in school, in my opinion, are projects. Data collection from the earliest age. First, second, third year, already collecting data and discussing according to the level of the pupils. And there are easy ways to collect data. For example, there is a workshop I do where there is a huge banner on which each pupil puts a mark of the rounded measure of the size of their right hand, from the thumb to the minimum. And after 100, 200 hands, you start to see the shape of the distribution of the size of the hand. Girls will mark with one colour and boys with another, and then you see the difference between male and female hands, and what is the most frequent value, and the variability of this measure, etc. And for what? At the beginning you raise a question: are men's and women's hands the same size? If I am going to make gloves, do I have to make gloves for men and gloves for women, or do I have to make only one size? So, **knowing that in statistics we always have a question to answer and that this question will be worked on in different ways, starting from data collection**.	$R_I(RSE, S_{KT\text{-}Sec})$	"Statistical modelling" Not I_3.

[*Topic 3*] Within the **teacher education**, there should be more emphasis on statistics throughout the course, in the sense that **there should be data collection projects every year of the course**. If there are data collection projects every year of the course, preservice teachers will internalise the statistical way of thinking and compare situations. The important thing **is to work with data and internalise the way of reasoning in statistics**. **When** preservice teachers **do this throughout the four years, they will feel strong enough to do projects with their future students** [...]. For example, in the first and second year, only descriptive data analysis. Then, in the third and fourth year, you start with inferential analysis, but then the student is already familiar with it, has already done a lot of research, has already collected a lot of data. "Ah, but statistics is not taught in the second year of the degree". It does not matter, but there may be a moment when, **based on a question, data collection is proposed and someone is willing to help them in this data collection and to do some analysis (even with a computer) with the data, starting for example with graphical analysis, measurement analysis, etc.**	R_I(SETE, $S_{KT\text{-}TE}$)	$I_{1\text{-FP}}$. No $I_{3\text{-TE}}$.
[*Topic 5*] One thing that needs to be done **is to bring the Statistics department of the university together with the Education department of the university**, so that these people can have continuous training. This is not the case, because statisticians stay in their "niche" and educators say: "No, this is not my area. You statisticians are very theoretical, and we want something applied". And we have to inaugurate this continuing education. I do not know if this happens elsewhere, but here we do not see this debate happening. What we have seen is a discussion **about whether statistics is going to change its name, whether it is data science**, whether it is not data science.... **But this is not what is going to improve the supply of statistics in compulsory school, but rather to develop projects with teachers so that they feel comfortable to develop projects with their students**.	R_I(SETE, $S_{KT\text{-}TE}$)	Constraints Conditions
[*Topic 5*] If the educator of the subject Probability and Statistics is presenting the definition of measures of position and variability, but some student asks "But how is that in the educational context? How do I teach that?", he/she may have difficulties. **And that is why I say that the education department and the statistics department must work together to know how to work on these concepts, where to start, how to present these concepts to the preservice teachers**. It is a tortuous path, but I think we must go down it. And I do not know where to start because we have already made several forays into this, but **we** always **come up against obstacles**, **one of them being the statistics department itself, which is not usually interested in this**. So, it is a difficult road to go down, I do not have the right formula. **I think one idea would be to bring together statisticians of good will with educators of good will and have them get rid of their preconceptions and decide to come together**. It is possible that this, little by little, is beginning to happen.	R_I(SETE, $S_{KT\text{-}TE}$)	Constraints Conditions

Interviewee X_6 - comments	**Relations**	**Manifestations**
[*Topic 1*] **Statistics is**, let's say, **looking at a set of numerical information, dealing with this numerical information and drawing conclusions. Sometimes this involves an inferential process, i.e. if this numerical information comes from a sample, under the conditions of a sample that allow inference, inferences are made for the whole population**. It is often characterised as **the science of numbers**, which deals with numbers, i.e. collects numbers, and concludes about something (some object, some variable, some interest) from a larger set which would be the population.	R_I(RSE, S_{SK})	"Science of numbers"
[*Topic 2*] [In secondary school] The probability part is a part that often, being closer to mathematics, **is more attractive to be taught by the mathematics teacher**. **But statistics sometimes takes a back seat**. So, the student comes to university **without a background in statistics**. Eventually they know algorithms, they know how to calculate the mean, they know how to do any calculation, but they have no notion of what it means. And I think an important part, **which is expected to change in a few years, is precisely the discussion of this part of statistics in high school**. So that students, when they enter university, start from a different level of conversation. But in recent years, this is not clear. What is clear: computational or graphical issues. It is clear that today we have a greater facility and availability to do mathematics and graphics with the computer.	R_I(RSE, $S_{KT\text{-}Sec}$)	Condition
The BNCC curriculum advocates a number of things that I have never stopped to give a critical opinion on whether it is good or bad. **But I would say that if we did what the BNCC says, we would have a much better standard by now**.	R_I(RSE, $S_{KT\text{-}Sec}$)	Commentary related to the curriculum
[*Topic 2*] So, I think an important issue is in relation to how mathematics teachers will connect with statistics in primary and secondary education. Because look, **in mathematics, there is a lot of this issue of determinism**. Sometimes, in casual conversation, there is this view that mathematics "is or is not", there is no more or less, it is right or wrong. And **the essence of statistics is variability, randomness. Chance, variability is the essence of statistics**.	R_I(RSE, S_{SK})	Not I_2. "Science of risk and variability"
[*Topic 2*] And sometimes, the mathematics teacher, as a consequence of the subjects he or she had in the degree, **does not emphasise this subject and this generates a deterministic view of statistics**. And then they think that everything happens **with mean calculations, with variance calculations, with calculations of another measure**, or, eventually, by doing hypothesis tests, or by calculating the confidence interval. In other words, "everything is deterministic"! I think that one issue that is important **for the mathematics teacher in compulsory education to highlight is the difference between statistics and mathematics, because statistics works with variability, with randomness**. And this is what statistical methods produce so that we can decide things, without knowing the population. **This for me is the essential thing and it is foreseen there in the BNCC, but of course it goes unnoticed because of the initial teacher education**, not understanding how to explore this issue and how to draw the preservice teachers' attention to it.	R_I(RSE, S_{SK}) or R_I(RSE, $S_{KT\text{-}Sec}$)	Criticising I_1. "Science of risk and variability"
[*Topic 3*] **In addition to the content**, which is **mainly mathematical**, it is very interesting for the trainer to give the preservice teachers **the opportunity to experience what they will be dealing with their students in the future**	R_I(SETE, $S_{KT\text{-}TE}$)	$I_{2.TE}$.

[*Topic 3*] So, for example, one of the things that I think is fundamental for preservice teachers to have **the opportunity to experience is the collection of data and the analysis of the data collected**, what I call projects. Because **the idea is that this is developed with primary and secondary school students, that they can collect data**, either from their own class, school, neighbourhood or region, depending on the time they have to do it. And, based on these data that are in a context of interest to the students, these data are worked on and measures are used, and the students get used to the idea of numbers in a given context, which is also what we usually talk about in statistics. That is, **statistics has numbers in certain contexts**. "5", for example, can be a big number or it can be a small number, depending on the context. If you say "I have 5 cars", to me that is a lot of cars for one person. Now, if you say you have 5 T-shirts, it does not seem so many to me. So this experience of collecting information and working with it, which is developing projects, is something I always do with my students. In general, our course here at USP is a two-semester course, so in the first semester, they work with a database that is already created. It is a collection made by the students themselves, who respond, and they do some analysis from this database. In the second semester, they have to develop a project. So they get together in groups, choose the topic, prepare a questionnaire. They have this experience of preparing a questionnaire, which seems silly, but it is an interesting practice of knowing "how do I ask?" Because when I ask, I have to be clear in the question I ask so that whoever is going to answer understands what I am asking. Then they will collect data, work with the statistical techniques we saw during the course, prepare a report and present it to the whole class. In addition, I always try to apply and develop activities in the classroom so that they can experience different situations, in **addition to the theoretical training**, where the subject needs to acquire a certain mathematical basis to understand the things they are doing. Anyway, this is a bit of what I try to do.	R_I(SETE, S_{KT-TE})	Not I_{3-TE}.
[*Topic 3*] I think **a mathematics teacher should be clear that statistics is not going to be treated in the same way as mathematics, with that determinism of an equation**. You can solve an equation today and the result of "x" will be the same tomorrow, if you change country, it is also the same. Now, if I am talking about variability, the question today may have one answer, tomorrow another, and so on. Anyway, I **think that randomness and variability is the main thing**, and that students can leave with these concepts clear in their heads. And then, of course, **there is the filling in of contents because, when you open a subject, you open fronts**: **there are people who are going to be teachers, but there are people who are going to continue studying statistics in the future**. So, **you introduce a little bit of the fundamentals**.	R_I(SETE, S_{KT-TE})	Not I_2. I_{1-TE}.
[*Topic 5*] Many of the **educators who teach statistics come from the statistics department and their training was, shall we say, more mathematical**. Often, **they see statistics almost as a part of mathematics**. So, when they come and teach a course in teacher education or wherever, they **go and present it theoretically**. Which for them is intuitive because, after all, they have a theoretical background in the subject. But, for the preservice, it is not. **The guy stands there memorising the exercises "type 1, type 2, type 3"**, waiting for one of these to appear in the exam and then, two minutes after the end of the semester, he has already put all this out of his head and said "let's move on to another subject". Anyway, there is nothing left. It is a real mess. So, really, **an important battle is that these teacher education courses actually consider the fact that the guy is going to be a teacher** and if he does not have a good course, he probably will not give a good course in secondary school. You will end up running away from this part of the statistic.	R_I(PEFP, E_{SE-FP})	Constraints Criticising I_2. Criticising I_{1-TE}.

APPENDIX C: SRPs designed by the four groups in the pilot SRP-TE

Propostas de sequências didáticas dos grupos

Grupo 1

Planejamento

Pergunta: Por que o consumo de água é limitado em certas regiões do Brasil?

Como se coloca a questão: Apresentar vídeos mostrando certas regiões que têm o consumo de água limitado, por exemplo em São Paulo que passa por problemas com escassez de água constantemente e muitas vezes é apresentado em reportagens de jornais nas emissoras de TV famosas.

O que se propõe fazer: Questionar e discutir sobre a falta de água em determinadas regiões, se o consumo é limitado durante todo o ano ou em determinadas estações do ano, quais são os fatores fundamentais para ocorrer essa escassez de água nessas regiões, além de compreender como calcular o quantitativo de água doce disponível em cada região.

Ferramentas estão disponíveis: Youtube; vídeos; celulares; computadores.

Como o estudo está planejado para ser administrado em classe: vamos dividir a turma em grupos de quatro alunos e em cada grupo definir a função de cada integrante. Após colocar a questão e realizarmos a discussão as questões que surgirem serão divididas entre os grupos de forma que cada um deles investigue questões distintas. Em cada aula um integrante do grupo deve apresentar as informações que encontrou em suas investigações, quais as fontes e quais novos questionamentos surgiram.

Horário aproximado: 9 aulas (3 semanas).

Como se espera que a atividade seja concluída: Respondendo à questão inicial e apresentando uma proposta que possa ajudar a tentar solucionar o problema com a falta de água das regiões.

Quais conteúdos curriculares (em matemática e outros): porcentagem; análise de gráfico; multiplicação; adição; Mudança de unidades.

Grupo 2 – PEP da Caixa d’água

Q_0: Dá para coletar a água da chuva para usar em casa?

Contexto:

Podemos usar o contexto visto em nossa experiência com esse PEP da distribuição da água, partimos de como está distribuída a água na nossa cidade, fazendo os alunos perceberem que na nossa região temos, relativamente, pouca água. Assim levantamos a ideia de tentarmos economizar a água visto que a distribuição é um processo pouco utilizado no Brasil. Para isso falaremos sobre o clima e como a chuva pode nos ajudar a amenizar este problema. Em seguida podemos introduzir a nossa Q_0 para eles. Durante o percurso os alunos poderão utilizar as ferramentas que tiverem disponíveis, sejam elas livros, celulares, etc.

OBS: Se a maioria dos alunos forem da zona rural geralmente eles devem utilizar a água da chuva, pois nem sempre há água encanada nas residências.

Perguntas que podem aparecer par responder Q_0

- É possível fazer isso?
- Como fazer isso?
- É permitido fazer isso?
- Quais os materiais que podemos utilizar para isso?
- Essa água pode ser utilizada em casa?
- Como podemos minimizar o uso de água?
- Quais tarefas domésticas podemos executá-las utilizando água da chuva?
- Como armazenar/ tratar a água da chuva e distribuir de modo que dê para utilizar na maior parte das atividades domésticas?*
- Eu posso utilizar a água da chuva para beber? É possível torná-la potável?*
- Como eu faço para captar a maior quantidade de água da chuva? Vou usar uma bica?*
- Quanto de água é possível captar?*
- Essa economia seria capaz de alterar o valor que eu pago na conta de água? *
- Como eu posso limpar a água da chuva para aproveitar o máximo possível?*
- Quantos litros de cloro eu terei que utilizar caso eu tenha armazenado 300 litros de água?*
- Quanto maior o telhado da minha casa mais água eu posso captar?
- O que é índice pluviométrico?
- Como eu posso medi-lo?
- O que ele me diz sobre a quantidade de chuvas na minha região?
- Essa quantidade de chuva da minha região é o suficiente para umas pessoas viver somente com a água da chuva?*
- Quanto de água é preciso para uma pessoas utilizar no seu dia a dia?*

Questões que darão início aos conteúdos

- Como armazenar/ tratar a água da chuva e distribuir de modo que dê para utilizar na maior parte das atividades domésticas?*
- Eu posso utilizar a água da chuva para beber? É possível torná-la potável?*
- Como eu faço para captar a maior quantidade de água da chuva? Vou usar uma bica?*
- Quanto de água é possível captar?*
- Essa economia seria capaz de alterar o valor que eu pago na conta de água? *
- Como eu posso limpar a água da chuva para aproveitar o máximo possível?*
- Quantos litros de cloro eu terei que utilizar caso eu tenha armazenado 300 litros de água?*
- Essa quantidade de chuva da minha região é o suficiente para umas pessoas viver somente com a água da chuva?*
- Quanto de água é preciso para uma pessoas utilizar no seu dia a dia?*

Disciplinas e conteúdos

Considerando as perguntas anteriores, esperamos adentrar em conteúdos da química, biologia, matemática e geografia. Pensamos inicialmente nos seguintes conteúdos:

- Matemática: razão, grandezas e medidas, funções, áreas de figuras, volume.
- Geografia: Clima da região.
- Química: Estequiometria.
- Biologia: Micro-organismos e bactérias, ciclo da água.

Caímos de certa forma em algumas discussões que foram feitas nas apresentações de alguns colegas como por exemplo quando for pesquisado sobre a quantidade média para um pessoa utilizar durante o dia e perceberem que essa média é bem menor que a das pessoas em modo geral estão utilizando, levando a discussões de por que pessoas sofrem de falta de água sendo que sua média de consumo é bem maior que a determinada pela Organização das Nações Unidas (ONU).

Tempo estimado: Acreditamos que seja necessário em torno de um mês de aula para a realização desse percurso, visto que prevemos duas semanas de discussões, uma para aprender os assuntos que forem surgindo no processo e outra para preparar e apresentar a resposta final.

Conclusão

Esperamos que o estudo seja concluído com uma resposta à pergunta inicial, nela os alunos irão usar os conhecimentos adquiridos ao longo das aulas para dizer se de fato é possível realizar o que é pedido ou não. Caso a resposta seja afirmativa seria necessário mostrar um projeto que na prática cumpra o que se pede na Q_0 com estimativas do quanto de água pode ser coletado. Caso não seja possível ou viável, os alunos devem argumentar o porquê. Uma outra discussão interessante, no caso de não ser possível ou viável, seria pensar e justificar quais condições poderiam tornar a coleta possível ou viável.

Grupo 3

Q$_0$: Diante dos problemas relacionados à degradação da qualidade das águas, qual seria a taxa percentual de recursos hídricos em degradação?

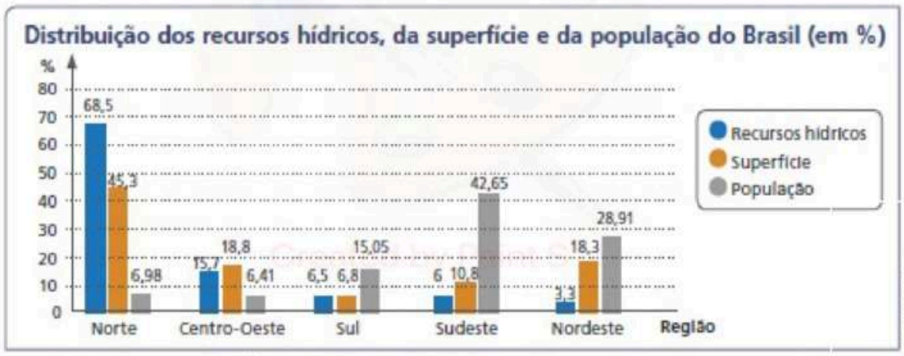
O texto (da atividade do livro didático) afirma que os recursos hídricos próximos às áreas urbanas são as principais vítimas de degradação. O que poderia gerar questões como:

- Todos os rios que passam pelas cidades estão sendo poluídos?
- Todas as regiões possuem a mesma taxa de degradação?
- A degradação dos recursos hídricos pode ser revertida?

Como se coloca a questão, o que se propõe fazer e que ferramentas estão disponíveis?

Iniciaremos com uma discussão sobre a questão e o gráfico da atividade do livro didático, com relação às quantidades de recursos hídricos e a degradação das águas.

1. O Brasil possui cerca de 13,7% do total de água doce do mundo, sendo considerado um território rico em termos hídricos. No entanto, o país vive sérios problemas, relacionados tanto à degradação da qualidade das águas, principalmente nas proximidades das áreas urbanas, quanto à falta de controle do excesso e da insuficiência de água, que atingem várias localidades brasileiras. Não são somente as enchentes que afetam as cidades brasileiras: a escassez hídrica também impõe sérias restrições e elevados custos ao desenvolvimento econômico e social de grandes cidades do Brasil.



Informações obtidas em: MINISTÉRIO DO MEIO AMBIENTE. Disponível em: <http://www.mma.gov.br/estruturas/sedr_proecotur/_publicacao/140_publicacao09062009025910.pdf>. Acesso em: 1º jul. 2018.

Em seguida, esperamos que surjam algumas perguntas derivadas como as que mencionamos acima. A depender dos tipos de perguntas, nós dividiremos a turma em cinco grupos, em que cada grupo irá trabalhar com uma região, ou trabalharemos somente com uma região específica, a depender dos questionamentos.

A principal ferramenta disponível a ser utilizada é a internet, mas os alunos podem utilizar outras fontes (livros, revistas, etc.)

Tempo estimado: entre 10 – 14 horas/aulas

Como se espera que a atividade seja concluída?

Com uma resposta que dê uma noção da quantidade de recursos hídricos em degradação.

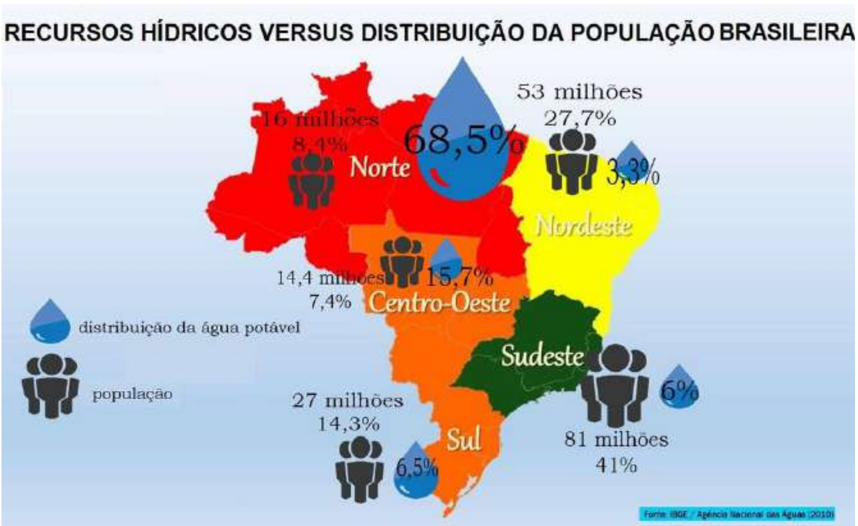
Conteúdos curriculares (em matemática e outros)

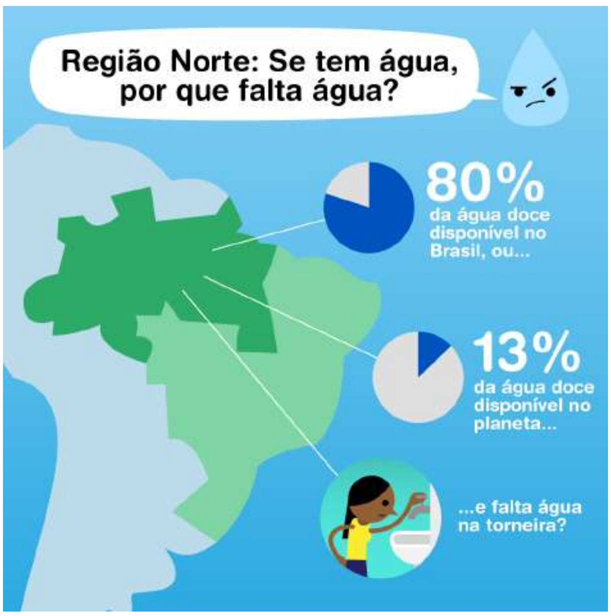
Porcentagem;
Análise de gráficos;
Geografia do Brasil;
Grupo 4

Q0: Se o Brasil é um país com grande disponibilidade de recursos hídricos, porquê existe escassez e/ou racionamento de água em todas as regiões do país?

Vídeo: https://www.youtube.com/watch?v=pYjg-sZDopg
Os desafios sobre o acesso à água potável do mundo.

Analisando as imagens que serão disponibilizadas bem como a pergunta inicial que será feita e o vídeo, os alunos deverão realizar pesquisas, elaborar perguntas e respondê-las de maneira que possam ajudar a entender melhor como funciona o sistema de recursos hídricos no Brasil, e o porquê de existir casos de escassez e/ou o racionamento de água no país. As imagens serão as seguintes:





A atividade deverá ser aplicada nos anos finais do ensino fundamental. O objetivo é que o questionamento inicial seja lançado na sala de aula e que os alunos possam realizar suas pesquisas tanto na escola, como em casa, nos casos em que existam a disponibilidade de acesso à internet e livros. A medida em que os alunos forem elaborando questionamentos, eles deverão ser respondidos, levados até a sala de aula e discutido entre os alunos. Esse processo de pesquisar, elaborar questões, respondê-las, discutir em sala de aula, e prosseguir até solucionar a QO, exigirá uma quantidade significativa de aulas, em que sua duração dependerá do desempenho dos alunos como pesquisadores.

Acredita-se que durante todo esse trabalho, os alunos sintam necessidade de conhecer e entender conteúdos geográficos do Brasil, tais como a população, suas distribuições, dados hídricos, bem como se aprofundar um pouco mais sobre a formação das regiões do país. Também se espera que surja a necessidade de estudar conteúdos matemáticos como a construção e interpretação de gráficos, e noções e representação de taxas de porcentagem.

Almeja-se que no final da atividade, os alunos consigam construir conhecimentos voltados aos conteúdos citados a cima, e que através deles encontrarem solução ou soluções para a Q0.

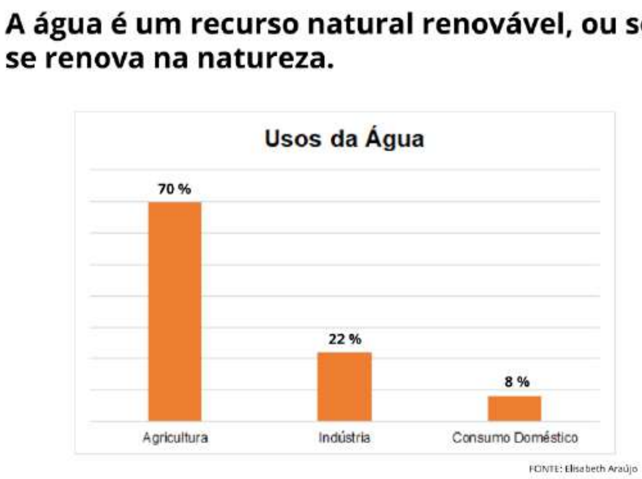
Visita às obras

- Nessa pesquisa, o aluno terá a oportunidade de rever e ampliar o seu conhecimento sobre conceitos relacionados à temática recursos hídricos. Além disso, o aluno tem a oportunidade de aprender estratégias que possam ser adotadas para a conservação da água potável.

- Questões geográficas
- Conteúdo matemático: porcentagem e análise dos gráficos.

Atividades

1-



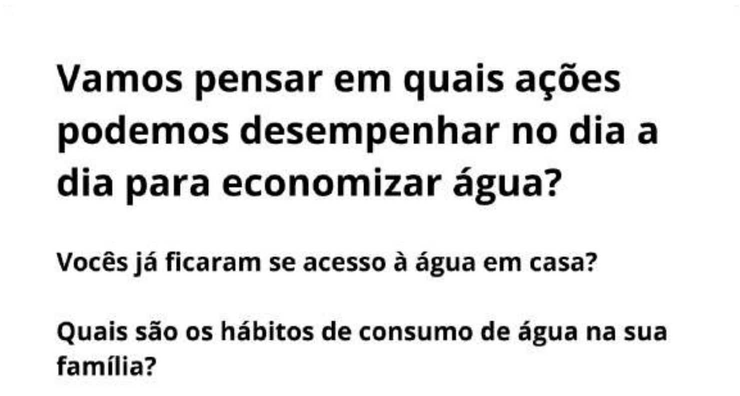
Tempo Sugerido: 3 minutos

P: Vocês acham que a água pode acabar?

R: A água é um recurso natural renovável, ou seja, que se renova na natureza, especialmente a partir da chuva. Apesar da água ser um recurso infinito, sua distribuição no mundo não é igual, pois em alguns lugares a quantidade de chuvas é inferior a outros, ou há a má utilização da água por parte das pessoas, seja na agricultura, na indústria ou nas casas. Projete ou desenhe no quadro o gráfico apresentado no slide, para que a turma possa ter uma noção da quantidade de água consumida pelos três segmentos anteriormente mencionados.

https://planosdeaula.novaescola.org.br/fundamental/3ano/geografia/o-uso-consciente-da-agua/5813

2-



Tempo: 10 minutos

Orientações: Poderia ser entregue um material falando sobre falta de água. Solicite que os alunos em grupos comentem sobre experiências pessoais com o uso da água, em suas casas. Pergunte a eles se já passaram por situações de falta de água, quais os hábitos de consumo da família, etc.

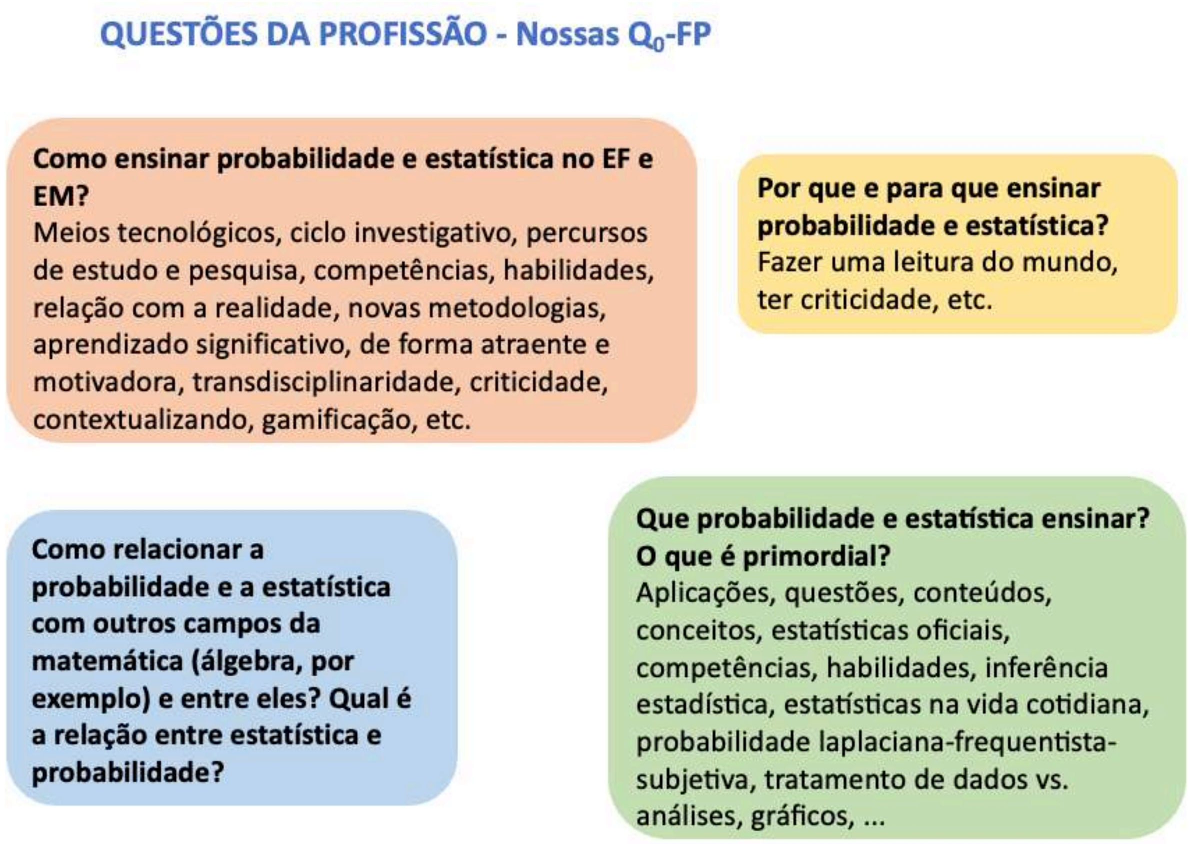
P: Vamos pensar em quais ações podemos desempenhar no dia a dia para economizar água?

https://planosdeaula.novaescola.org.br/fundamental/3ano/geografia/o-uso-consciente-da-agua/5813

Referências
https://agua-sua-linda.tumblr.com/post/157901041453/na-regi%C3%A3o-com-mais-%C3%A1gua-doce-do-brasil-falta-%C3%A1gua
https://www.reddit.com/r/brasil/comments/cv2yu5/recursos_h%C3%ADdricos_e_a_falta_de_%C3%A1gua_em_certas/

APPENDIX D.1: Report of the online SRP-TE for in-service teachers

Módulo 1 – Questões elaboradas pelos docentes



Módulo 2 – Análise coletiva do PEP vivido

<u>**Dialética questões e respostas**</u>

Q_0: Como analisar a veracidade desta notícia?
Resposta: Por meio das **fontes citadas** ao longo da reportagem! Fizemos um tratamento dos dados disponíveis no banco de dados e conseguimos (obtivemos) alguns resultados.
Mídia: https://mapbiomas.org/estatisticas

Q_1: Quais são os dados da noticia?
Resposta: Os dados são provenientes do Mapbiomas, uma vez que este instituto tem seu reconhecimento nacional e internacional no acompanhamento das questões ambientais e suas consequências para a população local, nacional e internacional.
R_1 Os dados são provenientes do Mapbiomas, uma vez que este instituto, tem seu reconhecimento nacional e internacional no acompanhamento das questões ambientais e suas consequências para a população local, nacional e internacional.

- - "Brasil perdeu 15% dos seus recursos hídricos em 30 anos, o equivalente a **3,1 milhões de hectares de superfície hídrica"**
- "O levantamento mostra que em 1991 a superfície hídrica do país era de 19 milhões de hectares. Em 2020, essa área foi reduzida para 16,6 milhões de hectares, uma redução equivalente a mais de uma vez e meia a superfície de água de toda região Nordeste em 2020".

- **"Rio Negro, na Amazônia, que perdeu 22%** da sua superfície desde os anos 90"
- **"Rio São Francisco, que corre por áreas de Cerrado e Caatinga, que perdeu 10%** em sua superfície nos últimos quinze anos"
- "O bioma Pantanal abriga o estado com a maior perda absoluta e proporcional de superfície de água na série histórica de 35 anos analisada pela equipe do MapBiomas: o Mato Grosso do Sul, com uma **redução de 57%**".
- "Se em 1985 o Mato Grosso do Sul tinha mais de 1,3 milhão de hectares cobertos por água, em 2020 eram apenas pouco mais de 589 mil hectares. Mais de 780 mil hectares de água foram perdidos no período".
- "O levantamento destaca a situação da perda de água no Rio Paraguai, no bioma pantaneiro. Os pesquisadores explicam que a construção de hidrelétricas é uma das principais causadoras da diminuição do nível da água, uma vez que estas obras afetam a biodiversidade e a dinâmica dos rios".
- "O segundo estado que mais perdeu água no período é o Mato Grosso, com uma perda de quase 530 mil hectares, seguido por Minas Gerais, com um saldo negativo de mais de 118 mil hectares de água perdida".
- "O Brasil abriga 12% das reservas de água doce da Terra e 53% dos recursos hídricos da América do Sul. Existem 83 rios fronteiriços e transfronteiriços no país, assim como bacias hidrográficas e aquíferos".
- "O bioma com a maior área coberta por água é a Amazônia, com mais de 10,6 milhões de hectares de área média, seguida pela Mata Atlântica (mais de 2,1 milhões de hectares) e pelo Pampa (1,8 milhão de hectares). O Pantanal ocupa a quinta posição, com pouco mais de 1 milhão de hectares de área média, atrás do Cerrado (1,4 milhão de hectares)".

$Q_{1.1}$ Existe bancos de dados oficiais que fornecem estes dados? Podemos acessar estes dados? $R_{1.1}$: Sim, banco de dados fornecidas na própria reportagem (MapBiomas). Podemos acessar o site do MapBiomas e fazer o download de diferentes estatísticas.

$Q_{1.2}$ Que informações podemos obter com eles (os dados do banco de dados)?

$R_{1.2}$: Os dados provenientes da reportagem, utiliza-se de um banco de dados governamentais. O Banco de dados do MapBiomas apresenta dados (por anos) por biomas, estados, municípios, regiões, bacias, sub-bacias e regiões hidrográficas.

$Q_{1.2.1}$ Como os dados desse banco de dados oficial são produzidos?

$R_{1.2.1}$: São produzidos por meio de registros de satélites que conseguem rastear as superfícies do Brasil.

$Q_{1.2.2}$ Quais dados utilizamos para realizar análises?

$R_{1.2.2}$ Decisões do grupo: tivemos que fazer escolhas, como: trabalhar com o banco de dados da própria notícia; no site do MapBiomas, tivemos que decidir em qual das planilhas olhar e escolhemos analisar os dados sobre "Regiões Brasil anual - área de superfície de água em cada região do Brasil de 1985 a 2020".

$Q_{1.2.3}$ O que representa os dados em porcentagem apresentados na reportagem com relação ao tamanho em hectares?

$R_{1.2.3}$ Para medir grandes espaços de terras, utiliza-se uma unidade de medida maior (hectares), no entanto, sempre buscar levar a comparação para campo de futebol. Não sendo, uma próximas dessas construções, como podemos citar a perda de 57% dos recursos hídricos na região do Pantanal?

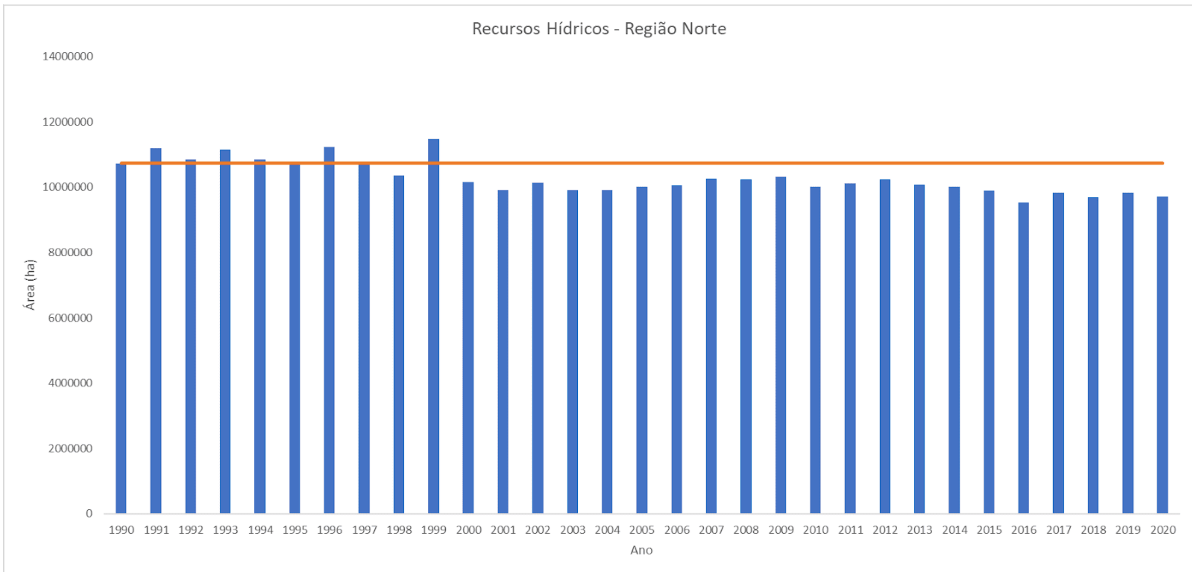
Q_2 Como podemos corroborar os diagnósticos mencionados no artigo?
R_2: **Aplicar conhecimentos estatísticos sobre os dados para obter resultados e ver se batem com a informação.** Construção de tabelas e gráficos, com base nos bancos de dados citados na reportagem, buscando novos bancos de dados para poder fazer comparações destes dados, bem como suas metodologias para a constituição destas estatísticas.

$Q_{2.1}$ Como se encontra a situação hídrica de cada região brasileira?
Construção de tabelas a partir do banco de dados da notícia, construção de gráficos com parâmetro de 1990 para comparar as perdas de recursos hídricos por regiões brasileiras: Norte. Nordeste, centro-oeste, sudeste e sul.

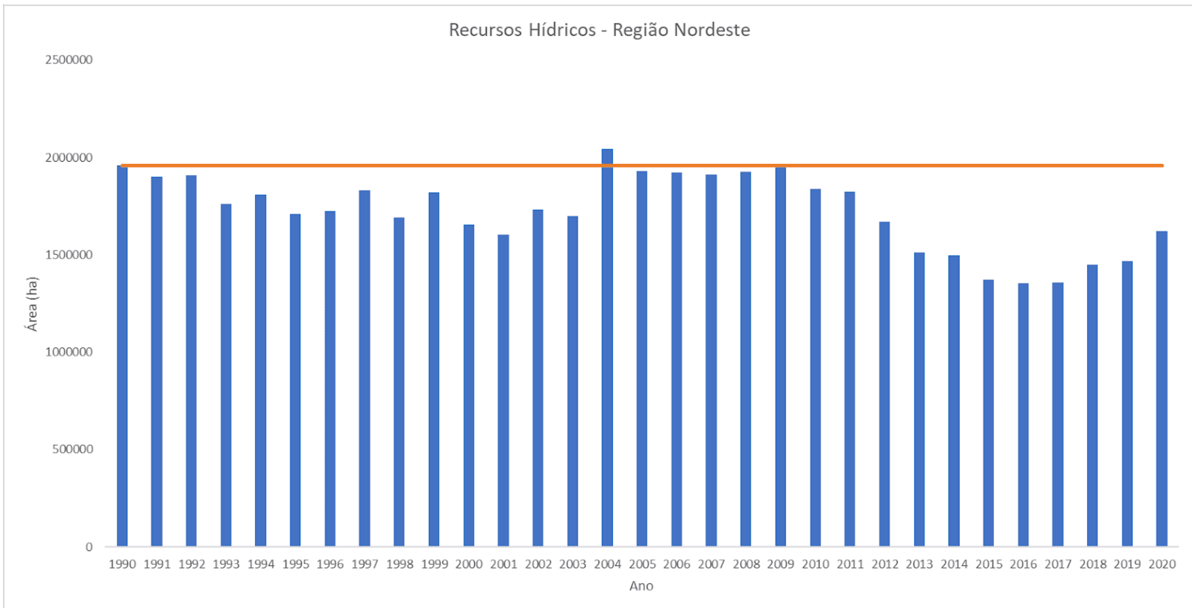
$R_{2.1}$:

Ano	Norte	Nordeste	Sudeste	Sul	Centro-Oeste	TOTAL
1990	10729683,6	1959873,15	1463823	2423997,69	2440949,404	19018326,82
1991	11187162,7	1901811,26	1556108,3	2419143,29	2654824,575	19719050,09
1992	10842121,8	1907527,95	1564099,96	2438036,53	2841410,473	19593196,67
1993	11149072,5	1763444,31	1562898,53	2440586,55	2743347,329	19659349,18
1994	10851602,2	1808188,48	1558718,21	2438563,47	2012902,269	18669974,66
1995	10740153,5	1711498,87	1554419,09	2433503,38	2250639,922	18690214,73
1996	11224947,5	1723254,17	1509885,93	2435822,63	2111116,347	19005026,59
1997	10777898,3	1830739,57	1556678,67	2431989,95	2241511,293	18838817,77
1998	10366763,2	1690269,83	1535079,27	2460938,65	2358200,184	18411251,13
1999	11472858,8	1821823,27	1545320,76	2479468,95	2301801,035	19621272,8
2000	10166407,1	1656109,31	1363386,87	2380849,84	2009313,67	17576066,78
2001	9910448,87	1602188,42	1294996,67	2388965,2	2073814,024	17270413,19
2002	10147105,6	1733216,74	1394524,42	2393402,34	2070126,704	17738375,83
2003	9912390,95	1698789,19	1414779,39	2397212,37	1978728,642	17401900,55
2004	9907514,46	2043707,13	1457953,3	2379619,54	2065932,911	17854727,34
2005	10021026,3	1930599	1471677,16	2384028,29	2071377,192	17878707,93
2006	10066067,3	1924576,49	1464550,85	2371906,69	2107615,914	17934717,28
2007	10268679,2	1911326,97	1499959,3	2396705,06	2022948,269	18099618,78
2008	10249130	1926148,54	1468254,19	2387516,83	1925415,922	17956465,51
2009	10321282	1966203,34	1484341,74	2388346,15	1827533,829	17987707,02
2010	10006179,2	1839291,73	1446590,02	2411143,62	1861197,402	17564401,98
2011	10117480,1	1824797,48	1451463,02	2405949,66	1861547,998	17661238,21
2012	10248685,4	1671759,78	1425350,29	2388633,47	1750949,84	17485378,78
2013	10077710,1	1513386,48	1353087,84	2408641,12	1613656,659	16966482,22
2014	10021440,3	1496975,49	1280465,6	2417527,97	1725683,88	16942093,25
2015	9887994,4	1373907,98	1249848,57	2421871,15	1699594,088	16633216,18
2016	9525771,22	1354460,28	1320773,39	2433277,68	1665714,371	16299996,93
2017	9840565,23	1357987,54	1282747,02	2424680,46	1663425,657	16569405,91
2018	9697253,31	1450685,1	1282146,74	2414525,93	1729522,916	16574134
2019	9842852,84	1468296,44	1303022,82	2417876,81	1697227,102	16729276,01
2020	9710126,95	1623029,38	1334258,42	2398945,42	1565211,689	16631571,85

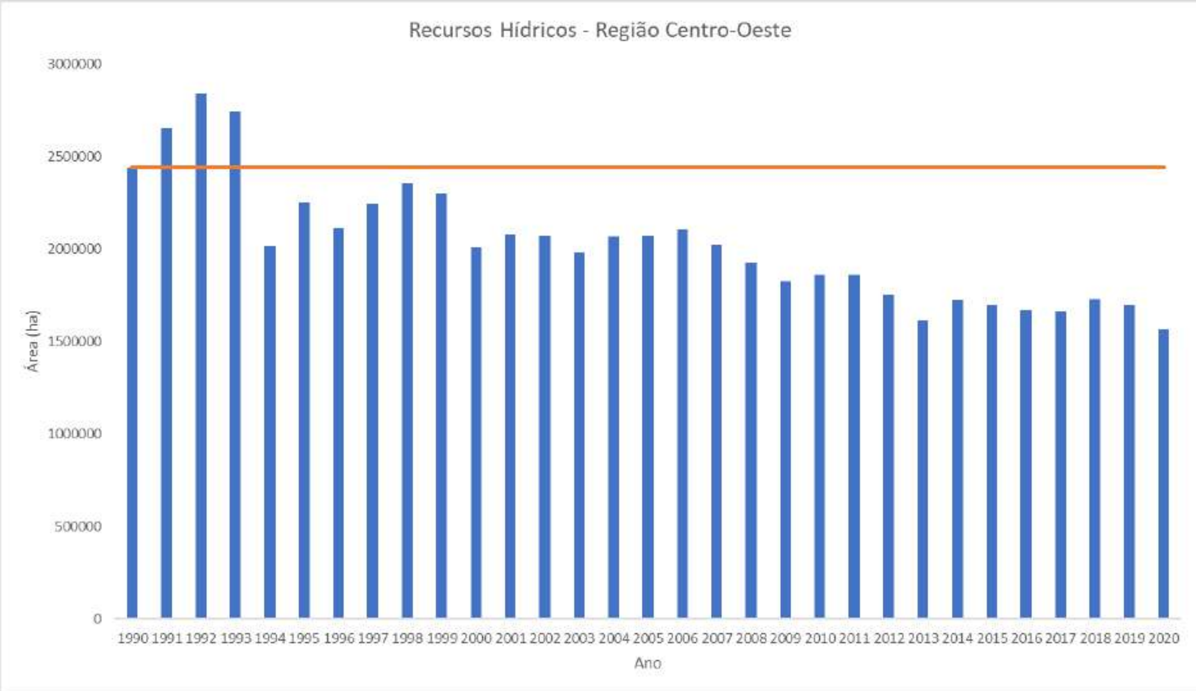
- Norte: a Região Norte é a região que teve o maior equilíbrio entre as regiões brasileiras com a maior regularidade das chuvas nas três décadas, embora tenhamos a primeira década com maior índice pluviométrico e as demais décadas abaixo da linha da primeira medição.



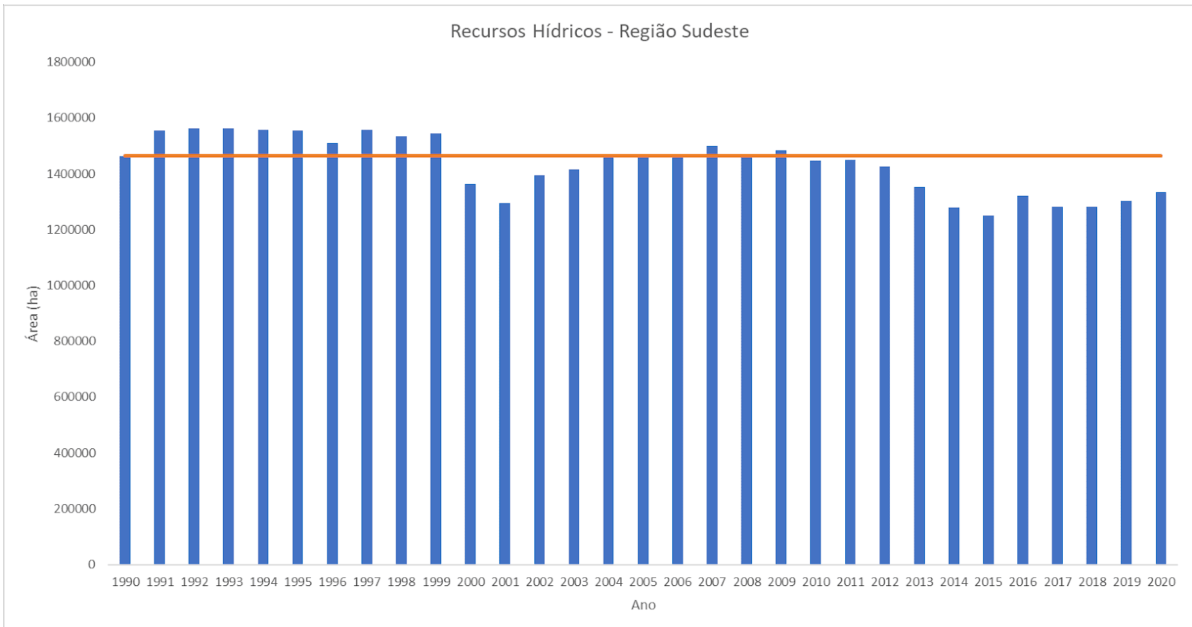
- Região Nordeste: Temos uma situação de grandes perdas hídricas históricas, ao longo dos tempos. De forma que a irregularidade das chuvas nessa região é histórica. Partindo apenas das três décadas em tela, podemos perceber que, na primeira década, temos uma irregularidade menos grave de chuvas, na segunda década um período melhor das chuvas (regularidade) e culminando na terceira década em que as precipitações foram as menores da região em seus índices pluviométricos.



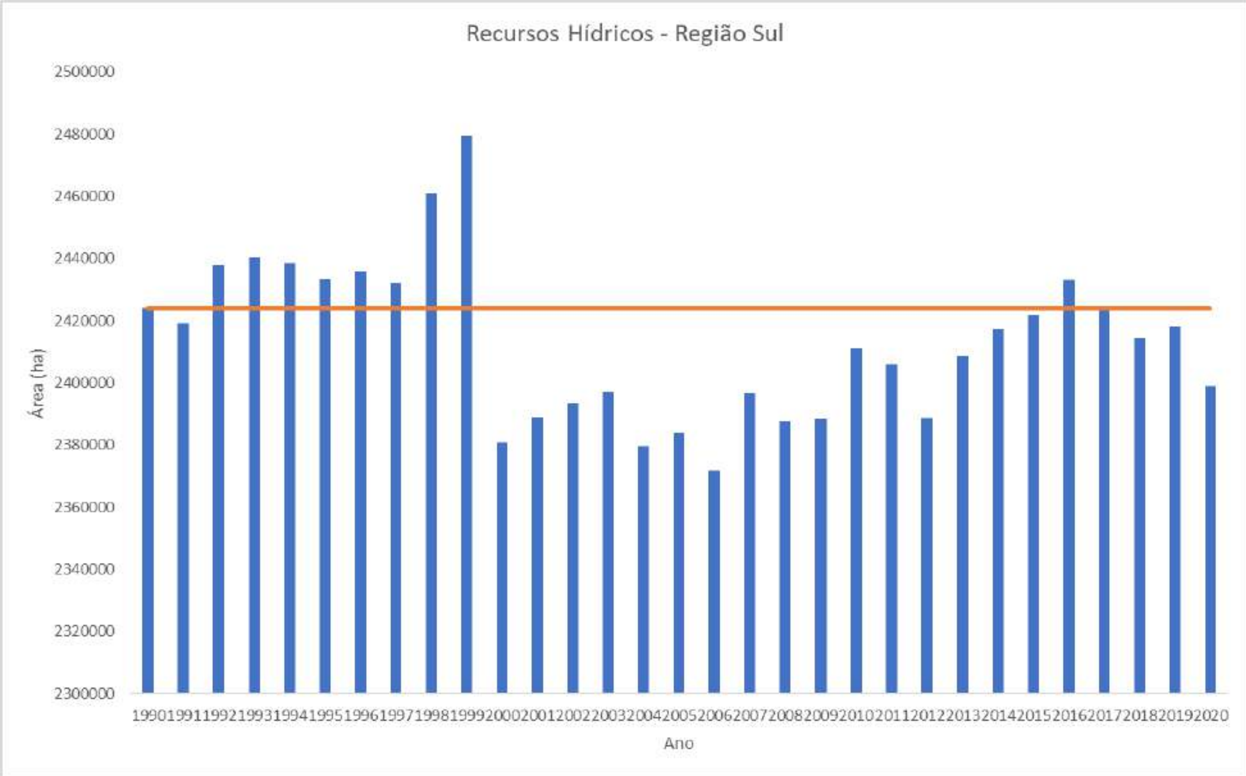
- Centro-Oeste: Nessa região, ocorre uma situação muito delicada, uma vez que não temos praticamente índices pluviométricos acima da primeira medição, e a situação só se agravou com uma redução considerável nestes índices pluviométricos, em especial na terceira década.



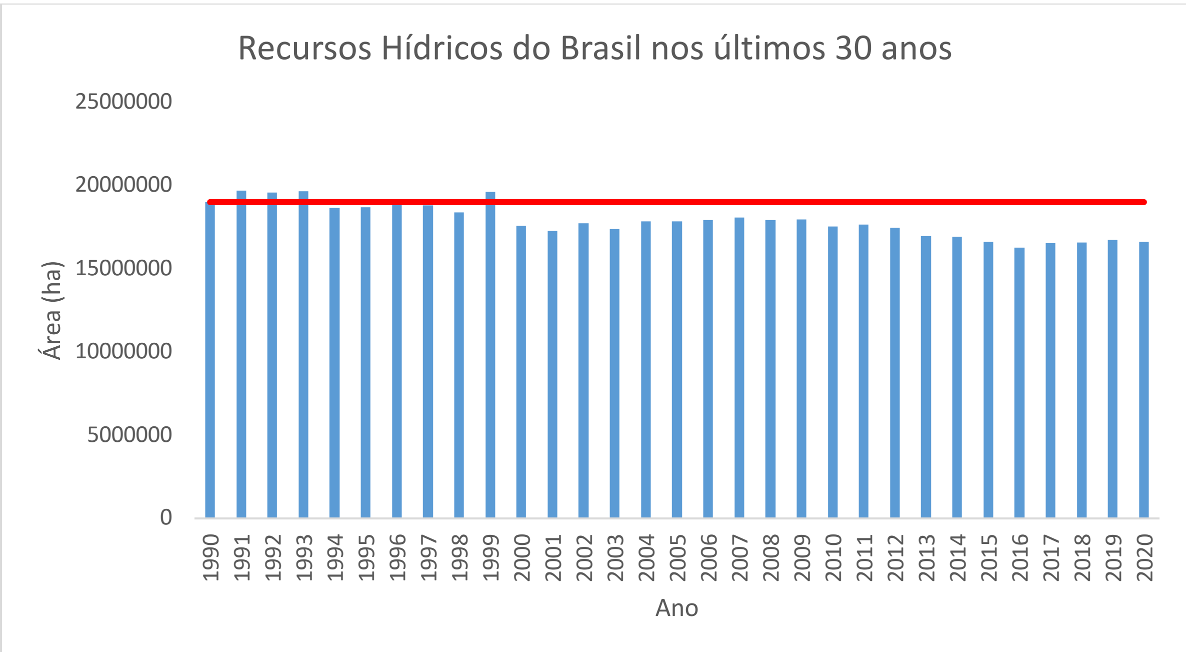
- Sudeste: é a região que teve, nas duas primeiras décadas, uma situação mais regular de chuvas, mas, na terceira década, sofreu com a diminuição dos índices pluviométricos.

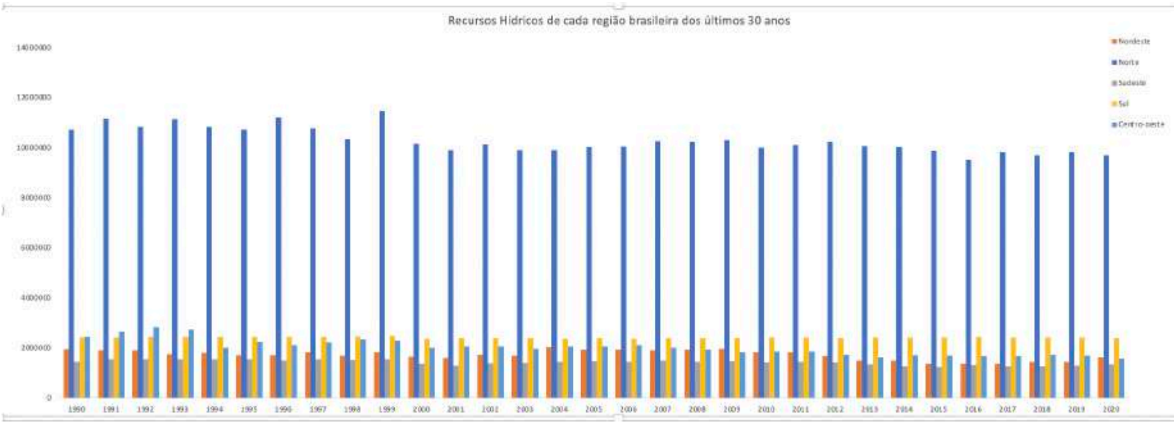


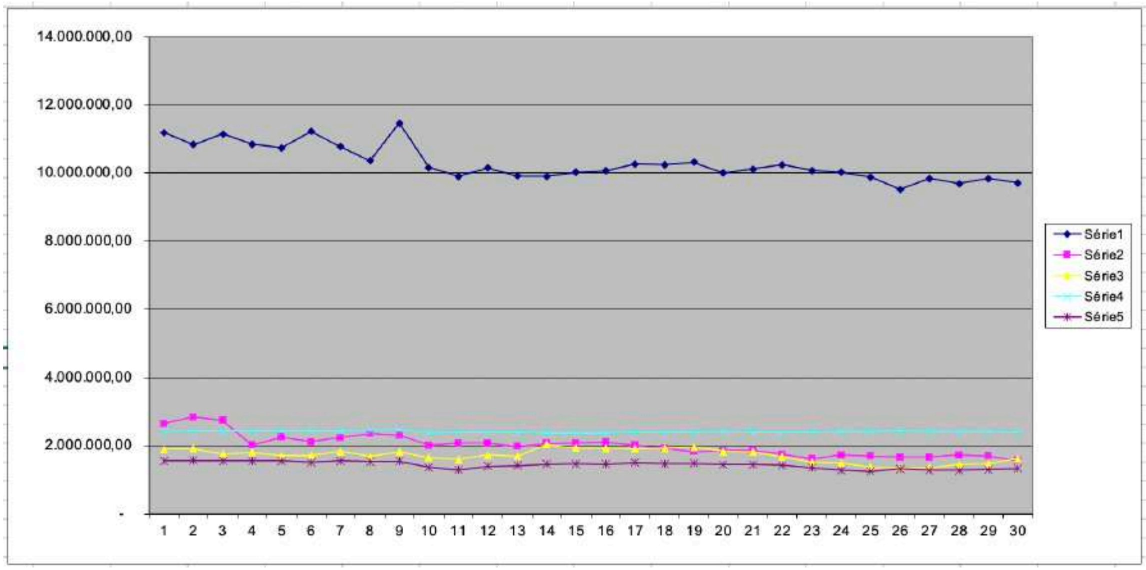
- Região Sul: nessa região, temos um desequilíbrio dos índices pluviométricos, pois apresenta a primeira década de chuvas acima da média, a segunda década com fortes perdas e uma recuperação na terceira década de seus índices pluviométricos.



Outros gráficos construídos pelo grupo:
Visão geral da variabilidade dos recursos hídricos das cinco regiões juntas




Recursos Hídricos de cada região brasileira dos últimos 30 anos


14.000.000,00
12.000.000,00
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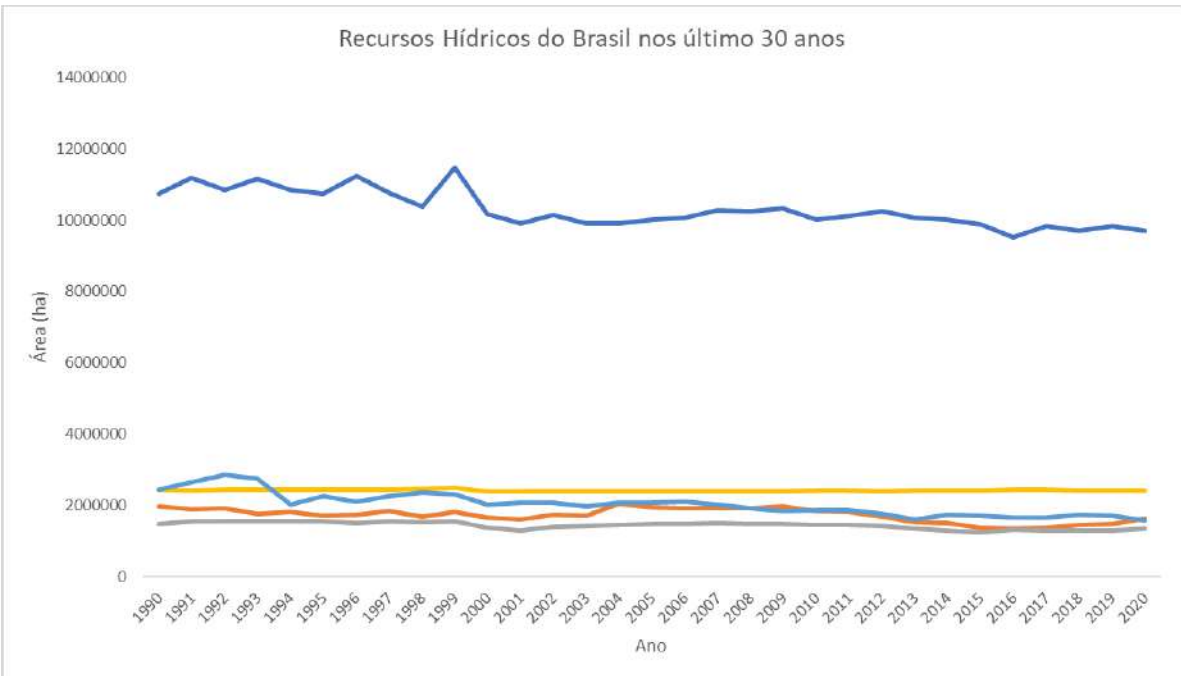
-
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30
Série1
Série2
Série3
Série4
Série5


Recursos Hídricos do Brasil nos último 30 anos
14000000
12000000
10000000
8000000
6000000
4000000
2000000
0
Área (ha)
1990 1991 1992 1993 1994 1995 1996 1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015 2016 2017 2018 2019 2020
Ano

Comentário: Vê-se muita diferença quando tiramos o Norte do comparativo.

Questões pendentes ou em aberto:

A partir dos dados apresentados na matéria do G1, como podemos comparar a situação hídrica dos lugares citados com sua região? Resposta: Consultando os dados citados na reportagem, fazendo um levantamento destes dados neste período de três décadas.
Calcular as perdas por regiões, olhar dados regionais (não tivemos tempo de olhar, …)

Como fazer a comparação das unidades de medidas (hectares)?
Quais são as alternativas para a produção das fontes de energias? As energias limpas (solar, eólicas e hidrelétricas? Incentivar a mudança da matriz energética para atenuar o uso de matrizes de energias poluentes.

Quais ações devem ser promovidas para a preservação dos recursos hídricos? Acompanhar os dados dos órgãos oficiais e trabalhar com a educação ambiental na escola, na formação crítica e consciente dos estudantes.

Que relações tem a variação de recursos hídricos com a população, a industrialização, a desflorestação (agricultura e hidroelétrica) e a superfície (livro didático e dados do nosso Excel)?

Outro banco de dados: Águas ao meu redor (Mídia)
https://www.snirh.gov.br/hidroweb/mapa
Podemos ver o impacto da crise hídrica ao redor de sua residência.
Não foi possível realizar o comparativo dos dados!

É muito desafiador falar desse assunto, uma vez que a questão hídrica e a questão agrícola é fundamental para a manutenção da vida, da forma conhecemos. Então, como não operar nessa oposição da binaridade? São dados relevantes, que podem suscitar boas discussões com outras áreas dos conhecimentos, tais como: química, física, biologia.

Curioso o artigo falar do Rio Negro, 22%, se segundo o próprio G1 comenta sobre as alagações provadas por essa hidrelétrica? Esta informação, precisaria de mais dados comparativos para fazermos as comparações a respeito destes impactos neste rio Negro e na Região Norte a construção das hidrelétricas.

A partir da análise geral, que efeitos podem ser observados na comunidade local? Fazer comparativo? Por termos um país continental e com muitos biomas e seus impactos em cada região é trabalho que demandaria mais tempo para que tais análises fossem realizadas. Olhar no site do INPA, nos repositórios sobre as hidrelétricas. Não conseguimos realizar o comparativo com este instituto.

A temperatura influencia na perda de água do planeta? Não conseguimos correlacionar com os dados do Mapbiomas com os dados das temperaturas neste período.

O que representa os dados em porcentagem apresentados na reportagem com relação ao tamanho em hectares? Resposta: Para medir grandes espaços de terras, utiliza-se uma unidade de medida maior (hectares), no entanto, sempre buscar levar a comparação para campo de futebol, não sendo, uma medida oficial o campo de futebol, tendo em vista que os campos oficiais são de tamanhos diferentes.

Análise das séries histórias, encontrados no site da ANA, para corroborar, ou não, com a notícia? Sim, pode-se ter um parâmetro comparativo com os dados analisados e poder propor novas ações de preservação e manutenção do meio ambiente.

Quais os impactos positivos da construção de hidrelétricas em nosso país? Verificar os impactos nesta matriz energética para o país, sem deixar de lado os impactos para as comunidades locais, bem como a fauna e flora.

Quais impactos negativos na construção de hidroelétricas em nosso país?
Resposta: Segundo a reportagem, podemos perceber que a construção de hidroelétricas, assim como a poluição, o uso excessivo e as mudanças climáticas têm impactado de forma preocupante a fauna, flora e a vida de pessoas que estão próximas dessas construções, como podemos citar a perda de 57% dos recursos hídricos na região do Pantanal.

Qual a resposta das comunidades afetadas? Fazer pesquisa nas comunidades afetadas e seus impactos para a população local, tendo em vista atenuar as mudanças provocadas nestas construções.

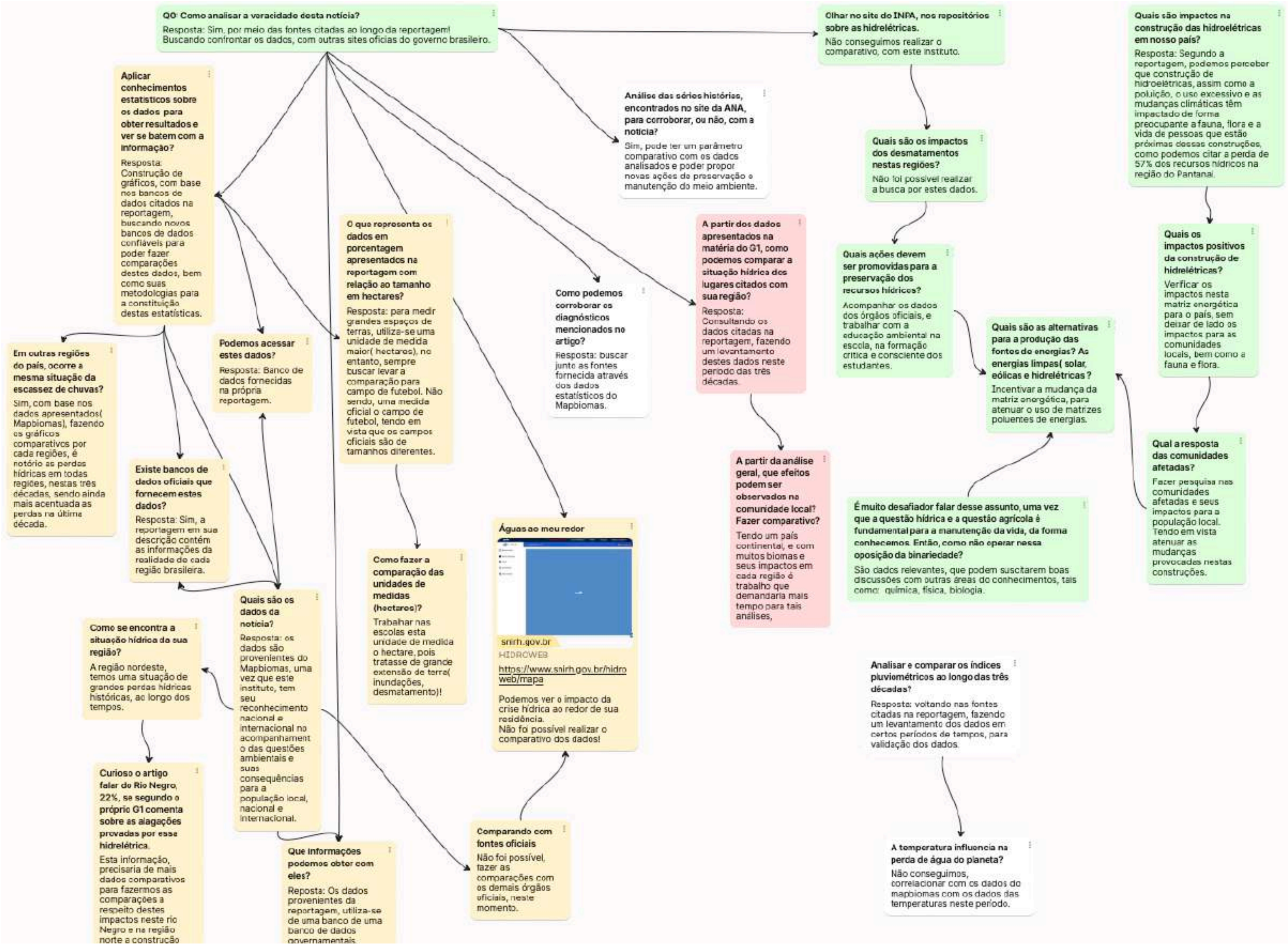
Quais são os impactos dos desmatamentos nestas regiões? Não foi possível realizar a busca por estes dados.

Analisar e comparar os índices pluviométricos ao longo das três décadas? Resposta: Voltando nas fontes citadas na reportagem, fazendo um levantamento dos dados em certos períodos de tempo para validação dos dados.

Em outras regiões do país, ocorre a mesma situação da escassez de chuvas? Sim, com base nos dados apresentados (Mapbiomas), fazendo os gráficos comparativos por cada regiões, são notórias as perdas hídricas em todas regiões, nestas três décadas, sendo ainda mais acentuadas as perdas na ultima década.

→ Reflexão sobre as questões que ficaram pendentes:
Dificuldades de mudar de paradigma; pouco tempo para avançar o estudo. Dificuldades em avançar sozinhos, com a dependência da intervenção da formadora para que o estudo avançasse.

Mapas de questões e respostas produzidos no Padlet



Q1 Quais são os dados da notícia?

R_1: Os dados são provenientes do Mapbiomas, uma vez que este instituto, tem seu reconhecimento nacional e internacional no acompanhamento das questões ambientais e suas consequências para a população local, nacional e internacional.
- "**Brasil perdeu 15% dos seus recursos hídricos em 30 anos,** o equivalente a **3,1 milhões de hectares de superfície hídrica**"
- "O levantamento mostra que em 1991 a superfície hídrica do país era de 19 milhões de hectares. Em 2020, essa área foi reduzida para 16,6 milhões de hectares, uma redução equivalente a mais de uma vez e meia a superfície de água de toda região Nordeste em 2020".
- "**Rio Negro, na Amazônia, que perdeu 22%** da sua superfície desde os anos 90"
- "**Rio São Francisco, que corre por áreas de Cerrado e Caatinga, que perdeu 10%** em sua superfície nos últimos quinze anos"
- "O bioma Pantanal abriga o estado com a maior perda absoluta e proporcional de superfície de água na série histórica de 35 anos analisada pela equipe do MapBiomas: o Mato Grosso do Sul, com uma **redução de 57%**".
- "Se em 1985 o Mato Grosso do Sul tinha mais de 1,3 milhão de hectares cobertos por água, em 2020 eram apenas pouco mais de 589 mil hectares. Mais de 780 mil hectares de água foram perdidos no período".
- "O levantamento destaca a situação da perda de água no Rio Paraguai, no bioma pantaneiro. Os pesquisadores explicam que a construção de hidrelétricas é uma das principais causadoras da diminuição do nível da água, uma vez que estas obras afetam a biodiversidade e a dinâmica dos rios".
- "O segundo estado que mais perdeu água no período é o Mato Grosso, com uma perda de quase 530 mil hectares, seguido por Minas Gerais, com um saldo negativo de mais de 118 mil hectares de água perdida".
- "O Brasil abriga 12% das reservas de água doce da Terra e 53% dos recursos hídricos da América do Sul. Existem 83 rios fronteiriços e transfronteiriços no país, assim como bacias hidrográficas e aquíferos".
- "O bioma com a maior área coberta por água é a Amazônia, com mais de 10,6 milhões de hectares de área média, seguida pela Mata Atlântica (mais de 2,1 milhões de hectares) e pelo Pampa (1,8 milhão de hectares). O Pantanal ocupa a quinta posição, com pouco mais de 1 milhão de hectares de área média, atrás do Cerrado (1,4 milhão de hectares)".

Q0 Como analisar a veracidade desta notícia ?

Brasil perdeu 15% dos seus recursos hídricos em 30 anos, uma perda de quase o dobro da superfície de água de todo o Nordeste

R_0: Por meio das **fontes citadas** ao longo da reportagem! Fizemos um tratamento dos dados disponíveis no banco de dados e obtivemos alguns resultados.
Mídia:
https://mapbiomas.org/estatisticas

Q1.1 Existe bancos de dados oficiais que fornecem estes dados? Podemos acessar estes dados?

$R_{1.1}$: Sim, banco de dados fornecidas na própria reportagem (MapBiomas). Podemos acessar o site do MapBiomas e fazer o download de diferentes estatísticas.

Q1.2 Que informações podemos obter com eles (os dados do banco de dados)?

$R_{1.2}$: Os dados provenientes da reportagem, utiliza-se de uma banco de uma banco de dados governamentais. O Banco de dados do MapBiomas apresenta dados (por anos) por biomas, estados, municípios, regiões, bacias, sub-bacias e regiões hidrográficas.

Q1.2.1 Como os dados desse banco de dados oficial são produzidos?

$R_{1.2.1}$: São produzidos por meio de registros de satélites que conseguem rastear as superfícies do Brasil.

Q1.2.2 Quais dados utilizamos para realizar análises?

$R_{1.2.2}$: **Decisões do grupo:** tivemos que fazer escolhas, como: trabalhar com o banco de dados da própria notícia; no site do MapBiomas, tivemos que decidir em qual das planilhas olhar e escolhemos analisar os dados sobre "Regiões Brasil anual - área de superfície de água em cada região do Brasil de 1985 a 2020".

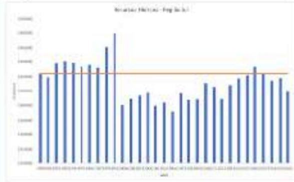
Q2 Como podemos corroborar os diagnósticos mencionados no artigo?

R_2: **Aplicar conhecimentos estatísticos sobre os dados para obter resultados e ver se batem com a informação.** Construção de tabelas e gráficos, com base nos bancos de dados citados na reportagem, buscando novos bancos de dados confiáveis para poder fazer comparações destes dados, bem como suas metodologias para a constituição destas estatísticas.

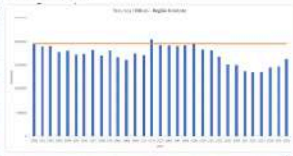
Q2.1 Como se encontra a situação hídrica de cada região brasileira?

$R_{2.1}$: Construímos gráficos de cada região para fazer as comparações de perda.

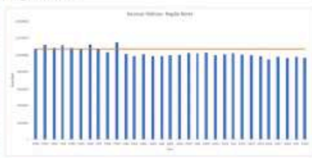
Região Sul



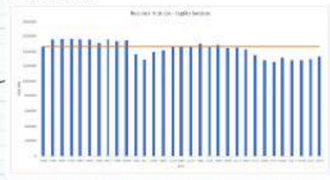
Região Nordeste



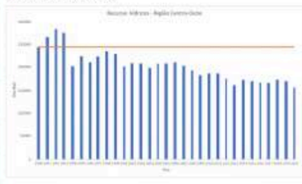
Região Norte



Região Sudeste



Região Centro-Oeste



Q2.1.1: É melhor construir gráficos de barras ou gráficos de linha para representar estes dados? Por quê? Com series temporais é mais habitual construir gráficos de linhas. Há informações sobre este tema?

Tabela com todos os dados organizados por regiões

Ano	Norte	Nordeste	Sudeste	Sul	Centro-Oeste	TOTAL
1990	10729683,6	1959873,15	1463823	2423997,69	2440949,404	19018326,82
1991	11187162,7	1901811,26	1556108,3	2419143,29	2654824,575	19719010,09
1992	10842121,8	1907527,95	1564099,96	2438036,53	2841410,473	19593196,67
1993	11140072,5	1763444,31	1562998,53	2440596,55	2743347,329	19659340,18
1994	10851602,2	1808188,48	1558718,21	2438563,47	2012902,269	18669974,66
1995	10740153,5	1711498,87	1554419,09	2433503,38	2250639,922	18690214,73
1996	11224947,5	1723254,17	1509995,93	2435822,03	2111118,347	19005020,59
1997	10777898,3	1830739,57	1556678,67	2431989,95	2241511,293	18838817,77
1998	10366763,2	1690269,83	1535079,27	2460938,65	2358200,184	18411251,13
1999	11472858,8	1821823,27	1545320,76	2479468,95	2301801,035	19621272,8
2000	10166407,1	1658109,31	1363386,87	2380849,84	2009313,67	17578066,78
2001	9610448,87	1602188,42	1294996,67	2388965,2	2073814,024	17270413,19
2002	10147105,6	1733216,74	1394524,42	2393402,34	2070126,704	17738375,83
2003	9912390,95	1698789,19	1414779,39	2397212,37	1978728,642	17401960,55
2004	9807514,48	2043707,13	1457953,3	2379619,54	2065932,911	17854727,34
2005	10021026,3	1930599	1471677,16	2384028,29	2071377,192	17878707,93
2006	10066067,3	1924576,49	1464550,85	2371996,69	2107615,914	17934717,28
2007	10268679,2	1911320,97	1499959,3	2390795,06	2022948,269	18099618,78
2008	10249130	1926148,54	1468254,19	2387516,83	1925415,922	17956465,51
2009	10321282	1966203,34	1484341,74	2388346,15	1827533,829	17987707,02
2010	10006179,2	1839291,73	1446690,02	2411143,62	1861197,402	17564481,98
2011	10117480,1	1824797,48	1451463,02	2405949,66	1861547,998	17661238,21
2012	10248685,4	1671759,78	1425350,29	2388633,47	1750949,84	17485378,78
2013	10077710,1	1513386,48	1353087,84	2408641,12	1613658,659	16966482,22
2014	10021440,3	1496975,49	1280465,6	2417527,97	1725683,88	16942093,25
2015	9887994,4	1373907,98	1249848,57	2421871,15	1699594,088	16633216,18
2016	9525771,22	1354460,28	1320773,39	2433277,68	1665714,371	16299996,93
2017	9840565,23	1357987,54	1282747,02	2424680,46	1663425,657	16569405,91
2018	9497253,31	1450685,1	1282146,74	2414525,93	1729522,916	16574134
2019	9842852,84	1468296,44	1303022,82	2417876,81	1697227,102	16729276,01
2020	9710126,95	1623029,38	1334258,42	2398945,42	1565211,689	16631571,85

Meio: Q_0 – reportagem de uma mídia
https://g1.globo.com/natureza/noticia/2021/08/23/brasil-perdeu-15percent-dos-seus-recursos-hidricos-nos-ultimos-30-anos-uma-perda-quase-o-dobro-da-superficie-de-agua-de-todo-o-nordeste.ghtml

Objetos: Todos os conhecimentos que já temos sobre os temas. Bancos de dados do MapBiomas, , outros sites de bancos de dados, Excel (fórmulas, gráficos de barras e de segmentos), calculadoras, celular (whatsapp), computador, internet, outros artigos sobre recursos hídricos, expertos ao nosso redor.

→ Obras que acreditamos ser importantes para avançar desse estudo:

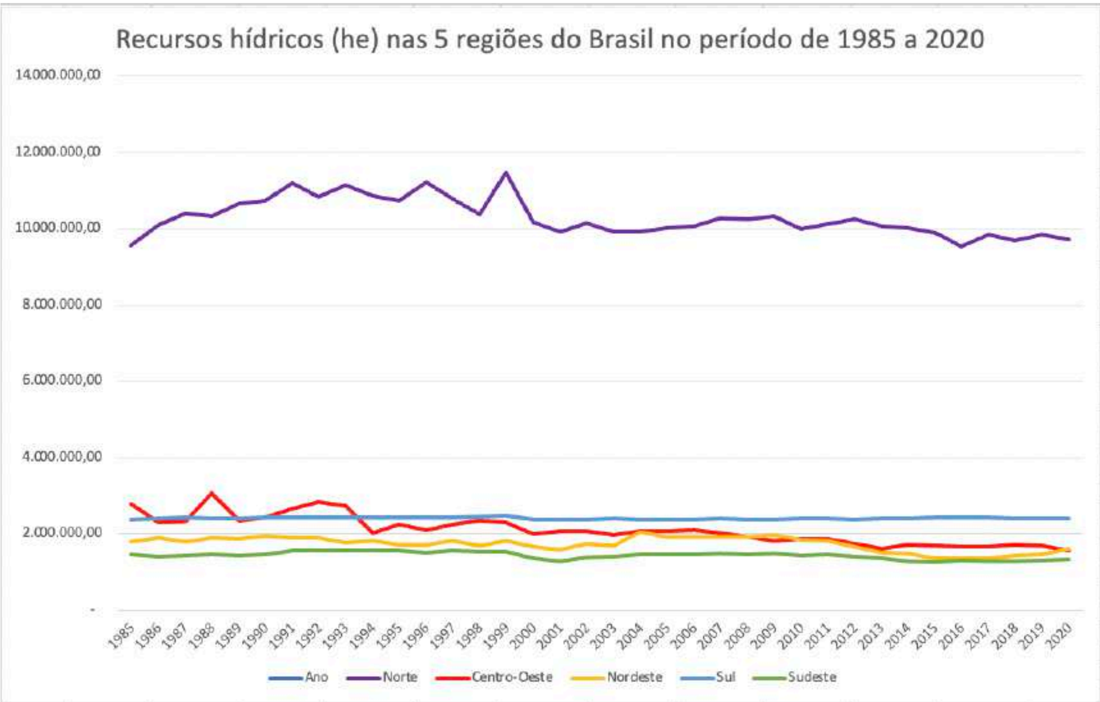
- compreender o funcionamento de hidrelétricas, índices pluviométricos;
- analisar sobre o desmatamento por regiões
- Recursos hídricos no Brasil
- Construção de tabelas: Quais os principais elementos de uma tabela?
- Gráficos estatísticos - analisar e decidir que tipo de gráficos construir para a representação dos dados; Quais os principais elementos de um gráfico?
- É melhor construir gráficos de barras ou gráficos de linhas (segmentos) para representar estes dados? Por quê? Com séries temporais é mais habitual construir gráficos de segmentos. Há informações sobre este tema?
- Gráficos com series tomam valores distintos. Como comparar?
- Comparações de perda em porcentagens (1990 - 2020), cálculo da variação anual (absoluta e em porcentagens)

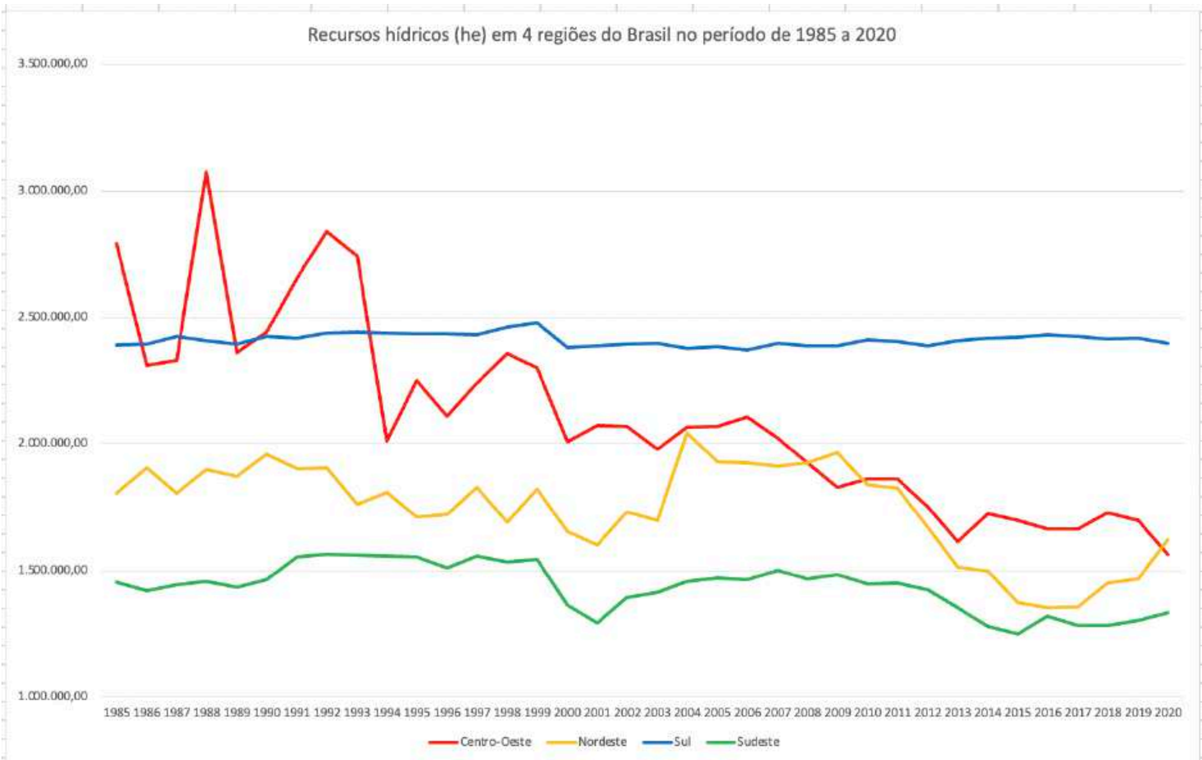
→ Conhecimentos estatísticos utilizados:

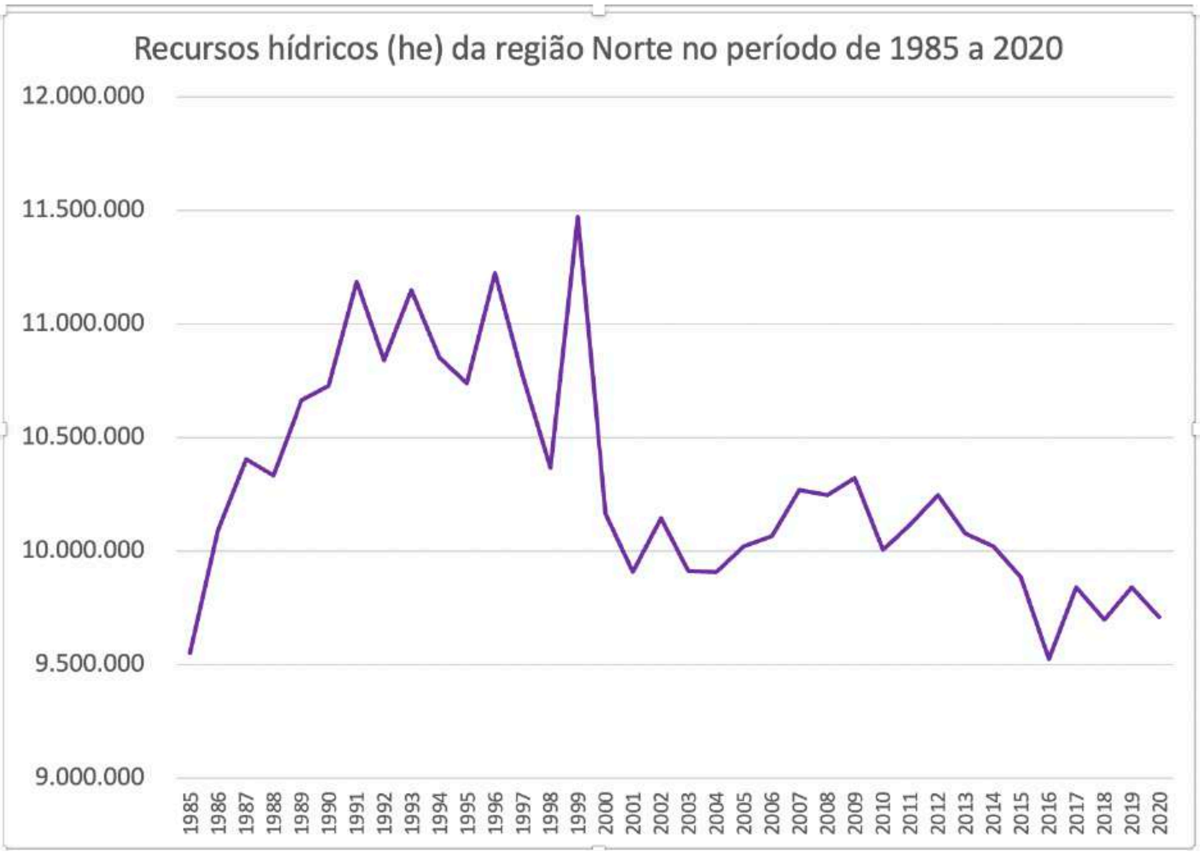
- Construção de tabelas.
- Quais são os elementos de uma tabela? Título (Recursos hídricos ….), fonte, períodos, unidade de medida.
- Construção de gráficos: Analisar e decidir que tipo de gráficos construir para a representação dos dados; a discussão sobre os elementos de um gráfico.
- É melhor gráfico de barras ou de linhas (segmentos)? Por quê? Com séries temporais é mais habitual construir gráficos de segmentos. Há informações sobre este tema?
- Gráficos com séries que tomam valores distintos. Como comparar?
- Comparações de perda em porcentagens (1990 - 2020), cálculo da variação anual (absoluta e em porcentagens)

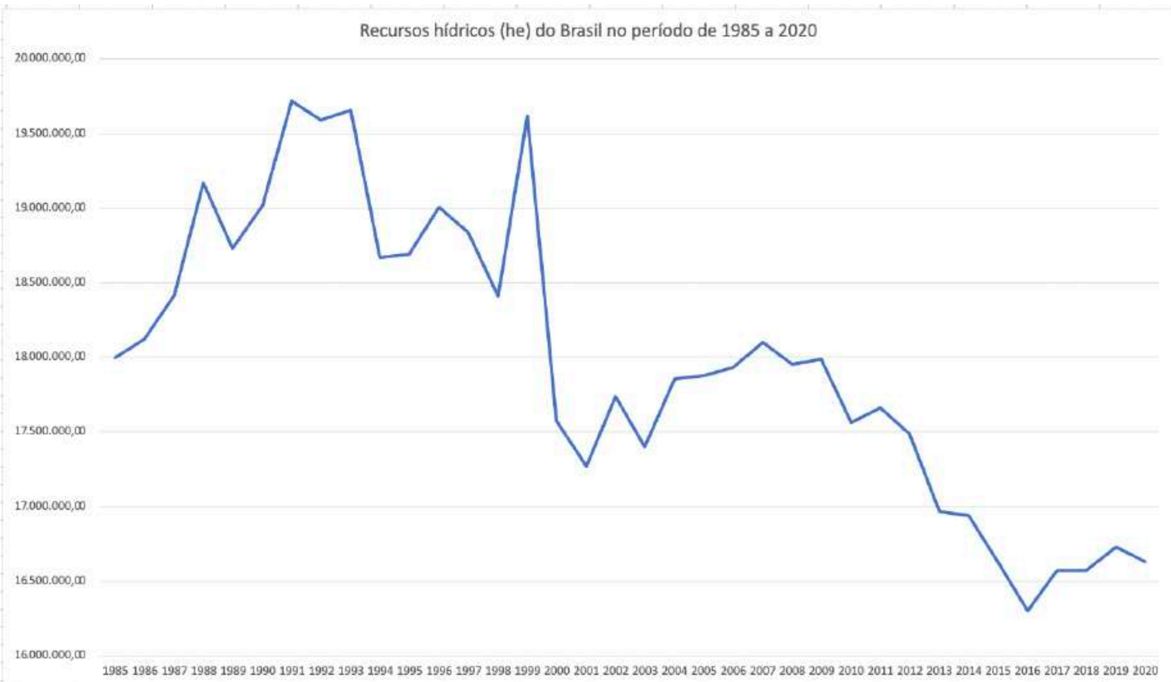
Reflexão sobre representação dos dados em gráficos de segmentos (apresentado pelas formadoras)

A seguir apresentamos outras maneiras de representar os dados sobre recursos hídricos por meio de gráficos de segmentos.









Comentário: Com esses tipos de representações se observa uma maior variabilidade dos recursos hídricos. Isso se dá por mudarmos as escalas dos eixos e por representar as regiões Nordeste, Centro-Oeste, Sul e Sudeste separadas da região Norte.

Módulo 3 – Desenhos/planejamentos de PEP

Atividade Módulo 3: Adaptações para implementação em sala de aula

▪ Trabalho (em seus grupos) para propor um projeto de atividade adaptado aos alunos do ensino fundamental ou ensino médio.
▪ Os grupos pensarão em:
    ▪ Questão inicial da atividade de investigação.
    ▪ Como se coloca a questão, o que se propõe fazer e que ferramentas estão disponíveis?
    ▪ Como o estudo está planejado para ser administrado em classe: horário aproximado
    ▪ Como se espera que a atividade seja concluída
    ▪ Qual conteúdos curriculares (em matemática e outros assuntos) pode ser abordado pela atividade proposta.
 	
Uma vez aprovada a proposta coletiva, prepare um documento como se você tivesse que explicá-lo à um colega da escola para substituí-lo durante à implementação.
Você terá que preparar todo o material e acrescentar indicações precisas para cada sessão.
Ah, e não se esqueça de lhe trazer um presente quando você voltar...

Planejamento de aula 1

ENSINO FUNDAMENTAL II – 6º ANO PLANEJAMENTO - 6 horas/aulas	
ÁREA DE CONHECIMENTO – MATEMÁTICA	
Professor: -----	Turma: 6º ano Turno: Tarde
OBJETO DE CONHECIMENTO	• Volume hidrográfico dos rios da região Norte • Medidas de capacidade (L) • Cálculo de volume (m^3) • Estatística
OBJETIVOS	- Propor aos alunos o reconhecimento e observação dos padrões geométricos como: reconhecer o metro cúbico como unidade padrão de volume e também conhecer o litro como unidade padrão de capacidade. - Explorar a função de tabelas e gráficos, assim como ler, interpretar, organizar e construir.
DESENVOLVIMENTO	• Propor aos alunos: leitura e interpretação de gráfico. • Atividade de pesquisa. • Trabalho em grupo.
UNIDADE TEMÁTICA	• Geometria • Medidas • Gráficos e tabelas
HABILIDADES	- Construir gráficos e tabelas para comunicar informações a partir da coleta e da organização dos dados obtidos e interpretar informações e dados apresentados.

	- Reconhecer e utilizar as unidades de medidas da capacidade presente no cotidiano.
AVALIAÇÃO	Observar a participação do aluno na resolução dos exercícios propostos, para que até o final do ciclo esse objetivo seja alcançado.

Planejamento de aula 2 (implementado)

Público-alvo: 6º ano do Ensino Fundamental **Duração**: 9 h/a
Unidade temática: Probabilidade e estatística

Conteúdos, habilidades e descritores:
- Identificação dos elementos de um gráfico (título, eixos, legenda, fontes). -Leitura das informações apresentadas em tabelas de dupla entrada e em gráficos de colunas. - Identificação das variáveis apresentadas nos diversos tipos de gráficos e tabelas.
- Coleta de dados, organização e registro. Construção de diferentes tipos de gráficos para representá-los e a interpretação das informações; diferentes tipos de representação de informações: gráficos e fluxogramas.

(EF06MA31) Identificar as variáveis e suas frequências e os elementos constitutivos (título, eixos, legendas, fontes e datas) em diferentes tipos de gráfico.

(EF06MA32) Interpretar e resolver situações que envolvam dados de pesquisas sobre contextos ambientais, sustentabilidade, trânsito, consumo responsável, entre outros, apresentadas pela mídia em tabelas e em diferentes tipos de gráficos e redigir textos escritos com o objetivo de sintetizar conclusões.

(EF06MA33) Planejar e coletar dados de pesquisa referente a práticas sociais escolhidas pelos alunos e fazer uso de planilhas eletrônicas para registro, representação e interpretação das informações, em tabelas, vários tipos de gráficos e texto.

(EF06MA34) Interpretar e desenvolver fluxogramas simples, identificando as relações entre os objetos representados (por exemplo, posição de cidades considerando as estradas que as unem, hierarquia dos funcionários de uma empresa etc.).
D37- Resolver problema envolvendo informações apresentadas em tabelas e/ou gráficos.

Metodologia:

Aulas 1 e 2 - Quinta-feira 17/11/22 e sexta-feira 18/11/22 (Laboratório de informação) – momento de exploração sobre o tema proposto

Estas duas primeiras aulas serão realizadas no laboratório de informática. Os alunos se organizarão em duplas ou trios para cada computador. Distribuir para cada dupla ou trio o Texto Base 1, impresso, e pedir para que leiam uma primeira vez entre eles. Depois, realizar uma leitura coletiva e questionar as informações apresentadas no texto junto com os alunos. Buscar aproximações com a realidade deles.

Após a leitura coletiva, propor que os alunos realizem buscas na internet para as questões iniciais:

Q_0: O que é considerado "estado de emergência" sobre estiagem? Que nível para chegar em "estado de emergência"? Quando se decreta o "estado de emergência?

Os alunos poderão acessar a internet para buscar respostas para estas questões. Também será orientado que as duplas ou trios levantem mais questões relacionadas ao tema. As questões levantadas poderão ser registradas na lousa. Pediremos que as duplas e trios escrevam as questões e possíveis respostas nos cadernos. Também é importante pedir para as duplas e trios escreverem as fontes (sites) que buscaram as informações.

Algumas questões que acreditamos que possam surgir:
→ Quais ações são implementadas para combater (amenizar) o estado de emergência?
→ O que o Estado deveria fazer para não chegar ao estado de emergência?
→ Quais as fontes de água da nossa região?
→ Toda água de rio e poços artesianos é potável?
→ Da nossa região, quais os rios ou poços artesianos com água potável?
→ Qual a bacia que banha a nossa região?
→ Quais são os mananciais (barragens) que abastecem nossa região?
→ Quais os níveis precipitação (chuva) da nossa região?

Após este movimento de levantamentos de questões, buscas na internet e possíveis respostas, será feita a **discussão coletiva** sobre tudo o que foi feito e o que as duplas e trios encontraram na internet.

- A professora fará os registros do desenvolvimento dos alunos por meio de fotos dos cadernos das duplas, fotos da lousa e dos computadores sempre que possível.

Recursos: Texto base 1 impresso para cada dupla ou trio; computadores; internet; cadernos para registros dos alunos; lápis/canetas; lousa para registro da professora.

Texto Base 1

Matéria Folha de Pernambuco.
Pernambuco decreta situação de emergência por estiagem em 61 cidades do Agreste; veja lista

Pernambuco decretou situação de emergência em 61 cidades do **Agreste** afetadas pela estiagem. O decreto assinado pelo governador Paulo Câmara e publicado na edição desta quarta-feira (15) do Diário Oficial do Estado tem validade de 180 dias contados a partir do último sábado (11). Com a medida, 116 dos 184 municípios pernambucanos, incluindo os 55 do Sertão anunciados em 6 de setembro, estão com medidas especiais de combate à seca. Órgãos estaduais localizados nas áreas atingidas e competentes para atuação específica devem adotar medidas necessárias para combater a situação de emergência, em conjunto com órgãos municipais. Entre as justificativas citadas pelo governo para a inserção das cidades no decreto estão a previsão de redução das chuvas e queda das reservas hídricas de superfície, os impactos na agropecuária e a situação socioeconômica desfavorável da região.

Veja a lista de cidades incluídas no decreto:

Agrestina	Águas Belas	Alagoinha	Altinho	Angelim	Belo Jardim
Bezerros	Bom Conselho	Bom Jardim	Brejão	Buíque	Brejo da Madre de Deus
Cachoeirinha	Caetés	Calçado	Canhotinho	Capoeiras	Caruaru
Casinhas	Cumaru	Cupira	Feira Nova	Frei Miguelinho	Gravatá
Iati	Itaíba	Jataúba	João Alfredo	Jucati	Jupi
Jurema	Lajedo	Limoeiro	Orobó	Panelas	Paranatama
Passira	Pedra	Pesqueira	Poção	Riacho das Almas	Sairé
Salgadinho	Saloá	Sanharó	Santa Cruz do Capibaribe	Santa Maria do Cambucá	São Bento do Una
São Caetano	São João	São Joaquim do Monte	São Vicente Férrer	Surubim	Tacaimbó
Taquaritinga do Norte	Terezinha	Toritama	Tupanatinga	Venturosa	Vertente do Lério
Vertentes					

Texto adaptado. Fonte: Portal Folha de Pernambuco, publicação em 15/09/21. https://www.folhape.com.br/noticias/pernambuco-decreta-situacao-de-emergencia-por-estiagem-em-61-cidades/197683/. Acesso em 20/10/2022.

Aulas 3 e 4 - Segunda-feira 21/11/22 (momento de construção/representação de gráficos)

Nesta aula, as duplas e trios continuarão trabalhando juntos. Será distribuída a Tabela 1 que apresenta dados sobre a precipitação média mensal (de um período de 30 anos) de municípios que estão ao redor de Pesqueira (município ao qual pertencem os estudantes). Primeiro, a professora discutirá como a tabela foi construída, o que significa **média mensal de precipitação, que dados são apresentados e qual a fonte destes dados**. Após este primeiro momento, e a partir dos dados da tabela, faremos um debate sobre a **construção de gráficos de barras**. Faremos uma construção coletiva de um gráfico de barras dos dados de Pesqueira.

Tabela 1 - PRECIPITAÇÃO MÉDIA POR MUNICÍPIO (em mm)

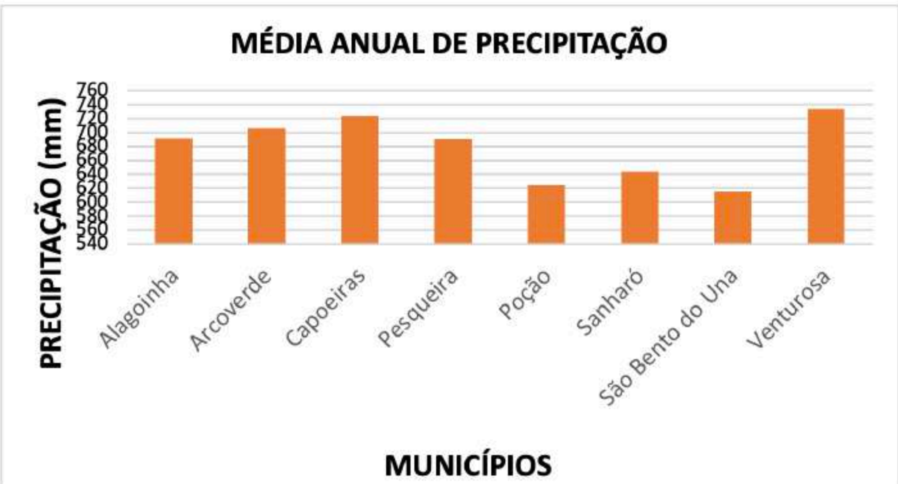
PRECIPITAÇÃO MÉDIA POR MUNICÍPIO (mm)													
MUNICÍPIO	JAN	FEV	MAR	ABR	MAI	JUN	JUL	AGO	SET	OUT	NOV	DEZ	ANUAL
Alagoinha	38	53	68	85	94	101	97	60	34	21	15	36	692
Arcoverde	42	71	122	116	77	77	77	39	21	14	27	22	706
Capoeiras	48	63	94	103	86	93	90	49	30	23	18	28	723
Pesqueira	39	70	93	105	96	89	73	38	18	16	24	31	691
Poção	42	66	103	102	79	71	57	30	14	14	19	28	625
Sanharó	40	55	121	102	95	78	65	33	15	17	11	33	644
São Bento do Una	42	54	100	85	81	76	64	33	19	17	19	26	616
Venturosa	47	73	112	111	91	86	78	42	22	18	24	30	734
Salgadinho													

"Os dados acima representam a climatologia (média) de cada município, para cada mês. Foram considerados as localidades com mais de 30 anos, e para as demais os valores foram estimados". Fonte: https://www.apac.pe.gov.br/193-climatologia/521-climatologia-por-municipio Acesso 20 de out 2022.

A professora que já conhece os estudantes acredita que eles nunca construíram gráficos de barra; então, durante a construção coletiva, será importante mostrar quais são os elementos essenciais para construir um gráfico de barras: O que será representado em cada eixo? Qual a legenda de cada eixo? Qual a unidade de medida dos dados apresentados? Qual será o Título? Qual a proporção do eixo vertical para desenhar as barras? Qual foi a fonte dos dados?

Gráfico 1 – Precipitação anual média dos municípios

Eixo vertical: precipitação média anual Eixo horizontal: municípios



Fonte: Dados obtidos em: https://www.apac.pe.gov.br/193-climatologia/521-climatologia-por-municipio Acesso 20 de out 2022.

Como será apenas uma aula, a professora acredita que será utilizada toda a aula para fazer a construção coletiva do gráfico e a discussão de todos os elementos. A professora registrará a construção do gráfico por meio de uma foto.

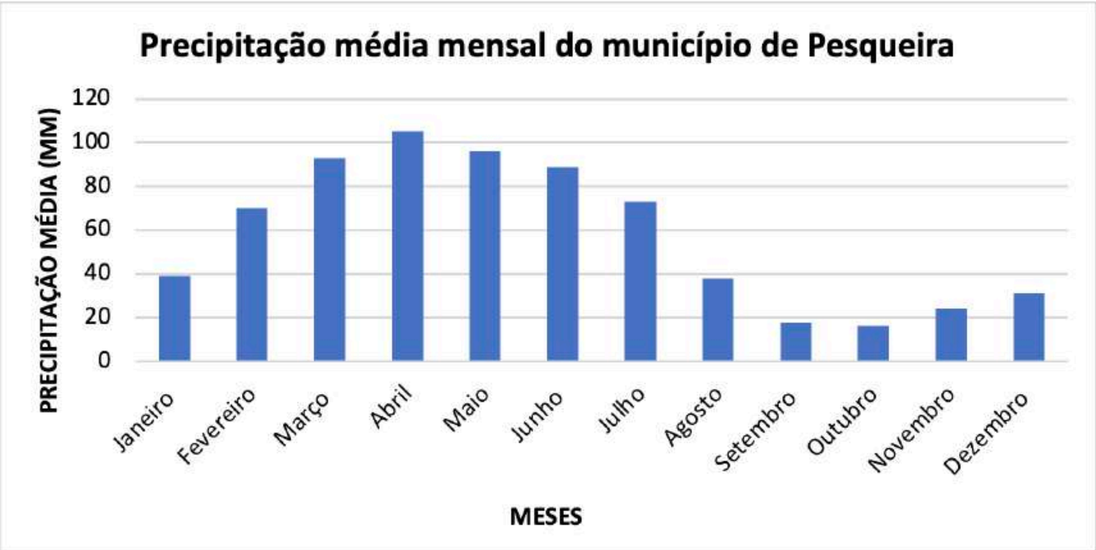
Aula 5 – Terça-feira 22/11/22 – Momento de interação e construção gráfica dos alunos

Recursos: Tabela 1 impressa, lousa, folhas quadriculadas, réguas, lápis, lápis de cor, giz de cera...

Nesta aula a professora fará mais uma construção coletiva de gráfico de barras com os estudantes, a partir dos dados representados na Tabela 1. Desta vez, será construído um gráfico de barras sobre a precipitação média mensal do município de Pesqueira (município ao qual os estudantes pertencem). Construção coletiva.

Gráfico 2 – Precipitação média mensal da cidade de Pesqueira

Eixo vertical: Precipitação média mensal (mm) Eixo horizontal: Meses.



Dados obtidos em: https://www.apac.pe.gov.br/193-climatologia/521-climatologia-por-municipio Acesso 20 de out 2022.

Aula 6 – Terça-feira 29/11/22

Após esta segunda construção coletiva, os estudantes trabalharão novamente em duplas ou trios para a construção de mais gráficos de barras. Propor para cada dupla ou trio que eles construam gráficos. Cada dupla e trio construirá dois gráficos. Para isso, serão distribuídas folhas quadriculadas para as duplas e trios. Pedir para as dupla e trio utilizar ao máximo do espaço da folha quadriculada.

Atividade – será realizada a construção de 19 Gráficos que serão construídos:

→ Cada gráfico será construído em uma folha quadriculada

Gráficos sobre precipitação média dos municípios

G1 – Precipitação média dos municípios no mês de janeiro
G2 – Precipitação média dos municípios no mês de fevereiro
G3 – Precipitação média dos municípios no mês de março
G4 – Precipitação média dos municípios no mês de abril
G5 – Precipitação média dos municípios no mês de maio
G6 – Precipitação média dos municípios no mês de junho
G7 – Precipitação média dos municípios no mês de julho
G8 – Precipitação média dos municípios no mês de agosto
G9 – Precipitação média dos municípios no mês de setembro
G10 – Precipitação média dos municípios no mês de outubro
G11 – Precipitação média dos municípios no mês de novembro
G12 – Precipitação média dos municípios no mês de dezembro

Gráficos sobre precipitação média mensal dos municípios

G1.1 - Precipitação média mensal da cidade de Alagoinha
G1.2 - Precipitação média mensal da cidade de Arcoverde
G1.3 - Precipitação média mensal da cidade de Capoeiras
G1.4 - Precipitação média mensal da cidade de Poção
G1.5 - Precipitação média mensal da cidade de Sanharó
G1.6 - Precipitação média mensal da cidade de São Bento do Uma
G1.7 - Precipitação média mensal da cidade de Venturosa
G1.8 - Precipitação média mensal da cidade de Salgadinho

Nesta aula, as duplas e trios começarão as construções, mas, como é apenas uma aula, não será suficiente para acabar. A professora guardará todos os trabalhos das duplas e trios ao finalizar a aula. Continuarão o trabalho na aula de sexta-feira.

Aula 7 - Quinta-feira 01/12/22

Nesta aula as duplas e trios continuarão a construção dos gráficos. A professora mediará as construções, tirando dúvidas à medida que apareçam.

Aulas 8 e 9 – Segunda-feira 05/12/22

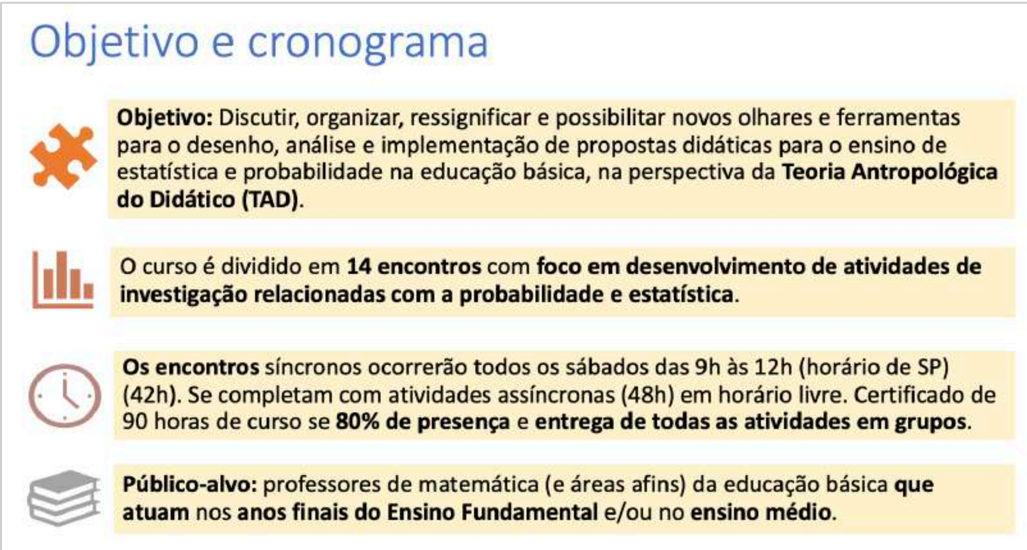
As duplas e trios construirão os gráficos. Nestas duas aulas, será o momento de finalizar as construções dos gráficos e expor/apresentar os resultados para toda a sala. Os gráficos serão fixados na lousa. Discussão coletiva.

Algumas questões para orientar a discussão:

→ Que comparações podemos fazer entre os gráficos construídos?
→ Quais conclusões podemos fazer a respeito dos níveis de precipitação (chuva) das diferentes regiões?
→ Em quais meses do ano há menos precipitações nos diferentes municípios observados?
→ ...

APPENDIX D.2: Slides of the online sessions

Sessão 1



Fundamentos e contextualização: Uma mudança de paradigma segundo a TAD

Paradigma da visita às obras (ou *monumentalismo*)

CURRÍCULO COMO CONJUNTO DE OBRAS OU CONTEÚDOS

- Esse paradigma pode ser entendido como o ensino tradicional ao qual pertencemos;
- Trata-se de um paradigma dominante e que se encontra em decadência;
- Papel do aluno e papel do professor.

Paradigma do questionamento do mundo

CURRICULUM COMO CONJUNTO DE CUESTIONES

- O objetivo principal não é mais estudar uma "obra" pronta e acabado e sim uma questão (*Q*);
- Papel do aluno e papel do professor mudam.

Chevallard (2013). Enseñar matemáticas en la sociedad de mañana: alegato a favor de un contraparadigma emergente

Fundamentos e contextualização

A formação de professores: visita ás obras ou questionamento do mundo?

Estamos imersos em um momento de debate sobre a profissão de professor e, em particular, sobre sua formação inicial e continuada.

Grande concordância de que a mudança de paradigma escolar deve andar de mãos dadas com a formação de professores como agente chave dessa mudança.

Vamos nos concentrar nas propostas dos Percursos de Estudo e Pesquisa (PEP) e da PEP para Formação de Professores (PEP-FP) (Ruiz-Olarría, 2015)

M0. Partirmos de uma **questão relacionada a profissão docente**: relacionada ao estatística nos anos finais do ensino fundamental
→ POSIÇÃO DE PROFESSOR

M1. Vivenciar um **processo de aprendizagem (PEP)** semelhante ao que poderia existir em aulas de matemática
→ POSIÇÃO DE ALUNO

M2. **Análise coletiva** do processo de ensino vivenciado com **ferramentas didáticas**
→ POSIÇÃO DE PROFESSOR-ANALISTA

M3. **Adaptar e implementar (análise *in vivo*)** o PEP experimentado a determinadas condições escolares
→ POSIÇÃO DE PROFESSOR-DESIGNER

M4. **Análise *a posteriori* coletiva** do PEP implementado e novas adaptações
→ POSIÇÃO DE PROFESSOR-ANALISTA-DESIGNER

Calendário de encontros

Sessões	Datas	Sessões	Módulos
1	03/09/2022	Introdução + Questões sobre o ensino	M0
2	10/09/2022	Investigação sobre Q_0	M0 e M1
3	17/09/2022	Investigação sobre Q_0: trabalho com dados	M1
4	24/09/2022	Investigação sobre Q_0 (preparação das respostas finais da Q_0)	M1
5	01/10/2022	Apresentação das respostas finais da Q_0	M1 e M2
6	08/10/2022	Análise do processo de aprendizagem vivenciado	M2
7	15/10/2022	Apresentação das análises + Adaptações para implementação em sala de aula	M2 e M3
8	22/10/2022	Adaptações para implementação em sala de aula	M3
9	05/11/2022	Adaptações para implementação em sala de aula	M3
10	12/11/2022	Trabalho em grupo: de um exercício escolar a um PEP	M3
11	19/11/2022	Implementações em sala de aula Trabalho em grupo: de um exercício escolar a um PEP	M4
12	26/11/2022	Implementações em sala de aula Apresentação dos PEP: de um exercício escolar a um PEP	M4
13	03/12/2022	Implementação em sala de aula. Análises finais – trabalho individual	M4
14	10/12/2022	Análises finais	M4

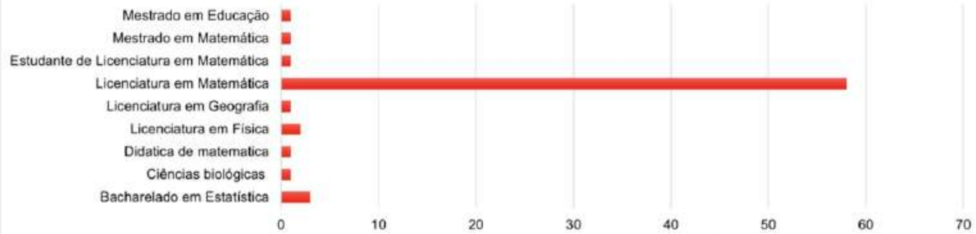
Organização dos grupos e equipes de trabalho

Equipes de trabalho: entre 4 e 6 pessoas

Formulário para indicar suas preferências

Conhecendo o perfil dos participantes

Formação dos PARTICIPANTES



	Bacharelado em Estatística	Ciências biológicas	Didatica de matematica	Licenciatura em Física	Licenciatura em Geografia	Licenciatura em Matemática	Estudante de Licenciatura em Matemática	Mestrado em Matemática	Mestrado em Educação
Série1	3	1	1	2	1	58	1	1	1

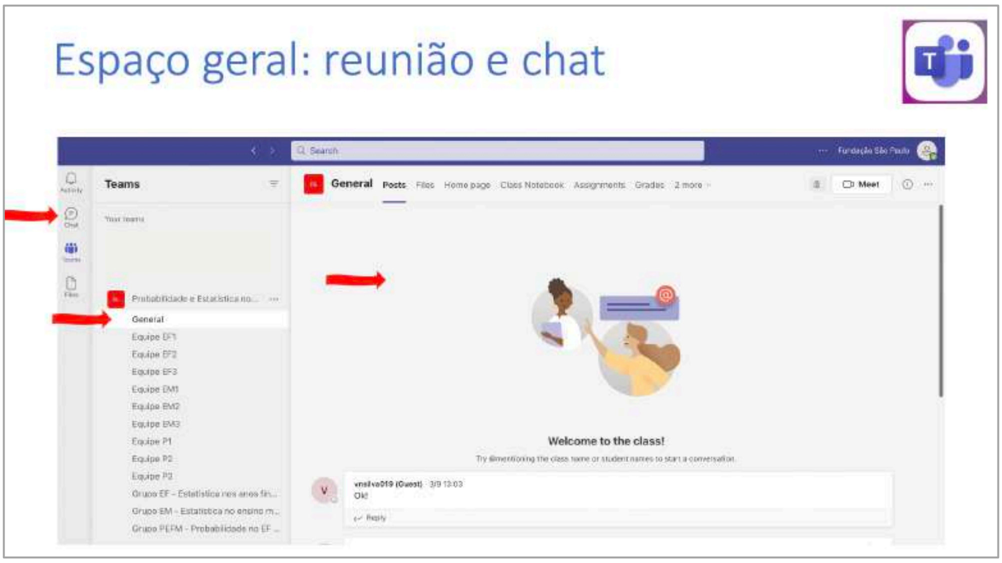

Conhecendo o perfil dos participantes
DE ONDE SÃO OS PARTICIPANTES
QUANTIDADE
35
30
25
20
15
10
5
0
30
7
13
2
1
3
3
4
2
1
1
1
1
2
1
1
1
1
1
Bahia
Ceará
Pará
Amazonas
Mato Grosso do Sul
Moçambique
Pernambuco
São Paulo
Alagoas
Mejía, Islay, Arequipa
Perú
Mali
Rondônia
Rio Grande do Sul
Rio de Janeiro
Rio Grande do Norte
Equador
Paraíba
Santa Catarina

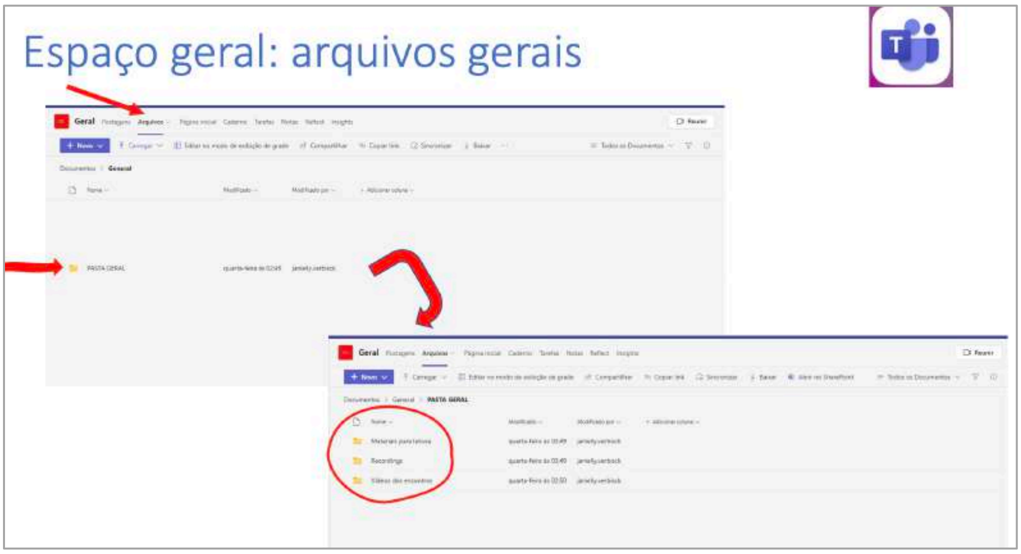

ATIVIDADE EM EQUIPES
Questões sobre o ensino de probabilidade e estatística
M0. Partir do objetivo deste curso de formação: questões relacionadas ao ensino de probabilidade e estatística na educação básica
→ POSICÃO DE PROFESSOR
→ Coluna 1: Estatística EF2
→ Coluna 2: Estatística EM
→ Coluna 3: Probabilidade EF2 e EM
Padlet: https://padlet.com/janiellyverbisck/xs7an51zndq0qazo

Sessão 3

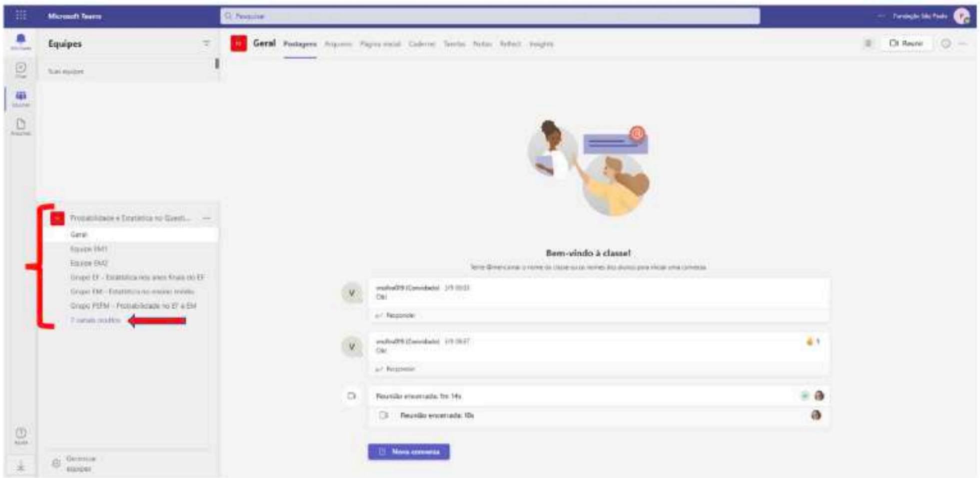

Ensino de probabilidade e estatística no questionamento do mundo
CURSO DE FORMAÇÃO SETEMBRO-DEZEMBRO 2022
SESSÃO 3, 17 de setembro de 2022
PUC-SP
UFBA
UFMS
FUNDAÇÃO UNIVERSIDADE FEDERAL DE MATO GROSSO DO SUL
UNIVERSITAT DE BARCELONA
CAPES


M1. Vivenciar um processo de aprendizagem (PEP) semelhante ao que poderia existir em aulas de matemática
→ POSIÇÃO DE ALUNO
M0. Partirmos de uma questão relacionada a profissão docente: relacionada ao ensino de probabilidade e estatística na educação básica
→ POSIÇÃO DE PROFESSOR
ESTRUTURA DA FORMAÇÃO
M2. Análise coletiva do processo de ensino vivenciado com ferramentas didáticas
→ POSIÇÃO DE PROFESSOR - ANALISTA
M3. Adaptar e implementar (análise in vivo) o PEP experimentado a determinadas condições escolares
→ POSIÇÃO DE PROFESSOR-DESIGNER
M4. Análise a posteriori coletiva do PEP implementado e novas adaptações
→ POSIÇÃO DE PROFESSOR-ANALISTA- DESIGNER

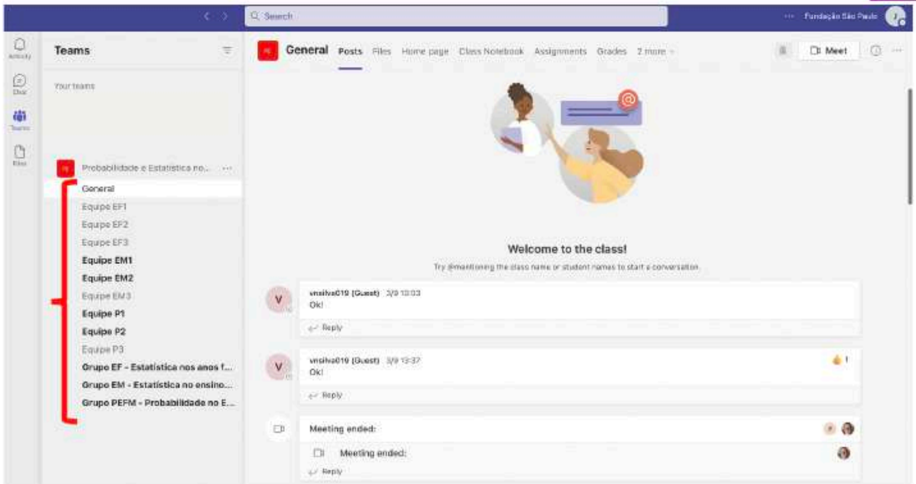

Espaço geral: reunião e chat
Teams
General
Posts
Welcome to the class!


Espaço geral: arquivos gerais

Canais



Canais

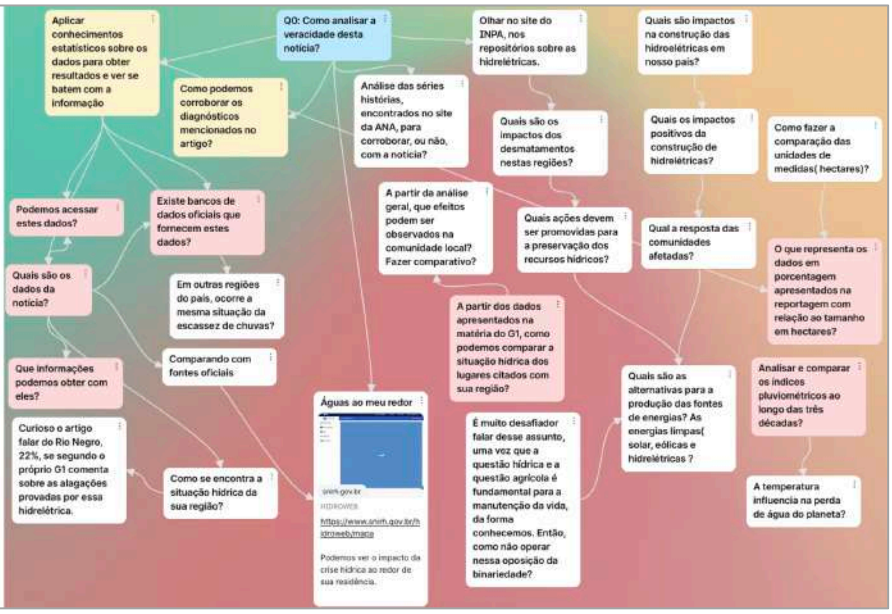


Grupo Estatística nos anos finais do EF - Q0

Observem a notícia publicada no G1 em 23/08/21: https://g1.globo.com/natureza/noticia/2021/08/23/brasil-perdeu-15percent-dos-seus-recursos-hidricos-nos-ultimos-30-anos-uma-perda-quase-o-dobro-da-superficie-de-agua-de-todo-o-nordeste.ghtml

QUESTAO GERATRIZ PEP: Como analisar a veracidade desta notícia?

Mapa da Q_0 e questões derivadas



Grupo Estatística no ensino médio - Q0

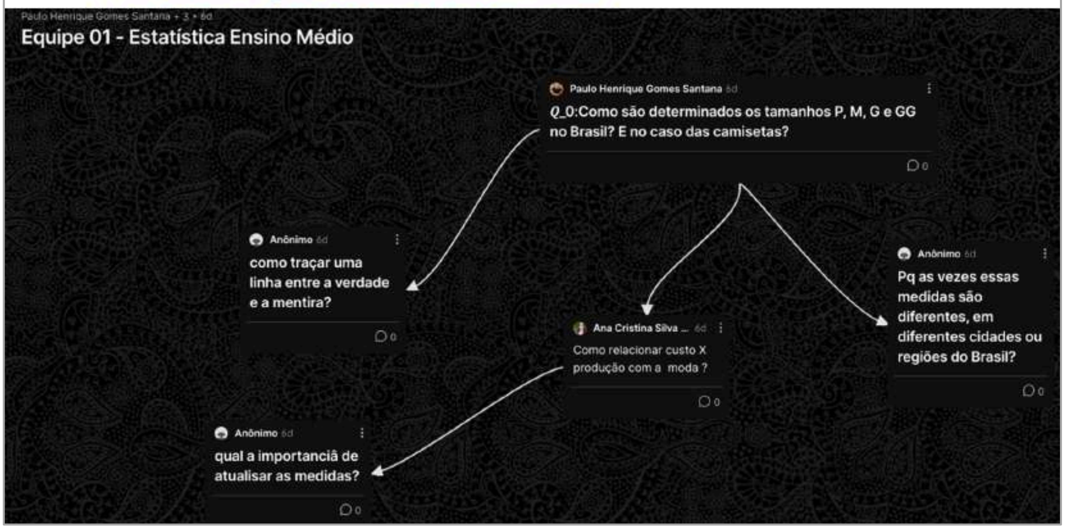
Observem a notícia publicada na Folha de São Paulo em 12/10/21:
https://www1.folha.uol.com.br/mercado/2021/10/brasil-discute-ha-quase-uma-decada-o-tamanho-das-roupas-das-mulheres.shtml

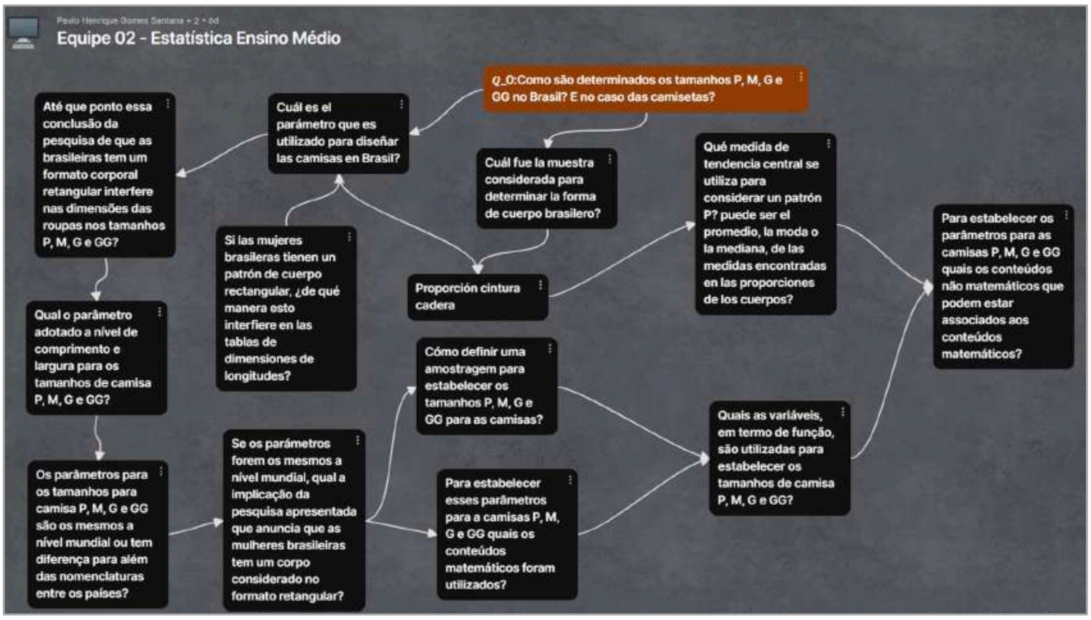
76% das brasileiras têm corpo retangular; entenda os cinco tipos que a moda adota

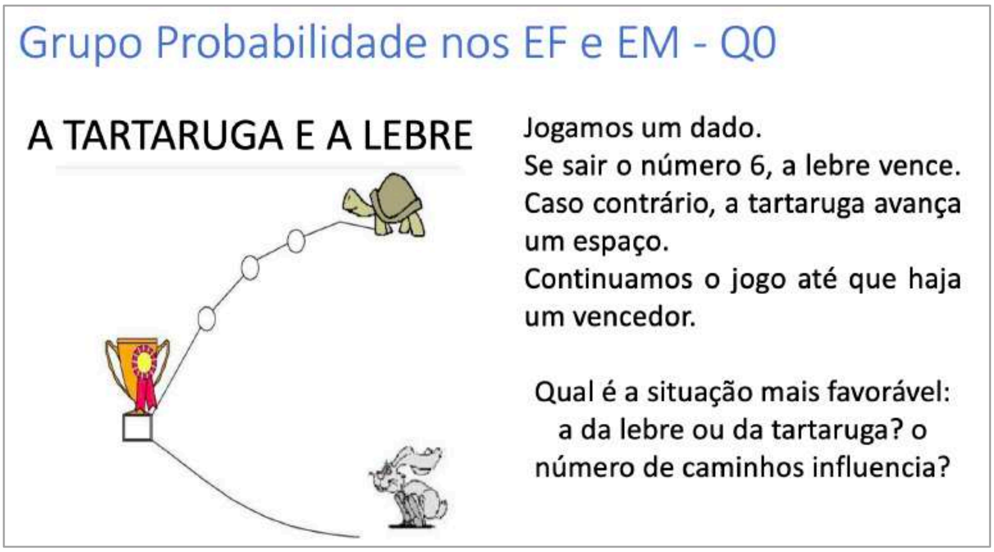
Embates no setor impediram país de adotar uma tabela de referência; ABNT espera publicar nova norma até dezembro

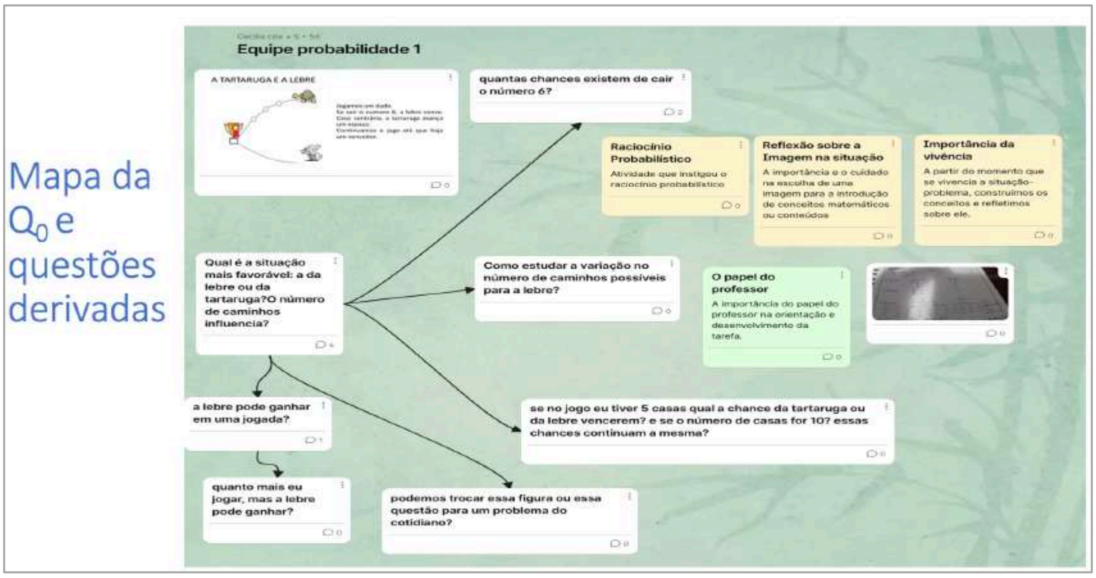
Q_0 :Como são determinados os tamanhos P, M, G e GG no Brasil? E no caso das camisetas?

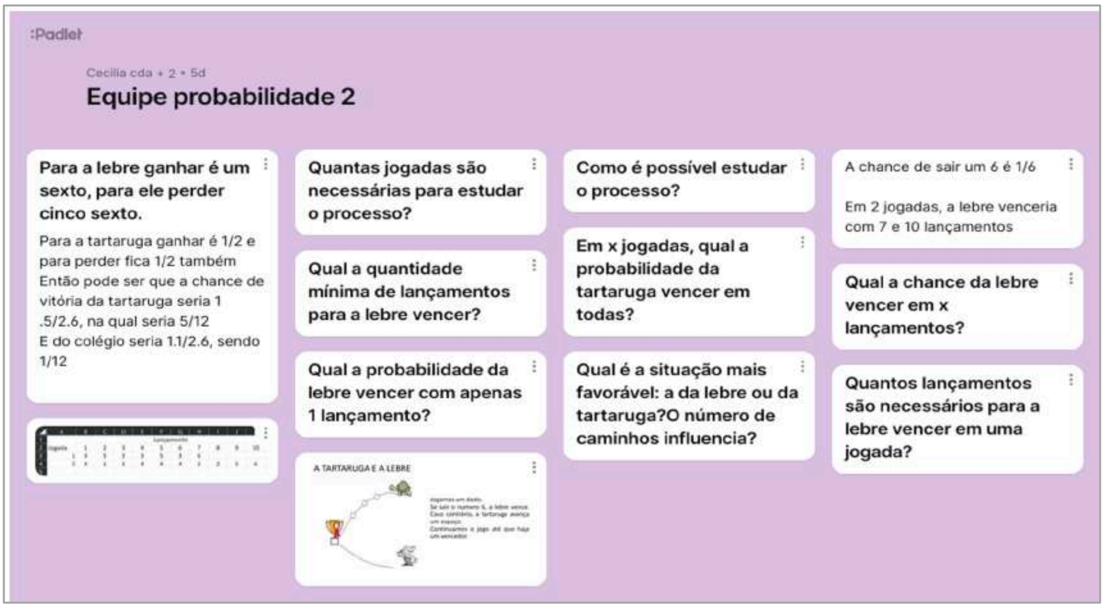
Mapa da Q_0 e questões derivadas

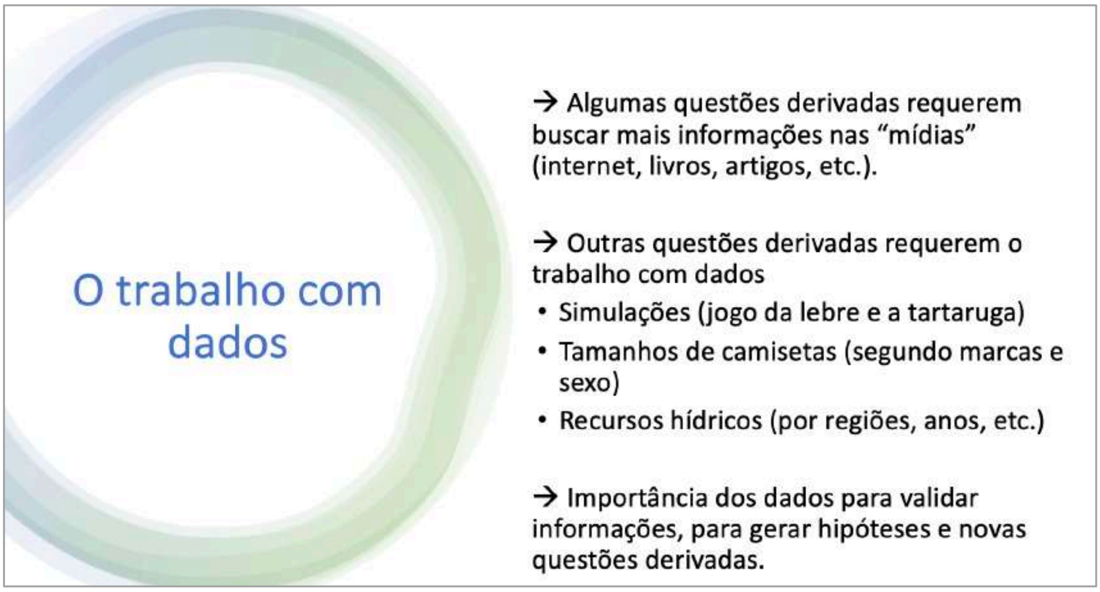



Equipe 02 - Estatística Ensino Médio
Q_0:Como são determinados os tamanhos P, M, G e GG no Brasil? E no caso das camisetas?
Até que ponto essa conclusão da pesquisa de que as brasileiras tem um formato corporal retangular interfere nas dimensões das roupas nos tamanhos P, M, G e GG?
Cuál es el parámetro que es utilizado para diseñar las camisas en Brasil?
Cuál fue la muestra considerada para determinar la forma de cuerpo brasilero?
Qué medida de tendencia central se utiliza para considerar un patrón P? puede ser el promedio, la moda o la mediana, de las medidas encontradas en las proporciones de los cuerpos?
Para estabelecer os parâmetros para as camisas P, M, G e GG quais os conteúdos não matemáticos que podem estar associados aos conteúdos matemáticos?
Si las mujeres brasileras tienen un patrón de cuerpo rectangular, ¿de qué manera esto interfiere en las tablas de dimensiones de longitudes?
Proporción cintura cadera
Qual o parâmetro adotado a nível de comprimento e largura para os tamanhos de camisa P, M, G e GG?
Cómo definir uma amostragem para estabelecer os tamanhos P, M, G e GG para as camisas?
Quais as variáveis, em termo de função, são utilizadas para estabelecer os tamanhos de camisa P, M, G e GG?
Os parâmetros para os tamanhos para camisa P, M, G e GG são os mesmos a nível mundial ou tem diferença para além das nomenclaturas entre os países?
Se os parâmetros forem os mesmos a nível mundial, qual a implicação da pesquisa apresentada que anuncia que as mulheres brasileiras tem um corpo considerado no formato retangular?
Para estabelecer esses parâmetros para a camisas P, M, G e GG quais os conteúdos matemáticos foram utilizados?


Grupo Probabilidade nos EF e EM - Q0
A TARTARUGA E A LEBRE
Jogamos um dado.
Se sair o número 6, a lebre vence.
Caso contrário, a tartaruga avança um espaço.
Continuamos o jogo até que haja um vencedor.
Qual é a situação mais favorável: a da lebre ou da tartaruga? o número de caminhos influencia?

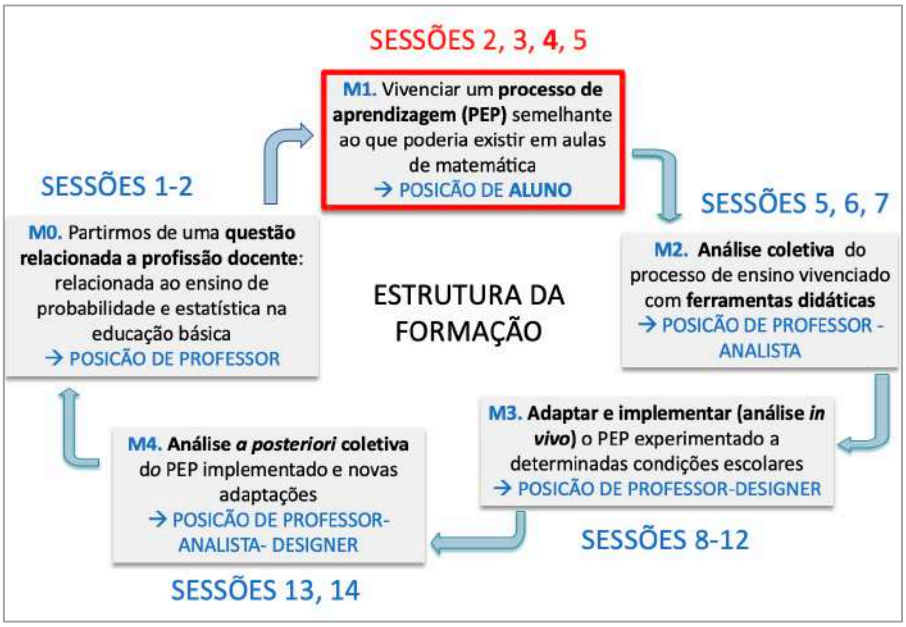

Mapa da Q_0 e questões derivadas
Equipe probabilidade 1
A TARTARUGA E A LEBRE
quantas chances existem de cair o número 6?
Raciocínio Probabilístico
Atividade que instigou o raciocínio probabilístico
Reflexão sobre a Imagem na situação
A importância e o cuidado na escolha de uma imagem para a introdução de conceitos matemáticos ou conteúdos
Importância da vivência
A partir do momento que se vivencia a situação-problema, construímos os conceitos e refletimos sobre ele.
Qual é a situação mais favorável: a da lebre ou da tartaruga?O número de caminhos influencia?
Como estudar a variação no número de caminhos possíveis para a lebre?
O papel do professor
A importância do papel do professor na orientação e desenvolvimento da tarefa.
a lebre pode ganhar em uma jogada?
se no jogo eu tiver 5 casas qual a chance da tartaruga ou da lebre vencerem? e se o número de casas for 10? essas chances continuam a mesma?
quanto mais eu jogar, mas a lebre pode ganhar?
podemos trocar essa figura ou essa questão para um problema do cotidiano?


:Padlet
Cecilia cda • 2 • 5d
Equipe probabilidade 2
Para a lebre ganhar é um sexto, para ele perder cinco sexto.
Para a tartaruga ganhar é 1/2 e para perder fica 1/2 também
Então pode ser que a chance de vitória da tartaruga seria 1 .5/2.6, na qual seria 5/12
E do colégio seria 1.1/2.6, sendo 1/12
Quantas jogadas são necessárias para estudar o processo?
Qual a quantidade mínima de lançamentos para a lebre vencer?
Qual a probabilidade da lebre vencer com apenas 1 lançamento?
A TARTARUGA E A LEBRE
Como é possível estudar o processo?
Em x jogadas, qual a probabilidade da tartaruga vencer em todas?
Qual é a situação mais favorável: a da lebre ou da tartaruga?O número de caminhos influencia?
A chance de sair um 6 é 1/6
Em 2 jogadas, a lebre venceria com 7 e 10 lançamentos
Qual a chance da lebre vencer em x lançamentos?
Quantos lançamentos são necessários para a lebre vencer em uma jogada?


O trabalho com dados
→ Algumas questões derivadas requerem buscar mais informações nas "mídias" (internet, livros, artigos, etc.).
→ Outras questões derivadas requerem o trabalho com dados
• Simulações (jogo da lebre e a tartaruga)
• Tamanhos de camisetas (segundo marcas e sexo)
• Recursos hídricos (por regiões, anos, etc.)
→ Importância dos dados para validar informações, para gerar hipóteses e novas questões derivadas.

Sessão 4


Ensino de probabilidade e estatística no questionamento do mundo
CURSO DE FORMAÇÃO SETEMBRO-DEZEMBRO 2022
Sessão 4 – 24 de setembro de 2022
PUC-SP
UFBA
UFMS
FUNDAÇÃO UNIVERSIDADE FEDERAL DE MATO GROSSO DO SUL
UNIVERSITAT DE BARCELONA
CAPES



O tratamento de dados

"[Dados] tem sido sempre um elemento básico da ciência. [...] A novidade é que **os dados estão agora à nossa volta**. A digitalização multiplicou-a e tornou-a ubíqua. Se gerir uma pequena loja, tem de ficar de olho no seu stock; se for um executivo de camionagem, tem de prever o preço da gasolina; e se for um jardineiro, e quiser que os seus canteiros pareçam verdes, tem de calcular quando regá-los. **Há poucos ofícios que não vão ser quantificados**. E mesmo que o seu não seja um deles, não importa, porque **os números também estão na sua vida diária**, como quando contrai uma hipoteca ou quando escolhe uma escola para os seus filhos. **Os dados estão à sua volta**."

Kiko Llaneras, *Piensa claro,* Debate, 2022.

O tratamento de dados

"[...] todos os ciudadãos precisam desenvolver habilidades **para coletar, organizar, representar, interpretar e analisar dados em uma variedade de contextos**, de maneira a fazer julgamentos bem fundamentados e tomar as decisões adequadas. Isso inclui raciocinar e utilizar conceitos, representações e índices estatísticos para descrever, explicar e predizer fenômenos."

(BNCC, p. 274)

Objetos de conhecimento e habilidades

Objetos de conhecimento	Habilidades
Coleta de dados, organização e registro	(EF06MA33) Planejar e **coletar dados de pesquisa** referente a práticas sociais escolhidas pelos alunos e **fazer uso de planilhas eletrônicas para registro, representação e interpretação das informações**, em tabelas, vários tipos de gráficos e texto.
Planejamento de pesquisa, coleta e organização dos dados, construção de tabelas e gráficos e interpretação das informações	(EF07MA36) Planejar e realizar pesquisa envolvendo **tema da realidade social**, identificando a necessidade de ser censitária ou de usar amostra, e **interpretar os dados** para comunicá-los por meio de relatório escrito, tabelas e gráficos, com o apoio de planilhas eletrônicas.
	(EM13MAT406) Construir e interpretar tabelas e gráficos de frequências **com base em dados obtidos em pesquisas** por amostras estatísticas, incluindo ou não o uso de softwares que inter-relacionem estatística, geometria e álgebra.

O tratamento de dados na escola

- Dimensão "banalizada" pela cultura matemática
- Pouca consideração, é um trabalho fácil e tedioso (como limpar a casa)
- Em qualquer caso, vem antes do trabalho estatístico "real" (resumos, medições, gráficos, etc.)
- Não ligado ao estudo de eventos relacionados com o acaso e a probabilidade: não são feitas simulações.

MAS:

- Encontrar, gerar e organizar dados é complicado
- É preciso ser confrontado com as tarefas para a realizar
- É preciso descobrir o criar novas técnicas e estratégias, adaptadas a cada situação
- Sem esta experiência prévia, não poderemos ensiná-la aos nossos alunos.

A proposta do curso

Três casos distintos e complementários:

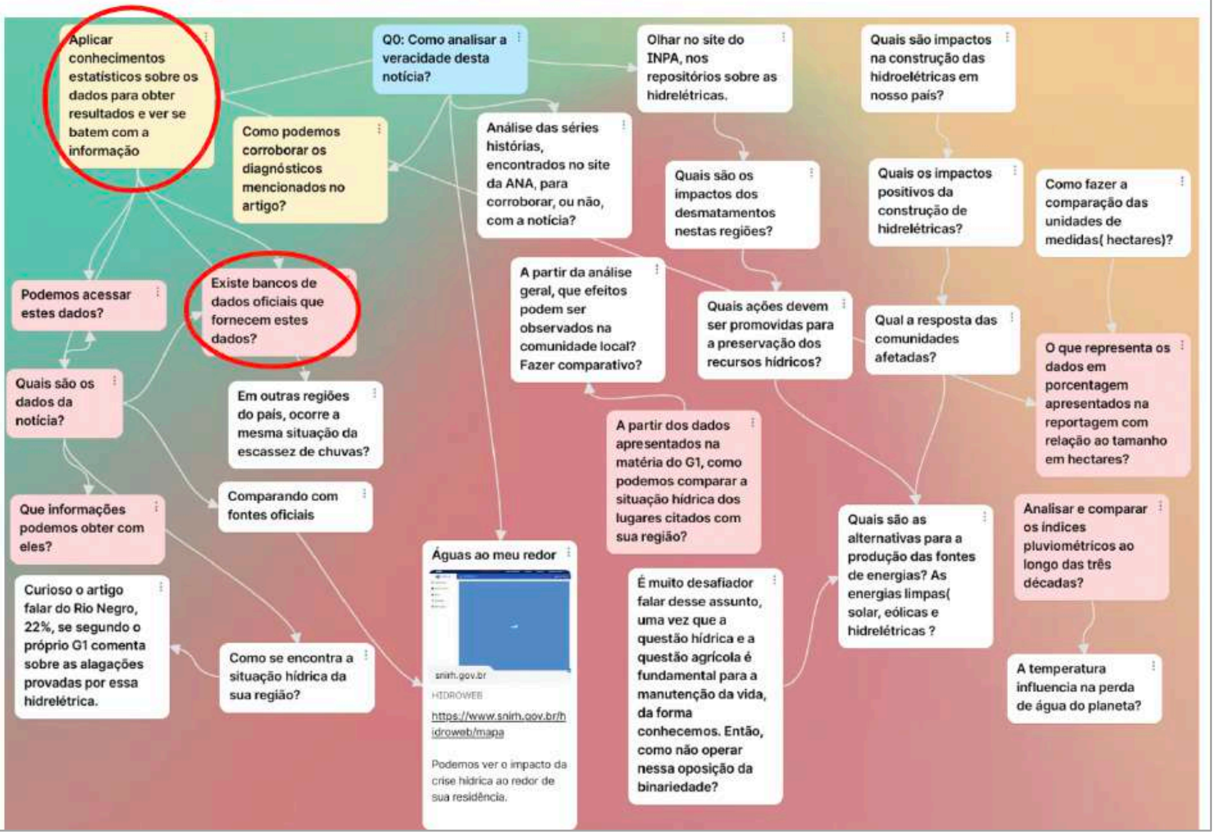
Estatística EF	Estatística EM	Probabilidade EF+EM
• *Recursos hídricos do Brasil* • Estudos e bancos de dados	• *Tamanho de camisetas* • Dados em web mas não organizados	• *Jogo da lebre e da tartaruga* • Simulações e análise de probabilidade empírica
JANIELLY	PAULO	CECILIA

Grupo Estatística nos anos finais do EF - Q0

Observem a notícia publicada no G1 em 23/08/21: https://g1.globo.com/natureza/noticia/2021/08/23/brasil-perdeu-15percent-dos-seus-recursos-hidricos-nos-ultimos-30-anos-uma-perda-quase-o-dobro-da-superficie-de-agua-de-todo-o-nordeste.ghtml

QUESTAO GERATRIZ PEP: Como analisar a veracidade desta notícia?

Mapa da Q_0 e questões derivadas



Existe bancos de dados que fornecem estes dados?

Maior seca registrada nos últimos 50 anos deixa Rio Paraguai em situação crítica e ameaça abastecimento de água em Cáceres — Foto: Ronivon Barros

O Brasil perdeu 15,7% de superfície de água nos últimos 30 anos, o equivalente a **3,1 milhões de hectares de superfície hídrica**, revela um levantamento inédito do MapBiomas, uma iniciativa que reúne cientistas e ambientalistas para mapear o país.

https://mapbiomas.org/estatisticas

SUPERFÍCIE DE ÁGUA (COLEÇÃO 1 - MAPBIOMAS ÁGUA) - dados de área (ha) da superfície de água por bioma, estado, município e bacias hidrográficas para cada ano entre 1985 e 2021.

	A	B	C	D
1	code	name	year	area_ha
2	1	Norte	1985	9550744,053
3	2	Nordeste	1985	1806214,425
4	3	Sudeste	1985	1455553,591
5	4	Sul	1985	2390959,458
6	5	Centro-Oeste	1985	2793413,433
7	1	Norte	1986	10091713,08
8	2	Nordeste	1986	1906156,546
9	3	Sudeste	1986	1421075,575
10	4	Sul	1986	2393714,835
11	5	Centro-Oeste	1986	2312419,493
12	1	Norte	1987	10406819,42
13	2	Nordeste	1987	1805553,472
14	3	Sudeste	1987	1445690,348
15	4	Sul	1987	2424560,489
16	5	Centro-Oeste	1987	2332338,183
17	1	Norte	1988	10335122,86
18	2	Nordeste	1988	1898275,393
19	3	Sudeste	1988	1456186,606

RESUMES E GRÁFICOS
↓
NOVAS QUESTÕES?
↓
MAIS DADOS?
↓
MAIS QUESTÕES?

Grupo Estatística no ensino médio - Q0

Observem a notícia publicada na Folha de São Paulo em 12/10/21: https://www1.folha.uol.com.br/mercado/2021/10/brasil-discute-ha-quase-uma-decada-o-tamanho-das-roupas-das-mulheres.shtml

76% das brasileiras têm corpo retangular; entenda os cinco tipos que a moda adota

Embates no setor impediram país de adotar uma tabela de referência; ABNT espera publicar nova norma até dezembro

Q_0: Como são determinados os tamanhos P, M, G e GG no Brasil? E no caso das camisetas?

Dados recolhidos e organizados

HOMENS CAMISETA BÁSICA

	TORAX	CINTURA	QUADRIL	TORAX	CINTURA	QUADRIL	TORAX	CINTURA	QUADRIL	TC
MARCA	HERING			NIKE			RENNER			
PP										82
P	94 - 98 cm	78 - 82 cm	94 - 98 cm	88 - 96 cm	73 - 81 cm	88 - 96 cm	88 - 96	68 - 76	84 - 92	86
M	98 - 104 cm	82 - 88 cm	98 - 104 cm	96-104cm	81-89cm	96-104cm	96 - 104	74 - 82	92 - 100	94
G	104- 110 cm	88 - 94 cm	104- 110 cm	104-112cm	89-97cm	104-112cm	104 - 112	82 - 90	100 - 108	102
XG	110 - 116 cm	94 - 100 cm	110 - 116 cm	112-124cm	97-109cm	112-120cm	112 - 120	90 - 98	108 - 116	110
XGG	116 - 123 cm	100 - 106 cm	116 - 123 cm	124-136cm	109-121cm	120 - 128	120 - 128	98 - 106	116 - 124	
Referência	https://www.hering.com.br/camiseta-masculina-manga-curta-comfort-em-algodao-basicos-do-brasil-4fbzwkmen/p?skuId=94087			https://atendimento.nike.com.br/hc/pt-br/articles/5342135441811#2			https://www.lojasrenner.com.br/p/camiseta-em-algodao-manga-curta-com-lettering-authenticity/-/A-551377744-br.lr?sku=551377752#			http

MULHERES CAMISETA PADRÃO

	TORAX	CINTURA	QUADRIL	TORAX	CINTURA	QUADRIL	TORAX	CINTURA	QUADRIL	TC
MARCA	HERING			NIKE			RENNER			
XP	81 - 85	63 - 67	92 - 96	76-83cm	60-67cm	84-91cm	78 - 82 cm	60 - 64 cm	88 - 92 cm	78 -
P	85 - 92	67 - 74	93 - 103	83-90cm	67-74cm	91-98cm	82 - 90 cm	64 - 72 cm	92 - 100 cm	84 -
M	92 - 99	74 - 81	103 - 110	90-97cm	74-81cm	98-105cm	90-98 cm	72 - 80 cm	100 - 108m	92 -
G	99 - 106	81 - 88	110 - 117	97-104cm	81-88cm	105-112cm	98 - 107 cm	80 - 89 cm	108 - 117	100
GG	106 - 113	88 - 95	117 - 124	104-114cm	88-98cm	112-120cm	107-117 cm	89 - 99 cm	117 - 127	109
GGG				114-124cm	98-108cm	120-128cm	117-127 cm	99 - 109 cm	127 - 137	
Referência	https://www.hering.com.br/blusa-cropped-feminina-canelada-basicos-do-brasil-4f8lmnen/p			https://atendimento.nike.com.br/hc/pt-br/articles/5342016402835			https://www.lojasrenner.com.br/p/blusa-alongado-em-algodao-com-lettering-yokohama-flower/-/A-633062906-br.lr?sku=633062922#			htt crop

"LIMPEZA"
↓
RESUMES E GRÁFICOS
↓
NOVAS QUESTÕES?
↓
MAIS DADOS O INFORMAÇÕES?
↓
MAIS QUESTÕES?

Grupo Probabilidade nos EF e EM - Q0

A TARTARUGA E A LEBRE

Jogamos um dado.
Se sair o número 6, a lebre vence.
Caso contrário, a tartaruga avança um espaço.
Continuamos o jogo até que haja um vencedor.

Qual é a situação mais favorável: a da lebre ou da tartaruga?
O número de caminhos influencia?

Resultados depois de 100 simulações

	CAMINHOS															1	A tartaruga ganha para ese número de caminhos													
Jogadas	1	2	3	4	5	6	7	8	9	1	11	12	13	14	15	1	2	3	4	5	6	7	8	9	1	11	12	13	14	15
1	3	5	2	3	5	3	6									1	1	1	1	1	1									
2	4	1	1	4	4	4	2	2	5	6						1	1	1	1	1	1	1	1	1						
3	4	1	3	5	3	2	4	1	2	6						1	1	1	1	1	1	1	1	1						
4	4	4	4	3	5	1	3	6								1	1	1	1	1	1	1								
5	3	6														1														
6	1	5	5	4	5	4	1	1	5	1	4	3	6			1	1	1	1	1	1	1	1	1	1	1	1			
7	4	4	2	1	5	3	6									1	1	1	1	1	1									
8	6																													
9	1	1	5	6												1	1	1												
10	4	1	5	4	1	6										1	1	1	1	1										
11	6																													
12	1	2	6													1	1													
13	6																													
14	4	6														1														
15	6																													
16	2	1	4	2	2	1	3	4	4	5	6					1	1	1	1	1	1	1	1	1	1					
17	3	6														1														
18	4	1	6													1	1													
19	5	6														1														
20	3	1	1	2	6											1	1	1	1											

Jogo com 4 caminhos
1 = ganha a tartaruga

PORCENTAGENS E GRÁFICOS → NOVAS QUESTÕES? → PROBABILIDADE?

Resultados depois de 50 simulações

	A	B	C	D	E	F	G	H	I
1									
2	Tabela com 50 jogadas		Tartaruga ganhando com 4 valores diferentes			lebre ganhando com 1 seis			
3									
4	equipe 1	4	10			equipe 2	9	7	
5	valor do dado	lebre	tartaruga			valor do dado	lebre	tartaruga	
6	3		x			1		x	
7	1		x			2		x	
8	2		x			6	x		
9	5		x	ganhou		3		x	
10	5		x			5		x	
11	3		x			2		x	
12	1		x			4		x	ganhou
13	2		x	ganhou		4		x	
14	4		x			3		x	
15	1		x			2		x	
16	2		x			4		x	ganhou
17	3		x	ganhou		1		x	
18	3		x			5		x	
19	1		x			6			
20	1		x			1		x	
21	3		x	ganhou		3		x	
22	6	x		ganhou		6			
23	5		x			4		x	
24	1		x			1		x	
25	4		x			4		x	
26	5		x	ganhou		6			
27	1		x			2		x	
28	3		x			5		x	
29	1		x			3		x	
30	6	x		ganhou		5		x	ganhou
31	4		x			3		x	
32	2		x			3		x	
33	4		x			1		x	

Trabalho nos três grupos

Vocabulario: "caminhos" o "paradas"

Simulación:

- Cómo organizar el Excel? Diferentes propuestas
- Si hacemos uma simulación com 4 caminhos, la misma simulación nos sirve para 3 caminhos y para 2 y para 1 → Excel final
- Hacemos 100 simulaciones com 15 caminhos: qué observamos?
- Calculamos el % de veces que gana la tartaruga

Probabilidades:

- Observamos la simulación y vemos: para ganhar com 1 caminhos, tiene 5/6
- Para ganhar com 2 caminhos, la liebre no gana em 1 (5/6) ni em 2 (5/6)^2
- Para ganhar com 3 camnihos, la lebre no gana (5/6)^3
- El cálculo de probabilidades lo comparamos com las frecuencias obtenidas (no iguales, pero sí parecidos)
    - Si pensamos em la lebre em lugar de la tartaruga?
    - Diferencia simulación-porcentaje: ¿de qué depende? ¿Cómo estudiarla?

Sessão 6

Ensino de probabilidade e estatística no questionamento do mundo

CURSO DE FORMAÇÃO SETEMBRO-DEZEMBRO 2022

Sessão 6 – 08 de outubro de 2022

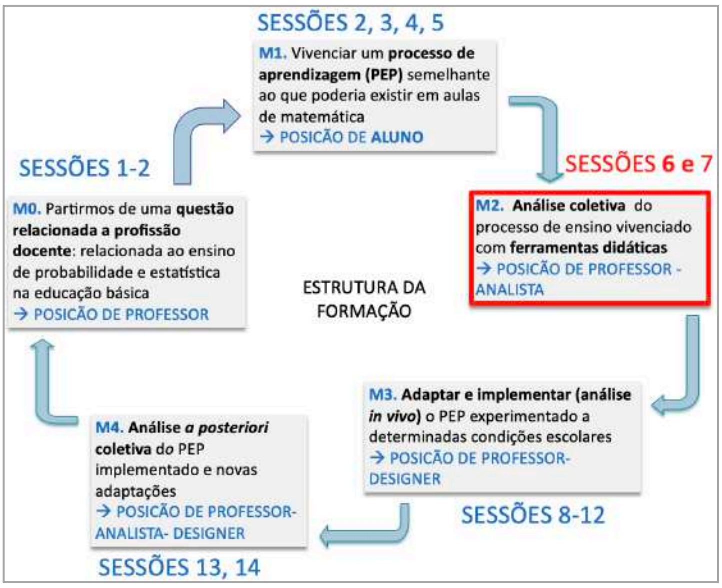
PUC-SP

CAPES

UFBA Universidade Federal da Bahia

UFMS FUNDAÇÃO UNIVERSIDADE FEDERAL DE MATO GROSSO DO SUL

UNIVERSITAT DE BARCELONA


SESSÕES 2, 3, 4, 5
M1. Vivenciar um processo de aprendizagem (PEP) semelhante ao que poderia existir em aulas de matemática
→ POSIÇÃO DE ALUNO
SESSÕES 6 e 7
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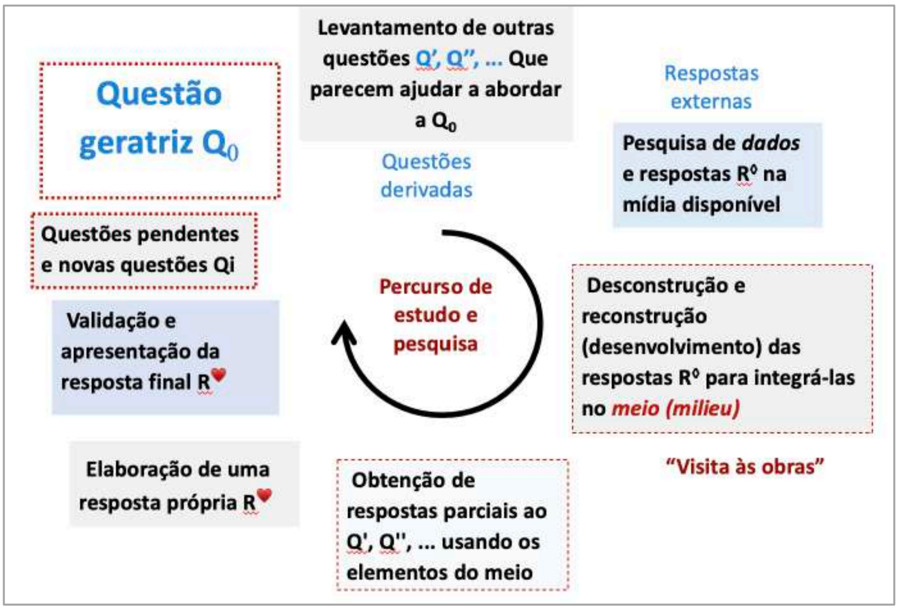
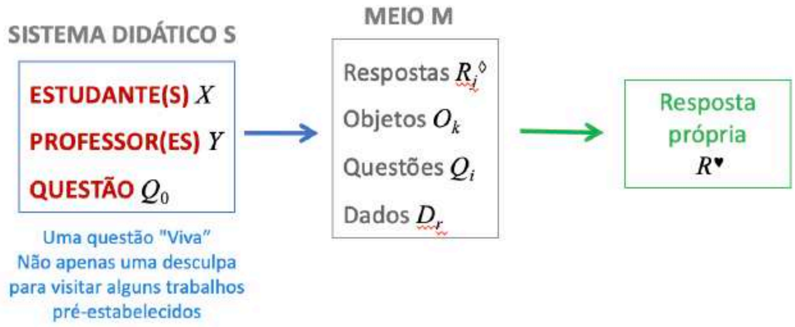
→ POSIÇÃO DE PROFESSOR-ANALISTA- DESIGNER
SESSÕES 13, 14


Uma metáfora sobre o que decidimos ensinar
Coleta e tratamento de dados em um PEP
Coleta e tratamento de dados na escola


Questão geratriz Q_0
Levantamento de outras questões Q', Q'', ... Que parecem ajudar a abordar a Q_0
Questões derivadas
Respostas externas
Pesquisa de dados e respostas $R^◊$ na mídia disponível
Questões pendentes e novas questões Qi
Validação e apresentação da resposta final R♥
Percurso de estudo e pesquisa
Desconstrução e reconstrução (desenvolvimento) das respostas $R^◊$ para integrá-las no meio (milieu)
"Visita às obras"
Elaboração de uma resposta própria R♥
Obtenção de respostas parciais ao Q', Q'', ... usando os elementos do meio

Esquema herbartiano



$$[S(X;Y;Q_0) \rightarrow \{R_j^\Diamond, O_k, Q_i, D_r\}] \hookrightarrow R^\heartsuit$$

Esquema herbartiano

- Uma **questão** Q_0 (a questão 'geratriz') e um grupo de **estudantes** X e **supervisor(es)** Y

  → sistema didático $S(X;\ Y;\ Q_0)$

- O objetivo é elaborar sua própria **resposta** (coletiva) $R^\heartsuit$ para Q_0

$$[S(X;\ Y;\ Q_0)] \hookrightarrow R^\heartsuit$$

- Para elaborar $R^\heartsuit$, o Sistema didático cria um *meio (milieu)* M:

$$[S(X;\ Y;\ Q_0) \curvearrowright \mathrm{M}] \hookrightarrow R^\heartsuit$$

- O **meio (milieu)** M **evolui** durante o processo de investigação.

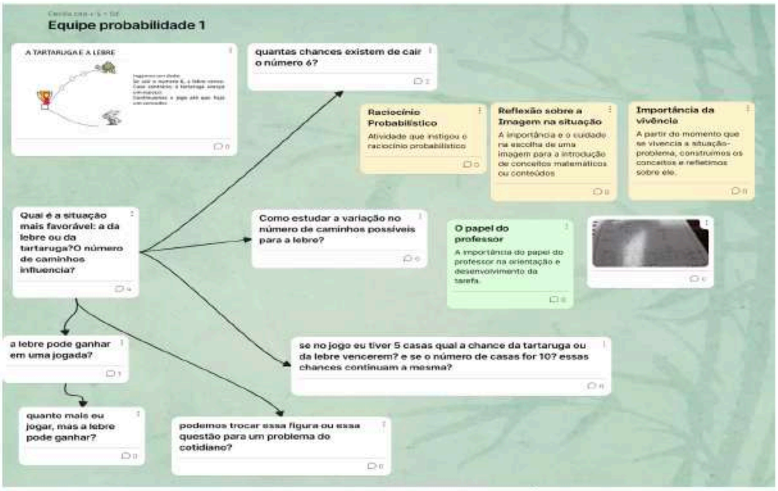
$$[S(X;Y;Q_0) \rightarrow \{R_j^\Diamond, O_k, Q_i, D_r\}] \hookrightarrow R^\heartsuit$$

EXEMPLOS DE S(X, Y, Q)

- Professor com alunos resolvendo um problema
- Estudante de doutorado e o problema de tesis
- Professores com educadores sobre o ensino de estatística e probabilidade no ensino médio
- Filhos com seus pais para aprender como prender seus sapatos
- Dois amigos, um ensinando português ao outro

$$[S(X;Y;Q_0) \rightarrow \{R_j^\Diamond, O_k, Q_i, D_r\}] \hookrightarrow R^\heartsuit$$

EXEMPLOS DE Q_0 e Q', Q'', etc...



$[S(X;Y;Q_0) \rightarrow \{R_j^{\lozenge}, O_k, \boldsymbol{Q_i}, D_r\}] \rightarrow R^{\heartsuit}$

EXEMPLOS DE DADOS

Nº	TORAX								
	TAMANHO	P		M		G		XG	
	INTERVALO	MIN	MAX	MIN	MAX	MIN	MAX	MIN	MAX
5	HERING	94	98	98	104	104	110	110	116
	NIKE	88	96	96	104	104	112	112	114
	RENNER	88	96	96	104	104	112	112	120
	C&A	86	94	94	102	102	110	110	118
	HM PERU	88	92	96	100	104	108	112	116
	TOTAL	444	476	480	514	518	552	556	584
	MEDIA	88,8	95,2	96	102,8	103,6	110,4	111,2	116,8

FEMININO	TORAX										
	TAMANHO	XP		P		M		G		XG	
	INTERVALO	MIN	MAX	MIN	MAX	MIN	MAX	MIN	MAX	MIN	MAX
5	HERING	81	85	85	92	92	99	99	106	106	113
	NIKE	76	83	83	90	90	97	97	104	104	114
	RENNER	78	82	82	90	90	98	98	107	107	117
	C&A	78	84	84	92	92	100	100	109	109	114
	HM PERU	76	80	84	88	92	96	100	104	110	116
	TOTAL	389	414	418	452	456	490	494	530	536	574
	MEDIA	77,8	82,8	83,6	90,4	91,2	98	98,8	106	107,2	114,8

$[S(X;Y;Q_0) \rightarrow \{R_j^{\lozenge}, O_k, Q_i, \boldsymbol{D_r}\}] \rightarrow R^{\heartsuit}$

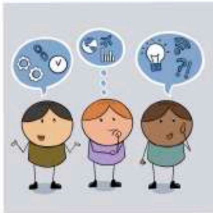
EXEMPLOS DE OBJETOS

- Todos os conhecimentos que já temos
- Excel, calculadoras, celular, computador
- Bases de dados
- Artigos, livros, expertos ao nosso redor

$[S(X;Y;Q_0) \rightarrow \{R_j^{\lozenge}, \boldsymbol{O_k}, Q_i, D_r\}] \rightarrow R^{\heartsuit}$

EXEMPLOS Respostas externas $R^\lozenge$

- Obras que acreditamos ser úteis e vamos estudar
  - Cálculo de probabilidades com eventos independentes ("teoria Kolmogorov")
  - Sistema brasileiro de medida de tamanhos de roupas
  - Recursos hídricos no Brasil
  - Medidas de dispersão de dados
  - Gráficos estatísticos
  - ...



$$[S(X;Y;Q_0) \rightarrow \{R_j^\lozenge, O_k, Q_i, D_r\}] \rightarrow R^\heartsuit$$

Esquema herbartiano

- Elementos do processo de estudo (investigação)

$$[S(X;Y;Q_0) \rightarrow \{R_j^\lozenge, O_k, Q_i, D_r\}] \rightarrow R^\heartsuit$$

Dialéticas

- Dinâmicas do processo de estudo (investigação)
  - Questões e respostas
  - Mídia e meio
  - Indivíduo e coletivo

Dialética questões e respostas

- Para abordar uma questão, geram-se questões derivadas
- Buscam-se respostas já disponíveis (em nosso meio ou em mídias acessíveis)
- Estas respostas são estudadas, validadas e desenvolvidas
- Durante o estudo surgem novas questões
- É um processo infinito!
- Obs.: no contexto escolar, são valorizadas muito mais as respostas já estabelecidas e pouco as perguntas. Precisamos tentar atuar em direção contrária, valorizando as questões que motivaram a resposta.

$$[S(X;Y;Q_0) \rightarrow \{R_j^\lozenge, O_k, Q_i, D_r\}] \rightarrow R^\heartsuit$$

Dialética mídia e meio

- Durante a pesquisa, utilizamos todos os conhecimentos e instrumentos que temos disponíveis em nosso meio. São coisas seguras, são coisas seguras, sobre as quais não temos (ou temos pouca) dúvida.
- Para avançar, necessitamos novas informações, respostas, conhecimentos, instrumentos, etc., que buscamos em distintas fontes de informação: as mídias.
- Estas informações, conhecimentos, instrumentos, etc. não são nossos. Temos que estuda-los, desconstruir, reconstruir, validar para integrá-los em nosso meio. Nos “apropriamos” deles para poder explorá-los na elaboração de respostas.

$[S(X;Y;Q_0) \rightarrow \{R_j^\lozenge, O_k, Q_l, D_r\}] \hookrightarrow R^\heartsuit$

Dialética mídia e meio

- Deste modo, o que encontramos nas mídias são incorporados (parcialmente) em nosso meio e este vai evoluindo.
- Também devemos ser capazes de obter novas informações de nosso meio, utilizá-las como mídia.
- Para analisar esta dialética, olhamos de onde saem as informações, dados e respostas externas e como seu acesso é gerenciado (mídia). Também perguntamos como são validadas e transformadas; e com que materiais elaboram-se as respostas próprias finais ou intermediárias (meio).

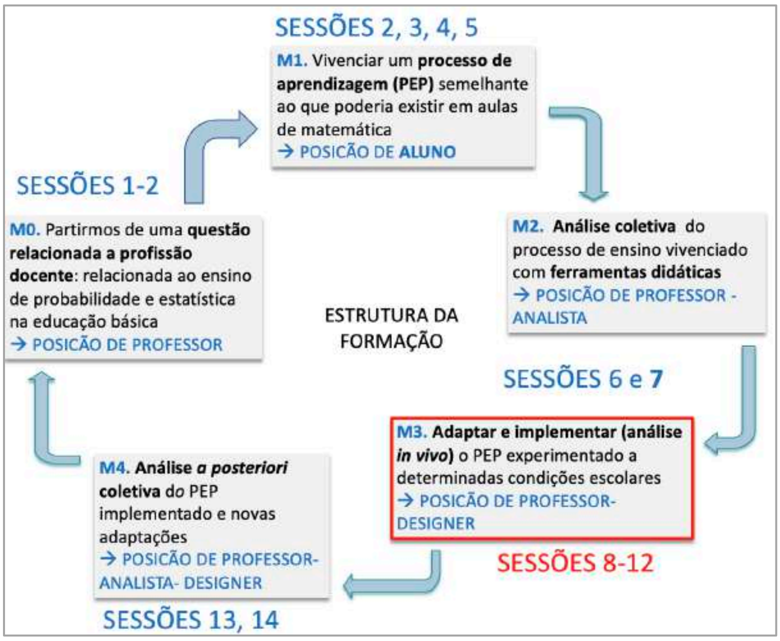
$[S(X;Y;Q_0) \rightarrow \{R_j^\lozenge, O_k, Q_l, D_r\}] \hookrightarrow R^\heartsuit$

Dialética indivíduo e coletivo

- A pesquisa é um processo coletivo realizado por X sob a direção de Y.
- Neste processo, pequenos grupos X_i são gerados e o trabalho individual x_i também é realizado. Os xi e os yi devem se organizar para trabalhar juntos
- Há muitos formatos de trabalho, desde o puramente individual (x_i coletando dados em seu computador) até o coletivo total (X e Y discutindo os resultados juntos).
- Também podemos falar de um meio coletivo (da sala/classe) e de meios individuais (de cada aluno): há aprendizado conjunto e aprendizado coletivo.
- Para analisar esta dialética, podemos perguntar sobre os papeis (quem faz o quê) nas descrições da dialética anterior.

Sessão 9


PUC-SP
CAPES
Ensino de probabilidade e estatística no questionamento do mundo
UFBA
Universidade Federal da Bahia
UFMS
FUNDAÇÃO UNIVERSIDADE FEDERAL DE MATO GROSSO DO SUL
CURSO DE FORMAÇÃO SETEMBRO-DEZEMBRO 2022
Sessão 9 -5 de novembro de 2022
UNIVERSITAT DE BARCELONA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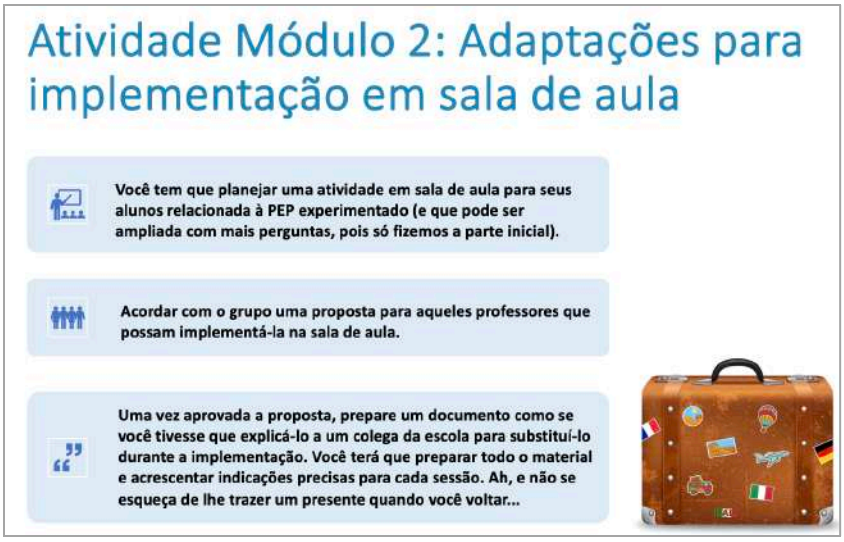
SESSÕES 8-12
SESSÕES 13, 14


Atividade Módulo 2: Adaptações para implementação em sala de aula
Você tem que planejar uma atividade em sala de aula para seus alunos relacionada à PEP experimentado (e que pode ser ampliada com mais perguntas, pois só fizemos a parte inicial).
Acordar com o grupo uma proposta para aqueles professores que possam implementá-la na sala de aula.
Uma vez aprovada a proposta, prepare um documento como se você tivesse que explicá-lo a um colega da escola para substituí-lo durante a implementação. Você terá que preparar todo o material e acrescentar indicações precisas para cada sessão. Ah, e não se esqueça de lhe trazer um presente quando você voltar...

Ferramentas de análise

- Contrato didático (Brousseau, 1980)
- Esquema herbartiano e dialéticas

Contrato didático (Brousseau, 1980)

WIKIPÉDIA
L'encyclopédie libre

Rechercher sur Wikipédia

Contrat didactique

O conceito de contrato didático foi introduzido por Guy Brousseau, um didata francês de matemática. Ele define o contrato didático como "o conjunto de comportamentos do professor que são esperados do aluno, e o conjunto de comportamentos do aluno que são esperados do professor". Brousseau (1980: 127).

Este contrato didático descreve as regras implícitas ou explícitas que regem a partilha de responsabilidades, em relação ao conhecimento mobilizado ou estruturado, entre o professor e o aluno. É, portanto, uma representação das expectativas de ambas as partes.

Contrato didático (Brousseau, 1980)

Conjunto de regras implícitas ou explícitas que regem as responsabilidades daqueles envolvidos nos processos de ensino e de aprendizagem em relação ao conteúdo em jogo

Os professores têm que fazer algo para que os alunos aprendam

Os alunos têm que fazer algo para aprender

As coisas que os alunos devem fazer não podem ser muito fáceis ou muito difíceis.

O contrato está sendo constantemente negociado

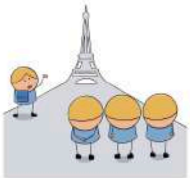
O aprendizado ocorre frequentemente quando há uma quebra de contrato

"Nunca se vanglorie de ser autodidata, repito, porque há pouco que se possa fazer sem a ajuda de outros. Não esqueça, porém, que este pouco é importante e que, além disso, ninguém pode ensiná-lo".

Antonio Machado (1936). *Juan de Mairena. Sentencias, donaires, apuntes y recuerdos de un profesor apócrifo*. Espasa-Calpe.

Mudanças de contrato na mudança de paradigma

Paradigma de la visita de las obras



Paradigma del cuestionamiento del mundo

Responsabilidades de professores e alunos? O que os professores podem administrar e o que os alunos podem administrar?

- Busca de definições ou novas noções
- Leitura de um texto
- Coleta de dados

Mudanças de contrato na mudança de paradigma

SISTEMA DIDÁTICO S

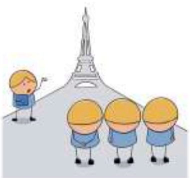
Estudantes X
Professores Y
Obra o questão $\mathcal{O}$ / Q_0

MEIO M

Respostas $R_j^\lozenge$
Objetos O_k
Questões Q_i
Dados D_r

Resposta própria
$R^\heartsuit$

Paradigma de la visita de las obras



$$[S(X;\ Y;\ \mathcal{O}) \rightarrow \{R_j^\lozenge, O_k, Q_i, D_r\}] \hookrightarrow R_Y^\heartsuit$$

Responsabilidades de professores e alunos?

Mudança de paradigma

Visita às obras	Questionamento do mundo	Novas condições
O progresso do tempo didático é marcado pelos trabalhos ou novos conteúdos que o professor introduz.	As questões que marcam o avanço do tempo didático	Como identificar as questões que estão sendo tratadas, nomeá-las, comentá-las no final de cada aula?
O professor geralmente apresenta as novas informações ou conteúdos. O professor é também o único que valida as respostas dos alunos.	É a comunidade de estudo que busca informações, estuda o novo conteúdo e as valida.	Como transferir progressivamente a responsabilidade de buscar informações e dados, validar suas respostas, etc., para os alunos.

Mudança de paradigma

Visita às obras	Questionamento do mundo	Novas condições
As perguntas dos alunos servem apenas para seu aprendizado individual, não para o avanço do estudo em classe.	É a classe (X, Y) que discute quais questões são importantes e quais não são. Os professores Y são líderes, mas não impostores.	Como administrar o compartilhamento, para que os estudantes façam perguntas, forneçam respostas e estas possam ser discutidas coletivamente.
O professor planeja as atividades com antecedência, independentemente do que os alunos façam.	O planejamento é compartilhado entre X e Y, embora Y tenha maior responsabilidade.	Como administrar o compartilhamento, para que os estudantes façam perguntas, forneçam respostas e estas possam ser discutidas coletivamente.

Ferramentas de análise

- Contrato didático (Brousseau, 1980)
- Esquema herbartiano e dialéticas
  - Cronogênese: como avançar, que questões
  - Mesogênese: como o meio evolui
  - Topogênese: partilha de responsabilidades → contrato didático

Vamos preparar as implementações

- Que dificuldades prevemos?
- O que procurar?
- Que informações coletar?

Sessão 11

Ensino de probabilidade e estatística no questionamento do mundo

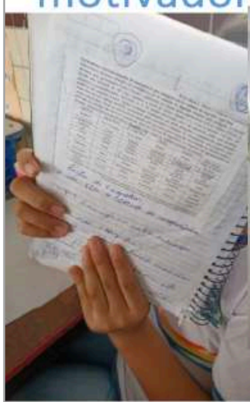
PUC-SP
CAPES
UFBA Universidade Federal da Bahia
UFMS FUNDAÇÃO UNIVERSIDADE FEDERAL DE MATO GROSSO DO SUL
UNIVERSITAT DE BARCELONA

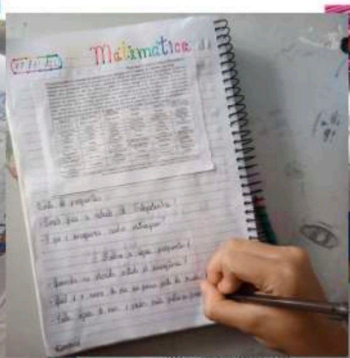
CURSO DE FORMAÇÃO SETEMBRO-DEZEMBRO 2022
Sessão 11 – 19 de novembro de 2022

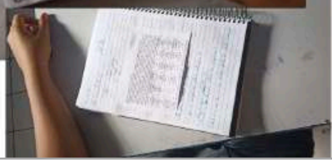
Organização da sala: disposição dos alunos

Aula 1

Leitura coletiva do texto motivador

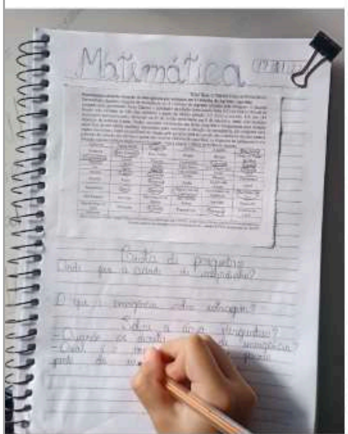




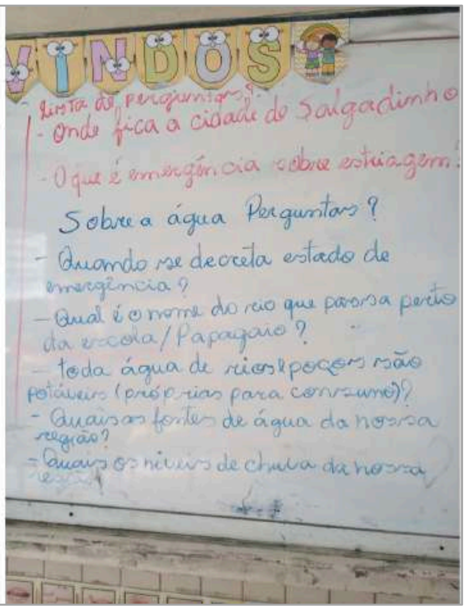


Aula 1

Elaboração de questões



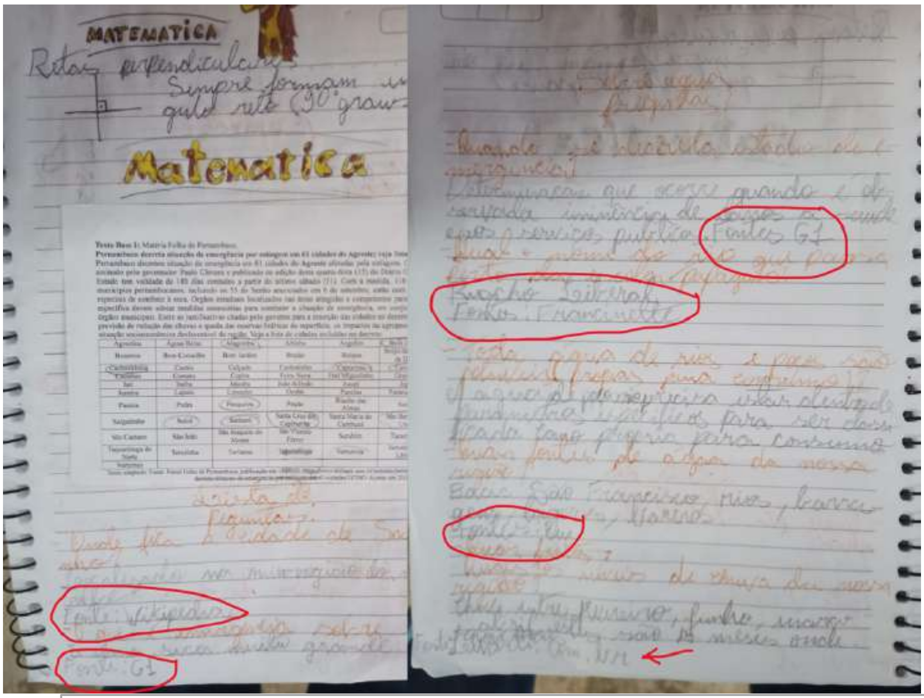
Aula 1

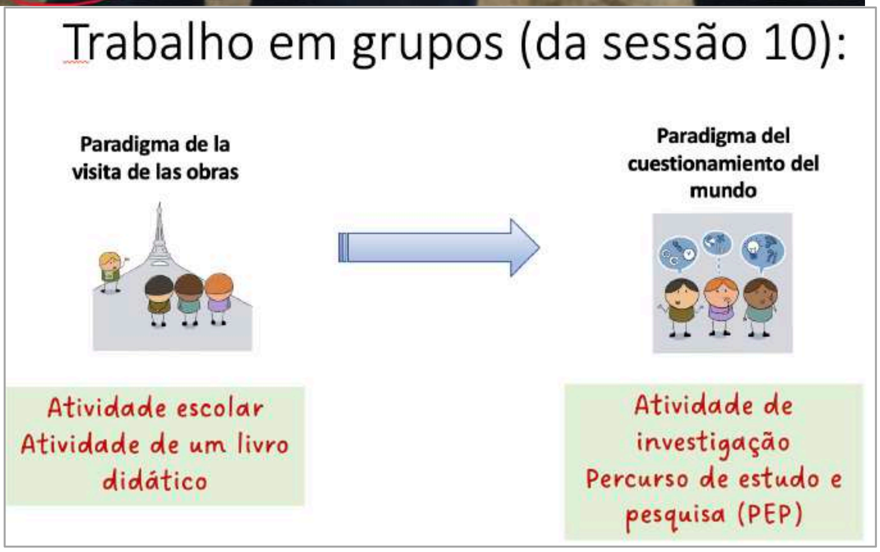



Elaboração de questões
Aula 1


Aula 2 –
laboratório
de
informática


SEJAM
BEM
VINDOS





"A atividade sobre as fichas e probabilidades é até um pouco 'boba'. Então como pegar esta situação e transformar em uma atividade dentro do paradigma do questionamento do mundo? Como transformar isso em um problema de investigação?"

"Qual é o objetivo desta atividade escolar?"
"Esta atividade pode ajudar o aluno a pensar em que?"

"Este exercício está incompleto? Por que não aparecem as pontuações relacionadas a cada cor das fichas?"
"Qual é o contexto desta questão? O que podemos questionar com este contexto?"

"E se pensarmos em pontuações para as fichas? Diferentes pontuações para as diferentes cores?"
"Quantos alunos jogariam?"
"Parece que o paradigma da visita às obras explica melhor o/um conteúdo"

"Parece que no Paradigma do questionamento do mundo a gente não tem muito controle"
"Que questão poderíamos pensar que não parte diretamente do conteúdo e que promova um processo investigativo?"

43 Valéria e Alexandre inventaram uma brincadeira em que retirariam ao acaso uma ficha colorida de uma sacola e, de acordo com a cor da ficha, cada um receberia uma pontuação. Quem fizesse mais pontos ganharia um prêmio. Eles colocaram na sacola 4 fichas amarelas, 3 fichas brancas e 2 fichas pretas.

Retirando aleatoriamente uma ficha da sacola, qual é a probabilidade de ela ser:

a) amarela? $\frac{4}{9}$ ou aproximadamente 44,4%.
b) branca? $\frac{1}{3}$ ou aproximadamente 33,3%.
c) preta? $\frac{2}{9}$ ou aproximadamente 22,2%.

"O que realmente é uma questão investigativa? É no sentido de o aluno avaliar a situação? De mudança da situação?... Ou seria mudar totalmente a questão? E como levar isso para o questionamento do mundo?"
"Essas fichas não têm significado nenhum para os alunos, mas se mudarmos para algum jogo real que faz parte do cotidiano dos alunos ..."

"O que podemos questionar quando trabalhamos com um jogo em sala de aula?"

"Mas daí qual é o propósito? Qual é a habilidade que eu quero desenvolver no meu aluno?"
"Mas aí você está pensando no paradigma da visita às obras.."

"O que a gente acha que os alunos vão questionar sobre um jogo?"
"Ele vai perguntar quem tem mais chance de ganhar?"
"Alguém se perguntou ou a gente sabe se essas fichas são idênticas? Ele não pode reconhecer pelo tato na hora de sortear?"

43 Valéria e Alexandre inventaram uma brincadeira em que retirariam ao acaso uma ficha colorida de uma sacola e, de acordo com a cor da ficha, cada um receberia uma pontuação. Quem fizesse mais pontos ganharia um prêmio. Eles colocaram na sacola 4 fichas amarelas, 3 fichas brancas e 2 fichas pretas.

Paulo Manzi/Arquivo da editora

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b) branca? $\frac{1}{3}$ ou aproximadamente 33,3%.

c) preta? $\frac{2}{9}$ ou aproximadamente 22,2%.

"Pensar em uma pergunta inicial? Nossa, que difícil!"

"Como a gente pode pensar uma questão inicial?"

"Como seria a organização da sala? Onde e como começar o PEP?"

"Como começar a questão inicial? Tenho computadores e internet a disposição para que os estudantes façam buscas?"

"Como introduzir a Q0? Com o próprio exercício do livro? Com alguma motivação para o tema?"

"E se a gente pudesse começar com um vídeo motivador?"

"Como organizar a sala? Os estudantes trabalhariam em grupos? De quantos membros?"

1. O Brasil possui cerca de 13,7% do total de água doce do mundo, sendo considerado um território rico em termos hídricos. No entanto, o país vive sérios problemas, relacionados tanto à degradação da qualidade das águas, principalmente nas proximidades das áreas urbanas, quanto à falta de controle do excesso e da insuficiência de água, que atingem várias localidades brasileiras. Não são somente as enchentes que afetam as cidades brasileiras: a escassez hídrica também impõe sérias restrições e elevados custos ao desenvolvimento econômico e social de grandes cidades do Brasil.

Observando o gráfico a seguir, responda no caderno:

Informações obtidas em: MINISTÉRIO DO MEIO AMBIENTE. Disponível em: <http://www.mma.gov.br/estruturas/sedr_proecotur/_publicacao/140_publicacao09062009025910.pdf>. Acesso em: 1º jul. 2018.

a) Que tipo de gráfico é este? Gráfico de colunas triplas ou gráfico de múltiplas colunas.

b) Indique a região brasileira:
- com a maior superfície; Região Norte: 45,30%.
- com mais recursos hídricos; Região Norte: 68,50%.
- com a segunda menor concentração de população. Região Norte: 6,98%.

c) Que região tem a menor taxa percentual de recursos hídricos do nosso país? Região Nordeste: 3,30%.

d) Em qual região há maior concentração de população? Região Sudeste: 42,65%.

e) Pode-se dizer que quanto maior a superfície da região, maior é o número de habitantes? Justifique sua resposta. Não. O Sudeste possui a maior população, porém possui a segunda menor superfície do Brasil.

f) Quantos por cento da água doce do mundo estão na região Sudeste brasileira? Explique como você pensou para responder. Cerca de 0,82%. Resposta pessoal.

g) Pode-se dizer que a região que dispõe de mais recursos hídricos é a que possui a maior população? Não.

1. O Brasil possui cerca de 13,7% do total de água doce do mundo, sendo considerado um território rico em termos hídricos. No entanto, o país vive sérios problemas, relacionados tanto à degradação da qualidade das águas, principalmente nas proximidades das áreas urbanas, quanto à falta de controle do excesso e da insuficiência de água, que atingem várias localidades brasileiras. Não são somente as enchentes que afetam as cidades brasileiras: a escassez hídrica também impõe sérias restrições e elevados custos ao desenvolvimento econômico e social de grandes cidades do Brasil.

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g) Pode-se dizer que a região que dispõe de mais recursos hídricos é a que possui a maior população? Não.

"Quando a gente pensa no tempo didático, ele precisa ser grande ou não?"

"Ainda em relação ao tempo didático, se a gente quisesse que ele fosse grande, é possível e faz sentido que os alunos visitem uma estação de armazenamento de água da cidade onde vivem, por exemplo?"

"Essa visita na estação de armazenamento de água poderia ser o início de um PEP? Ou essa visita só seria feita se surgissem dúvidas sobre este assunto?"

"Poderíamos iniciar um PEP com uma visita a uma hidrelétrica, por exemplo?"

"Nas sessões anteriores a gente tentou fazer planejamentos nos moldes do paradigma do questionamento do mundo. Na atividade de hoje é esta ideia também?"

"Os alunos (em sala de aula) têm que saber a teoria? Ou eles vão pesquisar sobre a teoria?"

"As vezes eu acho que precisa dar um direcionamento para as buscas que os alunos vão fazer, porque fica muito confuso essa busca. Deixar de buscar por conta própria ..."

"Os alunos não se questionam sobre as fontes [das buscas que fazem], por isso acredito que precisamos direcionar onde buscar informações. Por isso estou perguntando, a gente pode dar um direcionamento ou a gente os deixa livres?"

"Eu acho que quando a gente abre demais, a coisa fica solta"

"A gente busca alguma notícia para complementar a questão inicial?"

31 ‣ Em uma turma, as notas obtidas pelos alunos foram agrupadas da seguinte maneira:

- 0 ⊢— 2 (1 aluno);
- 2 ⊢— 4 (6 alunos);
- 4 ⊢— 6 (9 alunos);
- 6 ⊢— 8 (8 alunos);
- 8 ⊢— 10 (6 alunos).

Com esses dados, faça o que se pede no caderno.

a) Construa o histograma e marque o polígono de frequência. (MP)

b) Calcule a média, a moda, a mediana e o desvio-padrão dos dados. *MA = 5,8; Mo = 5,0; Me = 5,0; DP = 2,28.*

"Como que a gente faz uma pergunta para o aluno sem apresentar o "passo a passo" do que deve ser feito? Por exemplo, em relação a construção de um histograma? A gente não pode apresentar o "passo a passo" para eles? Como eles vão descobrir sozinhos?"

"A gente pode sugerir as mídias para os alunos olharem?"

"E sobre o tempo para desenvolver esta atividade: seria um mini curso? Seria na sala de aula? Pensando na minha realidade, na minha escola a estatística é trabalhado no segundo ano do EM em uma aula só. Então para mim isso deveria ser praticamente uma unidade letiva".

"Até onde eu vejo, o PEP não pode ser trabalho em poucas aulas porque pode surgir tanto coisa a partir de uma Q0. A gente não sabe o que vai acontecer..."

"É muito difícil implementar um PEP com todas estas restrições. Sem contar que não temos acesso à internet o tempo todo."

31 ‣ Em uma turma, as notas obtidas pelos alunos foram agrupadas da seguinte maneira:

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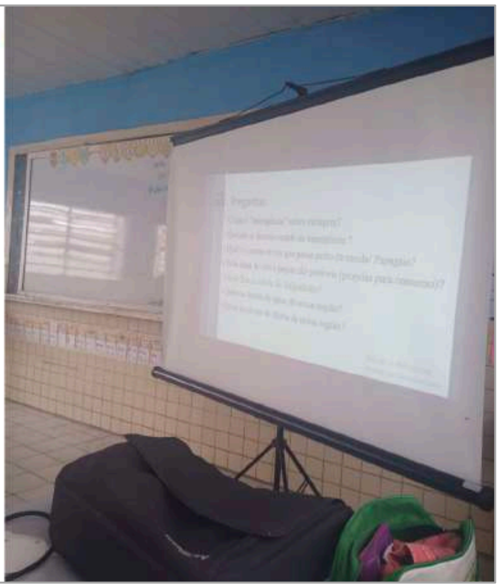
Sessão 12

Continuação do PEP no sexto ano – Aulas 3 e 4 Profa. Paola

- Resumo e tomada sobre as aulas 1 e 2

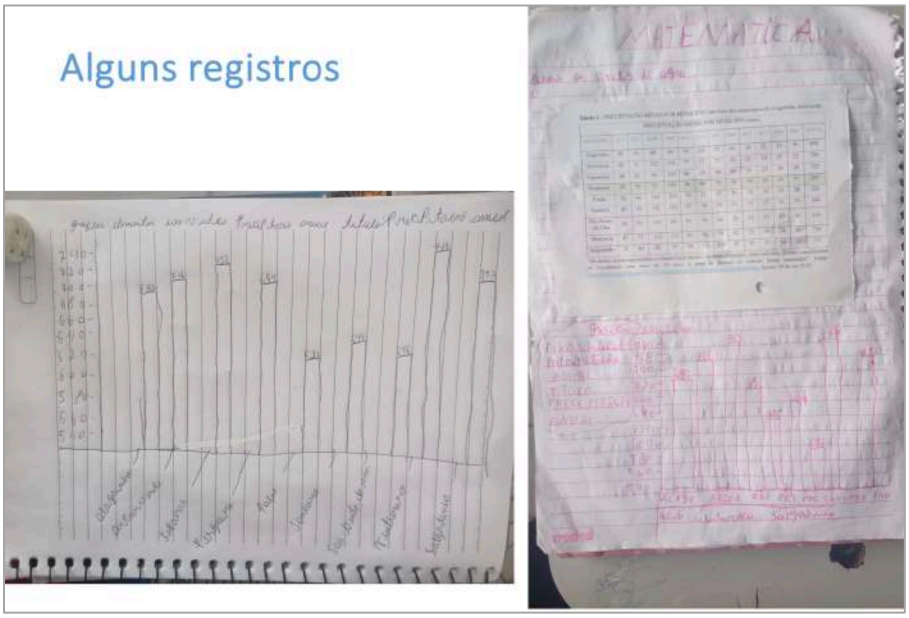
- Discussão sobre as precipitações médias → tabela com dados

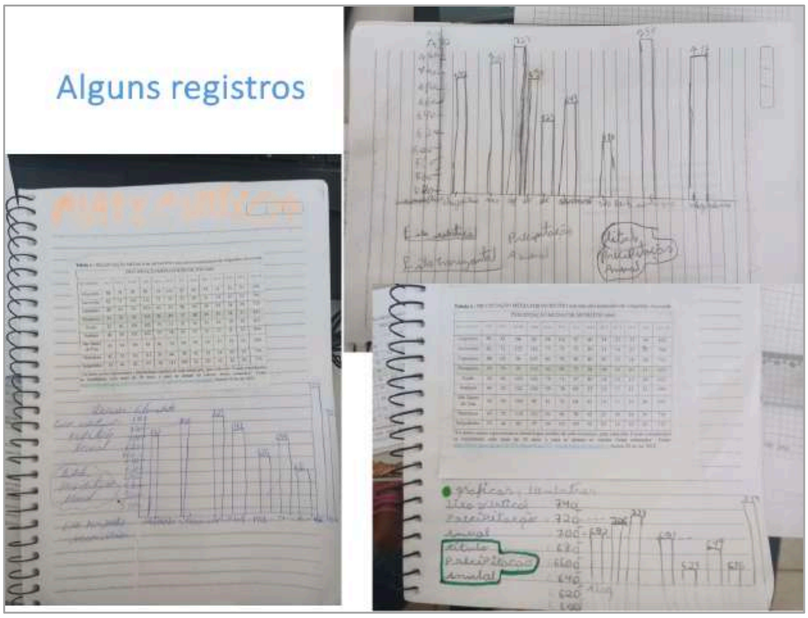


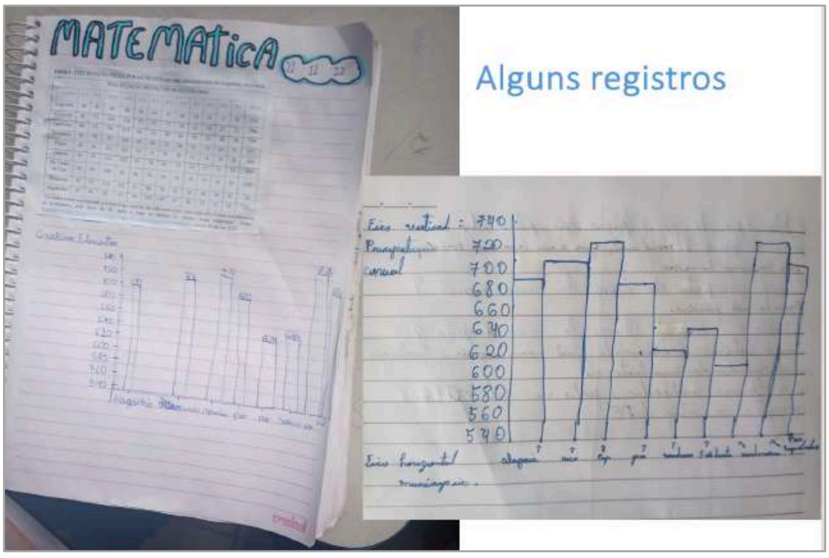
Continuação do PEP no sexto ano – Profa. Paola Aulas 3 e 4

- Discussão sobre as precipitações médias → tabela com dados
- Primeira construção coletiva: precipitação média anual dos 9 municípios (com a inclusão de Salgadinho)

Alguns registros

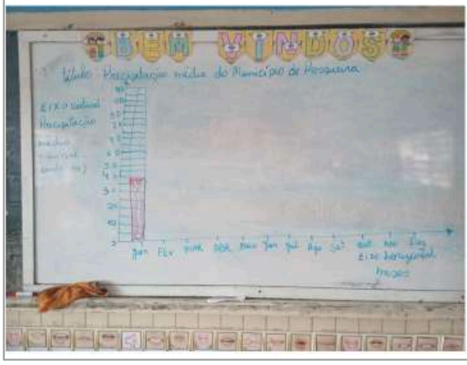

Alguns registros


Alguns registros


Alguns registros

Aula 5

- Segunda construção coletiva: precipitação média mensal do município de Pesqueira (município ao qual os estudantes pertencem)
- Nesta segunda construção coletiva foram distribuídas folhas quadriculadas



→ Tivemos apenas um registro pois os demais estudantes não terminaram as construções
→ ficou como tarefa a finalização da construção deste gráfico

Trabalho em grupos (sessões 10 e 11)
Apresentação das propostas

Paradigma de la visita de las obras

Atividade escolar
Atividade de um livro didático

Paradigma del cuestionamiento del mundo

Atividade de investigação
Percurso de estudo e pesquisa (PEP)

Exercício do livro didático:

31 Em uma turma, as notas obtidas pelos alunos foram agrupadas da seguinte maneira:

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b) Calcule a média, a moda, a mediana e o desvio-padrão dos dados. *MA* = 5,8; *Mo* = 5,0; *Me* = 5,0; *DP* = 2,28.

Notícia escolhida


ESTADO DE MINAS Educação

Maioria das redações nota mil no Enem 2019 são de Minas e escritas por mulheres

Em todo país, 53 participantes entre 16 e 28 anos conquistaram nota máxima ao dissertar sobre a 'Democratização do acesso ao cinema no Brasil'

Déborah Lima*

Informações interessantes nesse texto.

- Nota máxima e nota média em redação
- Nota média por área do conhecimento

Conteúdos matemáticos que podem ser explorados através desta notícia:

- Porcentagem
- Intervalos
- Média aritmética
- Gráficos


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Déborah Lima*

Q_0: COMO É CALCULADA A NOTA DO DESEMPENHO DO ENEM?

PLANEJAMENTO DA ATIVIDADE

1. Dividir a turma em grupos (máximo de 5)
2. Distribuir o texto e fazer a leitura
3. Fazer uma breve discussão sobre o texto, estimulando os grupos a expressar opiniões ou responder perguntas previamente estabelecidas.
4. Lançar a Q_0:
   4.1 - Solicitar que os grupos tracem estratégias para responder à pergunta Q_0;
   4.2 - Pedir aos grupos que formulem novas perguntas a partir da Q_0
5. Socializar os resultados obtidos em 4.1 e 4.2.

Obs. Os próximos passos serão determinados a partir do resultado obtido no item 5. Espera-se que os grupos pensem em pesquisar como responder a Q_0, assim como as questões derivadas, incluindo o que é **nota média, TRI**, dentre outros.

RESPONSABILIDADE DOS PROFESSORES	RESPONSABILIDADE DOS ALUNOS
• Fornecer meios que facilite chegar a resposta de Q_0. • Pensar no tempo cronológico e no tempo didático • Procurar desenvolver a autonomia dos alunos	• Traçar estratégias para solucionar a Q0 • Entender o que significa o processo de investigação • Fazer a pesquisa e trazer resultados

Exercício do livro didático:

43 ▸ Valéria e Alexandre inventaram uma brincadeira em que retirariam ao acaso uma ficha colorida de uma sacola e, de acordo com a cor da ficha, cada um receberia uma pontuação. Quem fizesse mais pontos ganharia um prêmio. Eles colocaram na sacola 4 fichas amarelas, 3 fichas brancas e 2 fichas pretas.

Paulo Manzi/Arquivo da editora

Retirando aleatoriamente uma ficha da sacola, qual é a probabilidade de ela ser:

a) amarela? $\frac{4}{9}$ ou aproximadamente 44,4%.

b) branca? $\frac{1}{3}$ ou aproximadamente 33,3%.

c) preta? $\frac{2}{9}$ ou aproximadamente 22,2%.

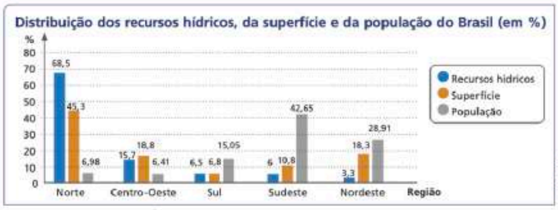
Questão proposta:

A lanchonete da escola trabalha com salgados e sucos e distribui aos estudantes um cartão fidelidade que funciona assim: quando um estudante realiza uma compra na lanchonete, ele tem direito a um registro no seu cartão. Quando ele completa dez compras ele passa a ter direito a um salgado. Se ele comprar um lanche completo (salgado + suco) tem direito a três fichas, se for só lanche tem direito a duas fichas e se for só suco a uma ficha.

Considerando que ele sempre compra na lanchonete. No final do mês qual a melhor opção de economia?

1. O Brasil possui cerca de 13,7% do total de água doce do mundo, sendo considerado um território rico em termos hídricos. No entanto, o país vive sérios problemas, relacionados tanto à degradação da qualidade das águas, principalmente nas proximidades das áreas urbanas, quanto à falta de controle do excesso e da insuficiência de água, que atingem várias localidades brasileiras. Não são somente as enchentes que afetam as cidades brasileiras: a escassez hídrica também impõe sérias restrições e elevados custos ao desenvolvimento econômico e social de grandes cidades do Brasil.

Observando o gráfico a seguir, responda no caderno:



Informações obtidas em: MINISTÉRIO DO MEIO AMBIENTE. Disponível em: <http://www.mma.gov.br/estruturas/sedr_proecotur/_publicacao/140_publicacao09062009025910.pdf>. Acesso em: 1º jul. 2018.

a) Que tipo de gráfico é este? Gráfico de colunas triplas ou gráfico de múltiplas colunas.

b) Indique a região brasileira:
- com a maior superfície; Região Norte: 45,30%.
- com mais recursos hídricos; Região Norte: 68,50%.
- com a segunda menor concentração de população. Região Norte: 6,98%.

c) Que região tem a menor taxa percentual de recursos hídricos do nosso país? Região Nordeste: 3,30%.

d) Em qual região há maior concentração de população? Região Sudeste: 42,65%.

e) Pode-se dizer que quanto maior a superfície da região, maior é o número de habitantes? Justifique sua resposta. Não. O Sudeste possui a maior população, porém possui a segunda menor superfície do Brasil.

f) Quantos por cento da água doce do mundo estão na região Sudeste brasileira? Explique como você pensou para responder. Cerca de 0,82%. Resposta pessoal.

g) Pode-se dizer que a região que dispõe de mais recursos hídricos é a que possui a maior população? Não.

Q0: Como são distribuídos os recursos hídricos brasileiros?

***Q0:** Como são distribuídos os recursos hídricos brasileiros?*

Q_1: quais países possuem os maiores recursos hídricos?

Q_2: qual é a sua superfície territorial desse país?

Q_3: qual é o total da população desse país?

Q_4: quais ações podem ou são feitas para garantir a água potável para as regiões brasileiras com menor índice de água potável?

Q_5: qual região com menor índice de recursos hídricos no Brasil?

Q_6: qual região com maior índice de recurso hídrico no Brasil?

Q_7: qual é a superfície territorial do Brasil?

Q_8: qual é a superfície territorial do Brasil por região?

Q_9: qual é a atual população brasileira?

- Primeiro, apresentação do vídeo motivador com a temática "recursos hídricos".
- Atividade em grupo com 4 ou 5 estudantes.
- Distribuição da atividade em sala e em laboratório de informática.
- ***Busca de dados:*** sites, matérias de jornais, artigos, reportagens, vídeos

Outro PEP que poderia ser desenvolvido:

→ Q0: Comparando o gráfico apresentado no exercício do livro didático, há mudanças em relação a dados mais atualizados sobre população, território/superfície, e recursos hídricos?

O exercício escolar apresenta o gráfico em porcentagens, então quais são estes dados? Quais variáveis são estas?

$Q_{POPULAÇÃO}$
$Q_{SUPERFÍCIE}$
$Q_{ÁGUA}$

O que mudou de 2018 para hoje (ou em relação aos dados mais atualizados)?

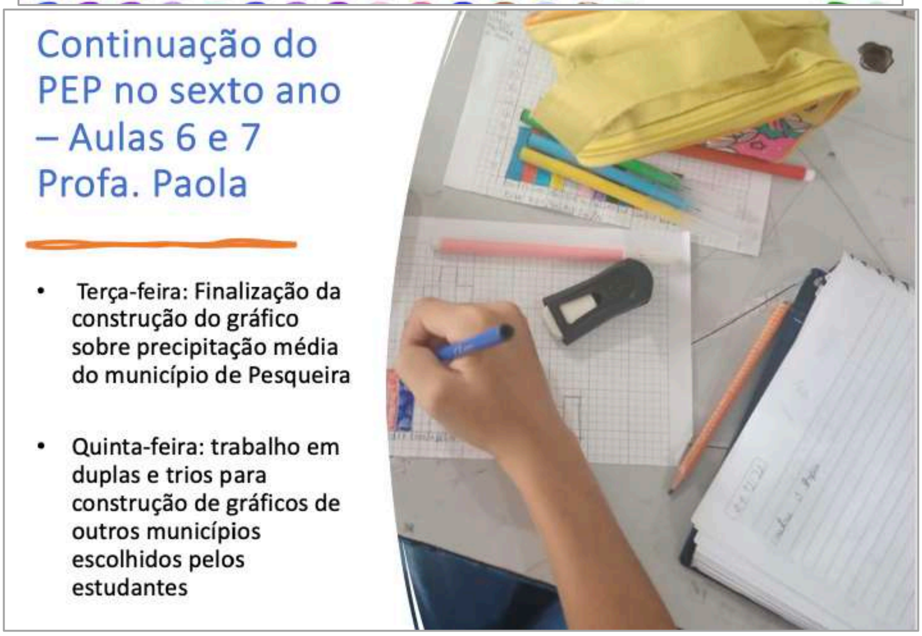
- Para isso será preciso construir um gráfico semelhante e fazer o comparativo:
- Recursos hídricos / superfície / população por Regiões: Norte/ Centro-Oeste/ Sul/ Sudeste/ Nordeste.
- O gráfico precisará ser feito em porcentagens para poder comparar com o gráfico do livro.

$Q_{POPULAÇÃO}$: Qual a população (número de habitantes) brasileira?
Qual a população (número de habitantes) brasileira por regiões?
Qual a unidade de medida?
Onde encontramos dados mais atualizados sobre isso (fonte dos dados)?

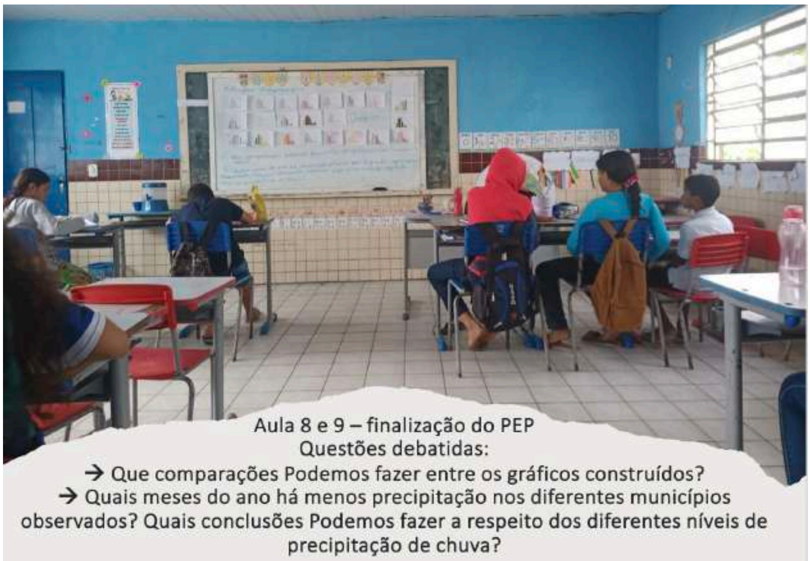
$Q_{SUPERFÍCIE}$: Qual a superfície territorial brasileira?
Qual a superfície territorial brasileira por regiões?
Qual a unidade de medida dos dados (em hectares ou km²?)?
Onde encontramos dados mais atualizados sobre isso (fonte dos dados)?

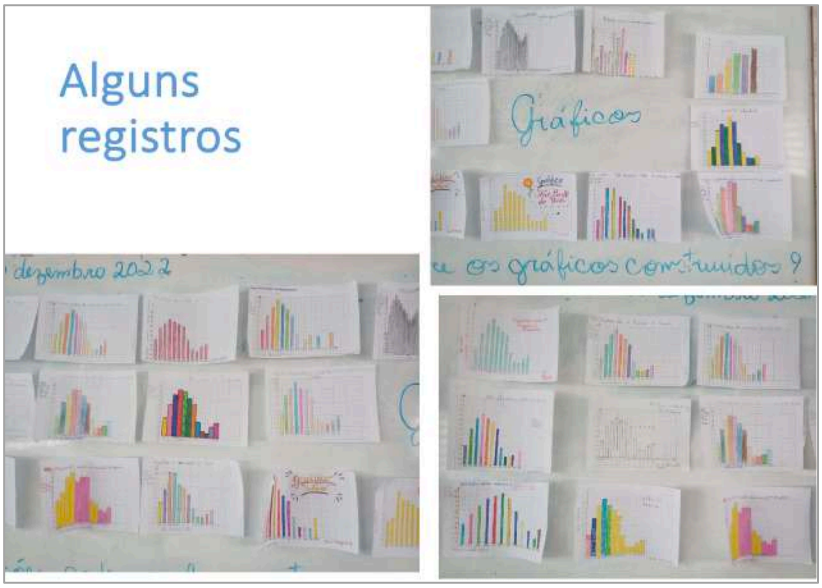
$Q_{ÁGUA}$: Qual a quantidade de recursos hídricos do Brasil?
Qual a quantidade de recursos hídricos do Brasil por regiões?
Qual a unidade de medida?
Onde encontramos dados mais atualizados sobre isso?

Sessão 14

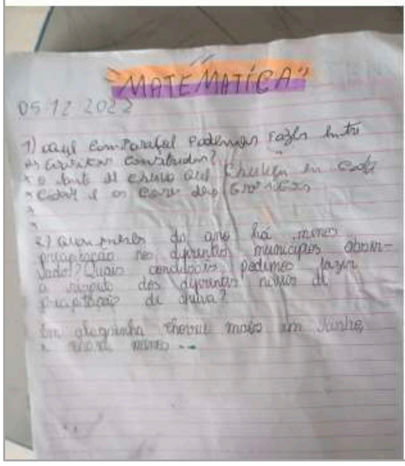


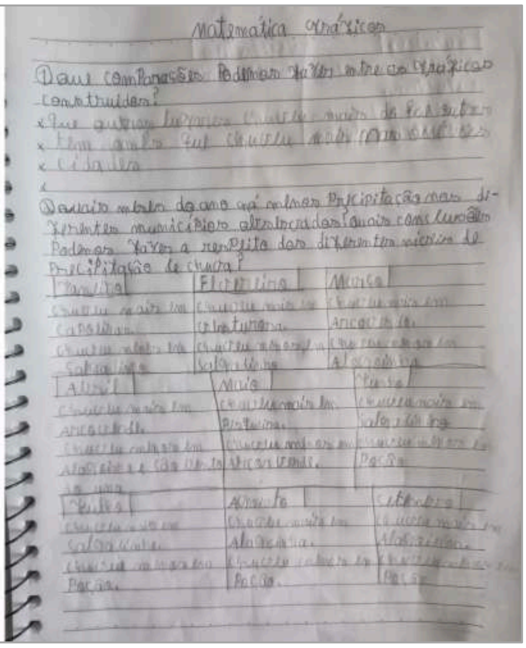

Alguns registros


Aula 8 e 9 – finalização do PEP
Questões debatidas:
→ Que comparações Podemos fazer entre os gráficos construídos?
→ Quais meses do ano há menos precipitação nos diferentes municípios observados? Quais conclusões Podemos fazer a respeito dos diferentes níveis de precipitação de chuva?

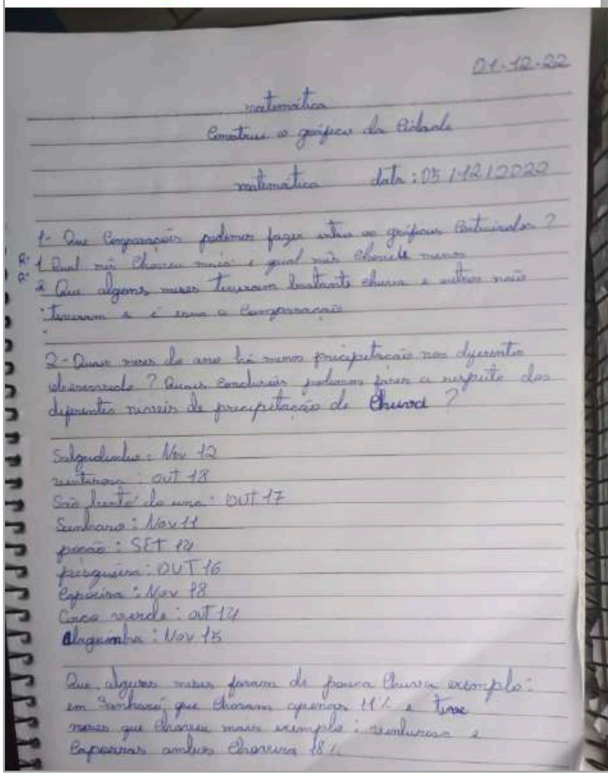

Alguns registros
Gráficos

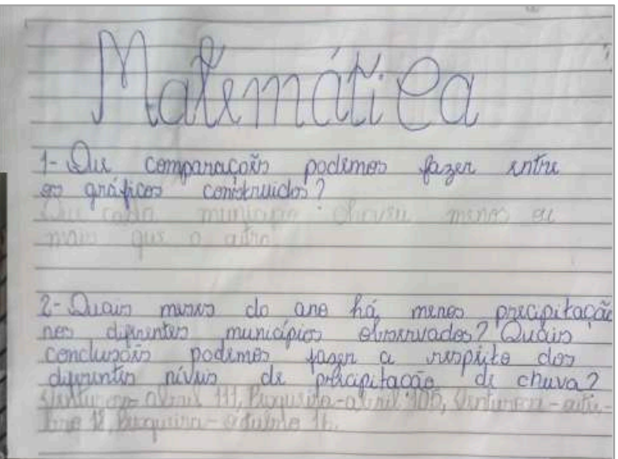
Alguns registros

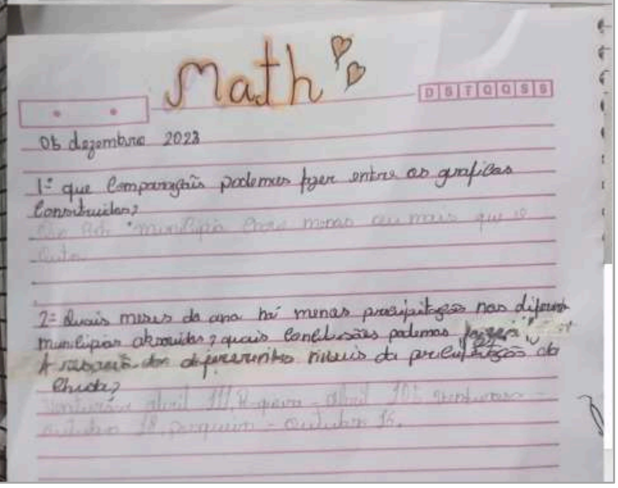




Alguns registros







APPENDIX E.1: Preservice teachers' reports of the face-to-face SRP-TE

Relatório final do Grupo 1

Parte I – PEP

- Questões formuladas Q_i

Pergunta 1:

Atualização Gráfica: o gráfico da distribuição de recursos tem certa atualização depois de tantos anos ou segue sem atualização desde a última disponibilização dos dados que foi em 2018? Que questionamentos podemos fazer em relação a esta comparação (sobre o gráfico do exercício do livro e o gráfico com dados atualizados)?

Pergunta:

Qual a quantidade de recursos hídricos do Brasil?
Qual a quantidade de recursos hídricos do Brasil por regiões?
Onde encontramos dados mais atualizados sobre isso (fonte de dados)?

Pergunta:

Qual a superfície territorial brasileira?
Qual a superfície territorial brasileira por regiões?
Onde encontramos dados mais atualizados sobre isso (fonte dos dados)?

Pergunta:

Em quais bases de dados encontramos dados atualizados?

- Mídias

Em quais bases de dados encontramos dados atualizados?
Onde encontramos dados mais atualizados sobre isso (fonte dos dados)?

Fontes da Pergunta 1

População – Censo 2022: https://www.ibge.gov.br/estatisticas/sociais/populacao/22827-censo-demografico-2022.html?=&t=resultados

Recursos hídricos: https://mapbiomas.org/estatisticas

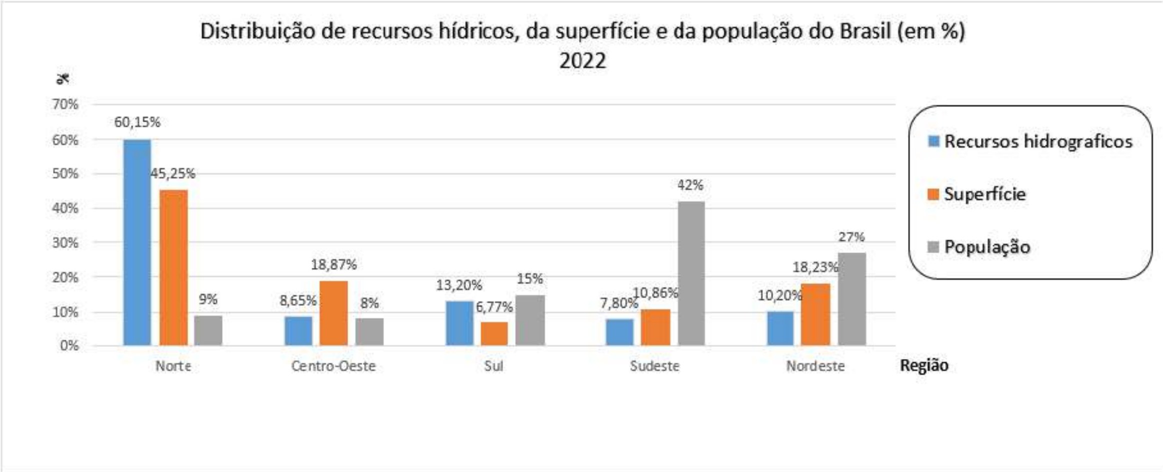
Território: https://www.ibge.gov.br/geociencias/organizacao-do-territorio/estrutura-territorial/15761-areas-dos-municipios.html?=&t=acesso-ao-produto

- Respostas preexistentes $R^\Diamond$

Encontramos os dados no IBGE em formato de tabelas separadas para cada tópico e depois reunimos as informações e montamos um gráfico mais atualizado.

- Respostas R_i

Pergunta 1: O grupo reuniu-se para juntar informação para pesquisar as tabelas; logo após reunir as tabelas, nós fizemos um gráfico atualizado de acordo com as tabelas mais atualizadas. Gráfico atualizado:



Pergunta 2: Encontramos a divisão de recursos hídricos/Qual a superfície territorial brasileira por regiões?

Região	Recursos hídricos	Coluna1
NORTE	10957539,31	60,15%
NORDESTE	1858373,667	10,20%
SUDESTE	1426681,627	7,80%
SUL	2407574,632	13,20%
Centro Oeste	1568226,971	8,65%
TOTAL	18218396,21	100,00%

Qual a superfície territorial brasileira?

Região	Território	Coluna1
NORTE	3.850.593,104	45,25%
NORDESTE	1.552.175,419	18,23%
SUDESTE	924.558,342	10,86%
SUL	576.736,821	6,77%
CENTRO-OESTE	1.606.354,086	18,87%
Total	8.510.417,772	100%

Qual a superfície territorial brasileira por regiões?

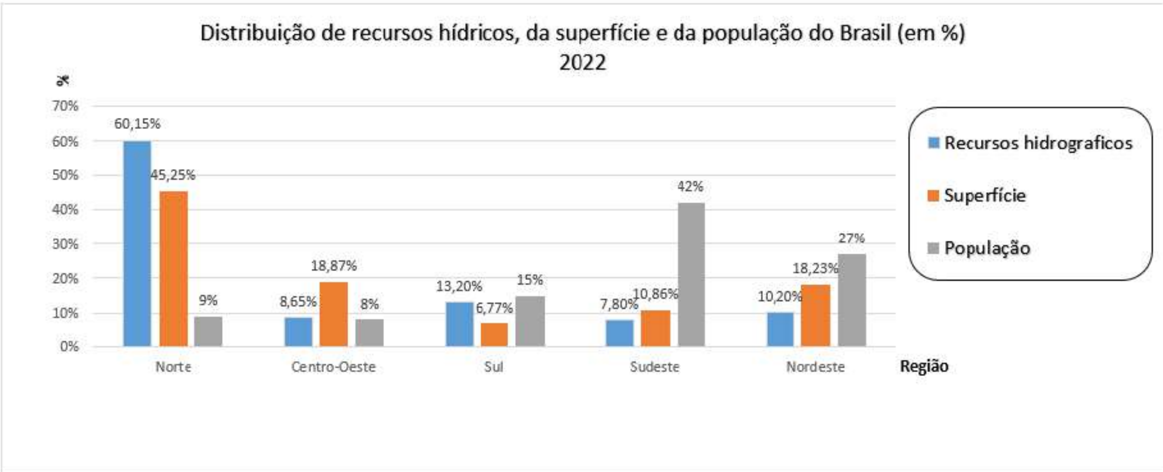
REGIÃO	Coluna1	POPULAÇÃO	Coluna2
Região Norte		17834762	9%
Região Nordeste		55389382	27%
Região Sudeste		87348223	42%
Região Sul		30685598	15%
Região Centro-Oeste		16492326	8%
	TOTAL	207750291	100%

- Resposta *R*♥

Apresentar a discussão da resposta final para o estudo realizado: análises, tabelas e gráficos construídos etc.

O grupo elaborou, depois das discussões, um gráfico atualizado de acordo com tabelas recentes.

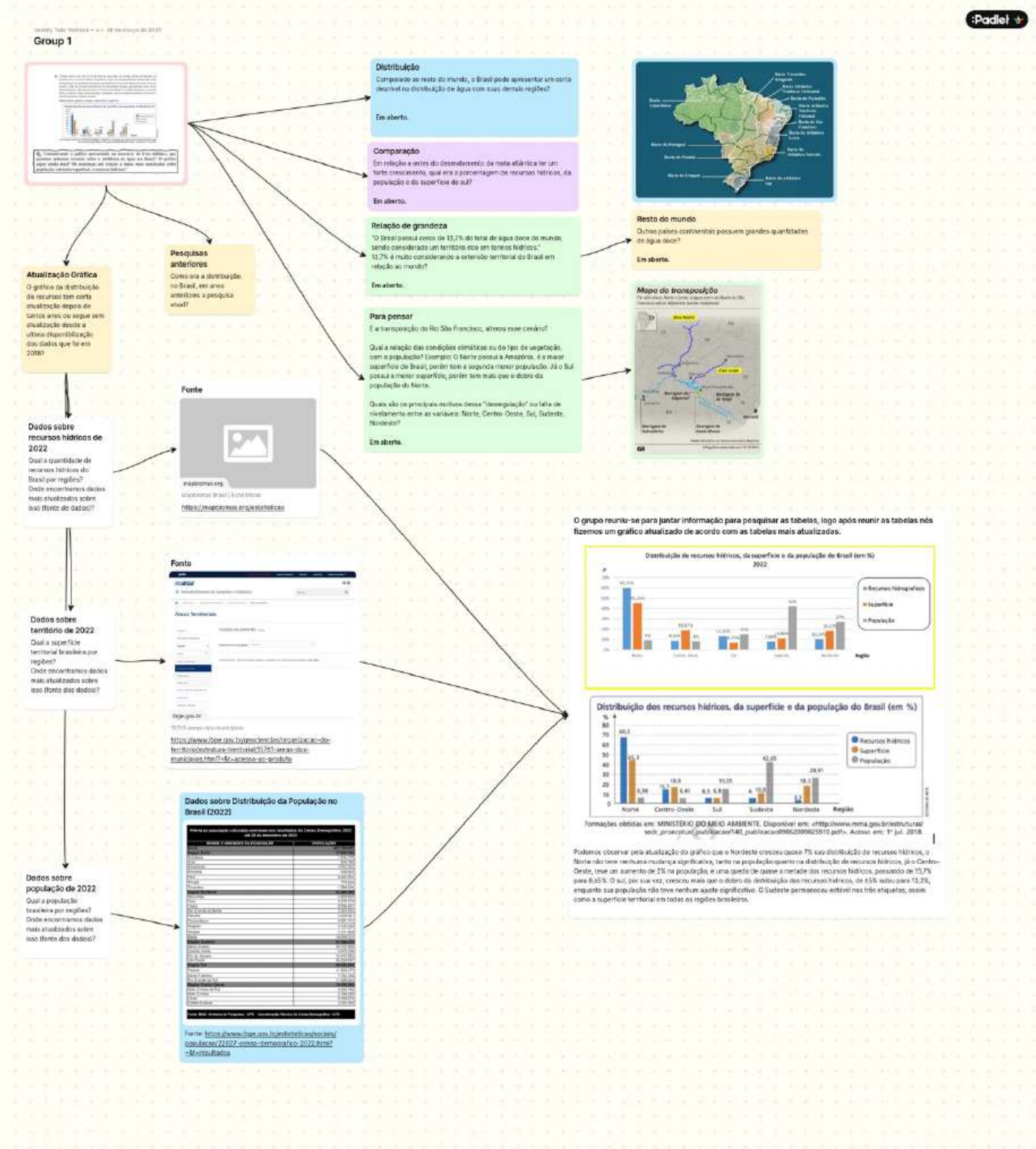
Região	Recursos hidrograficos	Superfície	População
Norte	60,15%	45,25%	9%
Centro-Oeste	8,65%	18,87%	8%
Sul	13,20%	6,77%	15%
Sudeste	7,80%	10,86%	42%
Nordeste	10,20%	18,23%	27%



- Questões em aberto

Em relação a antes do desmatamento da Mata Atlântica ter um forte crescimento, qual era a porcentagem de recursos hídricos, de população e de superfície do sul?

Mapa de questões-respostas do Padlet



Parte II – Análise do PEP

Dialética Questões-Respostas: Comentem sobre a utilidade desta ferramenta durante o PEP

- **Pontos positivos**:

Foi, de certa forma, mais organizado e mais limpo para o campo de visão para *linkar* as perguntas, formular perguntas, respondê-las e para agrupar perguntas que ficariam mais *linkadas* com perguntas formuladas por outros grupos.

- Pontos negativos:

- **Dificuldades encontradas?**

Aprender a manusear o campo de ideias, porque foi uma coisa inédita para os integrantes do grupo e ficamos meio perdidos também na proposta feita pela professora Janielly, precisando de um pouco de tempo para compreender e começar a formular as perguntas que nós respondemos.

Dialética mídia-meio:

- **Que mídia foram utilizadas durante o PEP?**

Foi mais usado o site do IBGE (para população e território) e Mapbiomas (para os recursos hídricos) disponibilizado pela professora Janielly.

- **Que objetos foram utilizados durante o PEP?**

Foi utilizado o *software* do Excel (para a produção do gráfico) e do *Google documentos* (para reunir todas as informações e formar o trabalho apresentado, *sites* de consulta para a atualização do gráfico (IBGE e Mapbiomas) e o gráfico inicial, de onde começamos a nossa busca para sua atualização.

- **Que conhecimentos estatísticos foram utilizados durante o PEP?**

Desde o início, utilizamos diversas ferramentas e conhecimentos estatísticos. Desde analisar o gráfico, ponderar as perguntas a se fazer, analisar dados, procurar dados, organizar essas informações, tudo faz parte da estatística.

- **Que dados foram utilizados durante o PEP?**

Um gráfico retirado de um livro que informava a distribuição de recursos hídricos, da população e do território, dados retirados do IBGE e do Mapbiomas para a consulta e formação de informações para a atualização do gráfico que foi uma proposta que tivemos.

- **Que estudos poderiam ou precisam ser aprofundados?**

Estudos referentes à distribuição hídrica e como melhor pode ser abordado esse estudo e mais bem distribuídos os recursos hídricos.

- **Dificuldades encontradas?**

Encontrar as informações do território e dos recursos hídricos, os dados da população foram mais fáceis de encontrar dentro da plataforma do IBGE; agora, da divisão territorial, tivemos que buscar mais a fundo para encontrar dentro do da plataforma do IBGE e a distribuição hídrica não conseguimos encontrar dentro da plataforma, sendo disponibilizado pela professora Janielly.

<u>Dialética Indivíduo-Coletivo:</u> Comentem sobre a organização do grupo durante o PEP

- **Discussões coletivas?**

O grupo teve umas discussões pelo *Whatsapp*, mas foram feitas reuniões presencialmente para a elaboração das pesquisas e a produção do gráfico.

- **Divisão de tarefas?**

A divisão ficou entre a procura de dados para a atualização do gráfico e a produção do gráfico no Excel.

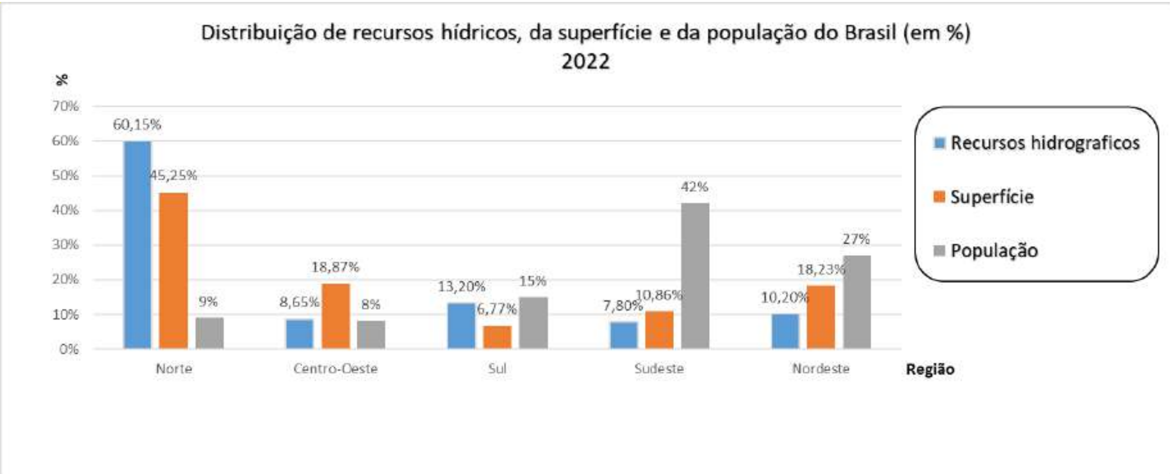
- **Dificuldades encontradas?**

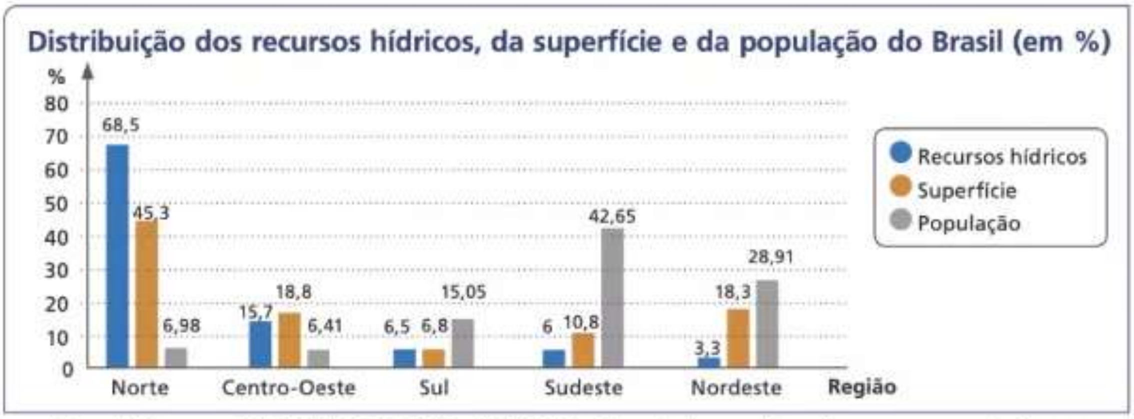
A compreensão e busca de informações/dados referente a pergunta "Em relação a antes do desmatamento da Mata Atlântica ter um forte crescimento, qual era a porcentagem de recursos hídricos, da população e da superfície do sul?" E antes da Professora Janielly disponibilizar os sites para os dados sobre os recursos hídricos.

- **O trabalho em grupo poderia ser melhorado? Em quais aspectos?**

Poderia ser melhor se todos os integrantes do grupo tivessem feito sua parte e ido atrás de saber sobre o trabalho e sobre como o grupo estava separando cada tarefa.

Que questionamentos podemos fazer em relação a esta comparação (sobre o gráfico do exercício do livro e o gráfico com dados atualizados)?





formações obtidas em: MINISTÉRIO DO MEIO AMBIENTE. Disponível em: <http://www.mma.gov.br/estruturas/sedr_proecotur/_publicacao/140_publicacao09062009025910.pdf>. Acesso em: 1º jul. 2018.

Podemos observar, pela atualização do gráfico, que o Nordeste cresceu quase 7% sua distribuição de recursos hídricos, o Norte não teve nenhuma mudança significativa, tanto na população quanto na distribuição de recursos hídricos, já o Centro-Oeste teve um aumento de 2% na população, e uma queda de quase a metade dos recursos hídricos, passando de 15,7% para 8,65%. O Sul, por sua vez, cresceu mais que o dobro da distribuição dos recursos hídricos, de 65% subiu para 13,2%, enquanto sua população não teve nenhum ajuste significativo. O Sudeste permaneceu estável nas três etiquetas, assim como a superfície territorial em todas as regiões brasileiras.

Módulo III - planejamento de atividade de investigação

Turma: 8º D/ 9º B/ 9º C/ 9º E
Escola: Hercules Maymone

- Questão inicial da atividade de investigação.
  R. No início da Residência Pedagógica, foi solicitado pelo professor preceptor da escola, e a professora orientadora da residência, que fosse aplicado nas turmas uma pesquisa socioeconômica/emocional, nos alunos, com foco no *bullying*. Aproveitamos dessa pesquisa que já havia sido realizada para trabalharmos o projeto de Estatística.

- Como se coloca a questão, o que se propõe fazer e que ferramentas estão disponíveis?
  R. A Pesquisa já havia sido realizada e os dados coletados. Foi necessário a organização dos dados, relembrar o conceito e a definição de tabelas e gráficos, bem como instruir como construir uma tabela e como relacionar ao gráfico.

- Como o estudo está planejado para ser administrado em classe: horário aproximado
  R. Foi planejado para que terminássemos de 4 a 6 aulas.

- Como se espera que a atividade seja concluída
  R. É esperado que cada grupo entregue a tabela e o gráfico, referente à pergunta que foi direcionada ao seu grupo, bem como uma conclusão sobre o tema.

- Qual/quais conteúdo(s) curricular(es) (em matemática e outros assuntos) pode(m) ser abordado(s) pela atividade proposta**.**
  R. Estatística/ Análise de Dados/ Porcentagem/ Multiplicação/ Senso Crítico.

Universidade Federal de Mato Grosso do Sul - UFMS
Instituto de Matemática
Curso de Matemática- Licenciatura
Disciplina: Residência Pedagógica – Projeto de ensino de graduação
Professora: Sonia Maria Monteiro da Silva Burigato

PLANEJAMENTO DE AULA

TURMAS: 8º D, 9º C, 9º D e 9º E.

RECURSOS: Sala de aula, folhas da pesquisa já realizada, quadro da sala, lápis, caneta, régua, folha sulfite, cartolina.

CONTEÚDO: Estatística

OBJETIVO DE ENSINO:
(EF06MA33) Planejar e coletar dados de pesquisa referente a práticas sociais escolhidas pelos alunos e fazer uso de planilhas eletrônicas para registro, representação e interpretação das informações, em tabelas, vários tipos de gráficos e texto.

(EF07MA36) Planejar e realizar pesquisa envolvendo tema da realidade social, identificando a necessidade de ser censitária ou de usar amostra, e interpretar os dados para comunicá-los por meio de relatório escrito, tabelas e gráficos, com o apoio de planilhas eletrônicas.

DATA DE REALIZAÇÃO:

METODOLOGIA: Preparando uma aula construtivista com foco em pesquisa socioeconômica e socioemocional, relacionada à situação econômica e questões ligadas ao *bullying* dos próprios alunos. A abordagem consistirá em orientar os alunos na análise e no agrupamento dos dados coletados, seguidos pela representação por meio de gráficos e tabelas. Para isso, será fornecida uma aula expositiva sobre estatística, abrangendo conceitos, tipos de gráficos, tabela de frequência e organização dos dados. Essa etapa permitirá que os alunos construam as tabelas e os gráficos com base na pesquisa realizada e, em seguida, realizem análises dos achados da pesquisa.

Para facilitar a organização e análise dos dados, dividimos os alunos em 4 grupos por sala de no máximo 7 pessoas. O trabalho ocorrerá nas quatro turmas de forma simultânea, temos 1 aula por dia em cada turma, de terça a sexta. Dessa forma, separamos as perguntas para cada sala da seguinte maneira:

<table>
<tr><td>8º D</td><td>Grupo 01
Pergunta 5: Possui acesso à internet em casa? sim/não
Título: Acesso à internet dos estudantes em casa.
Construir uma Tabela de Distribuição de Frequência.
Construir um Gráfico de Setores.

Grupo 02
Pergunta 5.1: Caso sim, sua internet provém de: ( ) wi-fi ( ) dados móveis ( ) outros.
Título: Provedor de internet dos estudantes.
Construir uma Tabela de Distribuição de Frequência.
Construir um Gráfico de Colunas.

Grupo 03
Pergunta 06: Qual seu meio de acesso à internet?
Título: Principal meio de acesso à internet dos estudantes.
Construir uma Tabela de Distribuição de Frequência.
Construir um Gráfico de Colunas.

Grupo 04
Pergunta 7: Quantas pessoas moram com você?
Título: Quantidade de pessoas que moram com os estudantes.
Construir uma Tabela de Distribuição de Frequência.
Construir um Gráfico de Colunas.</td></tr>
<tr><td>9°C</td><td>Grupo 01
Pergunta 14: Em qual ambiente você foi vítima de bullying?
Título: Ambiente onde os estudantes são vítimas de bullying.
Construir uma Tabela de Distribuição de Frequência.
Construir um Gráfico de Colunas.

Grupo 02
Pergunta 16: Você já presenciou alguém praticando bullying? sim/não
Título: Estudantes que já presenciaram bullying.
Construir uma Tabela de Distribuição de Frequência.
Construir um Gráfico de Setores

Grupo 03
Pergunta 16.1: Quem presenciou bulliyng, fez o que perante a isso?
Título: Atitudes dos estudantes que presenciaram bullying.
Construir uma Tabela de Distribuição de Frequência.
Construir um Gráfico de Colunas.

Grupo 04
Pergunta 17: Já se mutilou? sim/não
Título: Estudantes que já machucaram o próprio corpo propositalmente.
Construir uma Tabela de Distribuição de Frequência.
Construir um Gráfico de Setores</td></tr>
<tr><td></td><td>Grupo 01
Pergunta 09: Qual o tipo de residência dos estudantes?</td></tr>
</table>

9°D	Título: *Tipos de moradias dos estudantes.* Construir uma Tabela de Distribuição de Frequência. Construir um Gráfico de Colunas. **Grupo 02** Pergunta 10: Qual meio de transporte você utiliza para chegar à escola? Título: *Meio de transporte que os estudantes utilizam para chegar à escola.* Construir uma Tabela de Distribuição de Frequência. Construir um Gráfico de Colunas. **Grupo 03** Pergunta 11: Qual sua principal fonte de diversão em casa? Título: *Lazer dos estudantes.* Construir uma Tabela de Distribuição de Frequência. Construir um Gráfico de Colunas **Grupo 04** Pergunta 18: Você se sente confortável em sala? Título: *Os estudantes se sentem confortáveis em sala?* Construir uma Tabela de Distribuição de Frequência. Construir um Gráfico de Setores.
9°E	**Grupo 01** Pergunta 02: Qual a sua idade? Título: *Idade dos estudantes.* Construir uma Tabela de Distribuição de Frequência. Construir um Gráfico Histograma. **Grupo 02** Pergunta 12: Já sofreu *bullying*? Título: *Estudantes que já sofreram bullying.* Construir uma Tabela de Distribuição de Frequência. Construir um Gráfico de Setores **Grupo 03** Pergunta 12.1: Qual tipo de *bullying* você já sofreu? Título: *Tipos de bullying sofridos pelos alunos.* Construir uma Tabela de Distribuição de Frequência. Construir um Gráfico de Colunas. **Grupo 04** Pergunta 15: Já praticou *bullying*? Título: *Estudantes que afirmam ter praticado bullying.* Construir uma Tabela de Distribuição de Frequência. Construir um Gráfico de Setores

<u>Aula 01: Análise dos dados.</u>

<u>Objetivo:</u> Coletar dados da pesquisa realizada.

- Começaremos a aula relembrando os alunos sobre o questionário que que eles haviam respondido e sobre a importância de analisar os dados coletados.
- Distribuiremos os papéis com as respostas que foram realizadas em outra sala de aula, para essa turma, de forma que cada aluno tenha uma pesquisa em mãos. Como algumas respostas possuem informações delicadas sobre alguns alunos, essas serão analisadas pelos professores. É importante ressaltar que a pesquisa foi feita de forma anônima pelos alunos e que eles não terão acesso às respostas da própria sala.
- Em seguida, junto com os alunos, os auxiliaremos a organizarem os dados que estão brutos do questionário.
- Vamos abordar as perguntas uma de cada vez, permitindo que os alunos respondam de acordo com o que está indicado no questionário que está em sua posse. Em seguida, registramos no quadro os dados brutos de cada pergunta.
- **Ficará de tarefa** uma pesquisa sobre o que são tabelas de frequências e para que servem, o que são gráficos de setores, colunas e histograma e para que serve cada um deles.

<u>Aula 2 e 3: Aula expositiva sobre estatística e conceitos básicos</u>.

<u>Objetivo:</u> Introduzir os conceitos estatísticos necessários para a compreensão e análise dos dados coletados.

- Comece a aula revisando brevemente o objetivo da pesquisa e relembrando os dados coletados pelos alunos.
- Com base na pesquisa dos alunos, exemplifique e explique sobre a tabela de distribuição de frequência, para que serve, como interpretar e extrair informações dela. Apresente os diferentes tipos de gráficos e tabelas utilizados na representação de dados estatísticos, como gráfico de colunas, gráfico de setores e histograma.
- Demonstre como construir cada tipo de gráfico e como interpretar as informações apresentadas. Mostre exemplos práticos relacionados ao tema da pesquisa, se possível.
- Se possível, inicie a construção das tabelas de frequência com os dados levantados.
- **Possíveis tarefas para essas aulas:** Cada aluno, construir a tabela de distribuição de frequência da pergunta direcionada ao seu grupo no próprio caderno. Pesquisar qual o gráfico que seu grupo precisa construir e tentar construir no caderno.

<u>Aula 4 e 5: Organização dos dados e criação de tabelas e gráficos.</u>

<u>Objetivo:</u> Orientar os alunos na organização dos dados coletados e auxiliá-los na criação de tabelas e gráficos.

- Inicie a aula revisando com os estudantes os conceitos aprendidos anteriormente sobre estatística e representação gráfica.
- Ajude-os a organizar os dados em uma tabela de frequência, destacando as categorias principais e as respectivas contagens.

- Auxilie-os na construção dos gráficos, fornecendo orientações sobre escalas, títulos e legendas. Os gráficos e tabelas devem ser desenhados em folhas sulfite e posteriormente fixados em cartolinas para a finalização do trabalho.
- Encoraje a discussão e a análise dos resultados obtidos por meio dos gráficos.
- Peça aos alunos que identifiquem padrões, tendências e possíveis relações entre os dados apresentados.

Aula 06: Análise dos resultados e reflexão.

Objetivo: Promover a reflexão e discussão dos resultados obtidos, relacionando-os aos conceitos socioeconômicos e socioemocionais abordados na pesquisa.

- Inicie a aula revisando os resultados apresentados pelos grupos.
- Promova uma discussão em sala de aula sobre as conclusões obtidas a partir dos gráficos e tabelas construídos.
- Incentive os alunos a refletirem sobre os dados coletados, questionando-os sobre possíveis razões para os resultados observados.
- Estimule o debate e a troca de ideias entre os alunos.

AVALIAÇÃO:

- Participação ativa dos alunos: será verificado se os alunos se envolveram ativamente na atividade, desde a coleta de dados até a análise dos resultados. Será observado se houve troca de ideias, debate e construção coletiva de conhecimento.

- Capacidade de análise e síntese: será avaliada a habilidade dos alunos em analisar os dados coletados e identificar padrões, tendências e relações. Se foram capazes de sintetizar as informações e chegar a conclusões fundamentadas.

- Compreensão dos conceitos estatísticos: será verificado se os alunos demonstraram compreensão dos conceitos apresentados na aula expositiva sobre estatística, como população, amostra, variável, gráficos e tabela de frequência, observando também se conseguiram aplicar esses conceitos na criação dos gráficos e tabelas da pesquisa.

- Pensamento crítico: será observado se os alunos foram capazes de refletir criticamente sobre os resultados da pesquisa, relacionando-os aos conceitos socioeconômicos e socioemocionais abordados, verificando também se conseguiram fazer perguntas pertinentes, levantar hipóteses e desenvolver argumentos embasados em evidências.

- Colaboração e trabalho em grupo: avaliação sobre a capacidade dos alunos de trabalhar em equipe, compartilhando responsabilidades, ouvindo diferentes perspectivas e colaborando para alcançar os objetivos da atividade.

- Expressão oral e escrita: será observada a habilidade dos alunos em expressar suas ideias de forma clara e coesa, tanto oralmente, durante as discussões em sala de aula, quanto por escrito na organização dos dados, construção de gráficos e elaboração de conclusões.

A avaliação construtivista valoriza o processo de aprendizagem, o envolvimento ativo dos alunos e a construção do conhecimento; portanto, é importante considerar esses aspectos ao avaliar a atividade.

Anexo 1: Questionário já aplicado.

01) Em qual ano você está?
☐ 8º ano ☐ 9º ano

02) Qual a sua idade?
☐ 12 anos ☐ 13 anos ☐ 14 anos ☐ 15 anos ☐ 16 anos ☐ 17 anos
Outra idade:_____

03) Use três palavras que defina: **Matemática.**
_______________, _______________ e _______________.

04) Caso você tenha dificuldade em matemática, em casa, quem ajuda você?
☐ **Mãe/Pai** ☐ **Irmã(o)** ☐ **Outra pessoa da família.** Quem? _______________ ☐ **Alguém de fora da família.** Quem? _______________ ☐ **Ninguém**

05) Possui acesso à internet em casa?
☐ **Sim** ☐ **Não**
<u>Caso sim, sua internet provem de:</u>
☐ **Wi-fi** ☐ Dados móveis *(internet 3G/4G/5G)* ☐ **Outros:** ___________

06) Qual seu meio de acesso à internet?
☐ Celular próprio ☐ Celular dos pais ☐ Computador ☐ **Notebook**
Outro meio de acesso: _______________________

07) Quantas pessoas moram com você?
☐ **01 pessoa** ☐ **02 pessoas** ☐ **03 pessoas** ☐ **04 pessoas** ☐ **05 pessoas** ☐ **06 pessoas ou mais**

08) Você mora com:
☐ **Pai** ☐ **Mãe** ☐ **Avô** ☐ **Avó** ☐ **Irmão(a)** Quantos:_______ ☐ **Padrasto/Madrasta**
Outras pessoas: ____

09) Você mora em:
☐ **Moradia própria.** ☐ **Moradia cedida.** ☐ **Moradia alugada.** ☐ **Não sei dizer.**

10) Qual meio de transporte você utiliza para chegar na Escola?
☐ **Ônibus** ☐ **Carro/moto** ☐ **Bicicleta** ☐ **A pé** ☐ **Outros:** _______________

11) Qual a sua principal fonte de diversão em casa?
☐ **Assistir TV** *(Séries/anime/filme)* ☐ **Praticar Esportes** ☐ **Frequentar praças** ☐ **Ler**
Outras coisas que gosto de fazer:_______________________________________

12) Você já **sofreu bullying?**
☐ **Sim** ☐ **Não**
<u>**Se sim, qual tipo de bullying você sofreu?**</u>
☐ **Físico** *(Ocorre por meio de agressões físicas, socos, chutes, empurrões e outras ações que envolvam contato físico.)*
☐ **Verbal** *(Xingamentos, piadas, apelidos maldosos, ou qualquer tipo de humilhação verbal.)*
☐ **Psicológico** *(Perseguição, manipulação, chantagem, discriminação, ameaças, isolamento intencional.)*
☐ **Escrito** *(Bilhetes, cartas, pixações ou desenhos depreciativos são usados para atacar um colega.)*
☐ **Material** *(Ter seus pertences danificados, furtados ou objetos atirados contra você.)*
☐ **Moral** *(Quando você já foi difamado, intimidado, caluniado, ou imitado de forma depreciativa por terceiros.)*
☐ **Social** *(Rumores, fofocas, excluir a pessoa com a intenção de humilhar.)*
☐ **Virtual** *(Pode ser qualquer uma das violências citadas a cima, se caracteriza por acontecer na internet, qualquer ato produzido por terceiros que te cause sofrimento, constrangimento ou temor social.)*

13) Em alguma situação você procurou **ajuda de adultos ou responsáveis?**
☐ **Sim** ☐ **Não**
Caso **não**, porque? _______________________________________

14) Em qual ambiente você foi vítima de bullying?
☐ **Sala de aula** ☐ **Pátio da escola** ☐ **Terminal de Ônibus** ☐ **Outro local:** __________

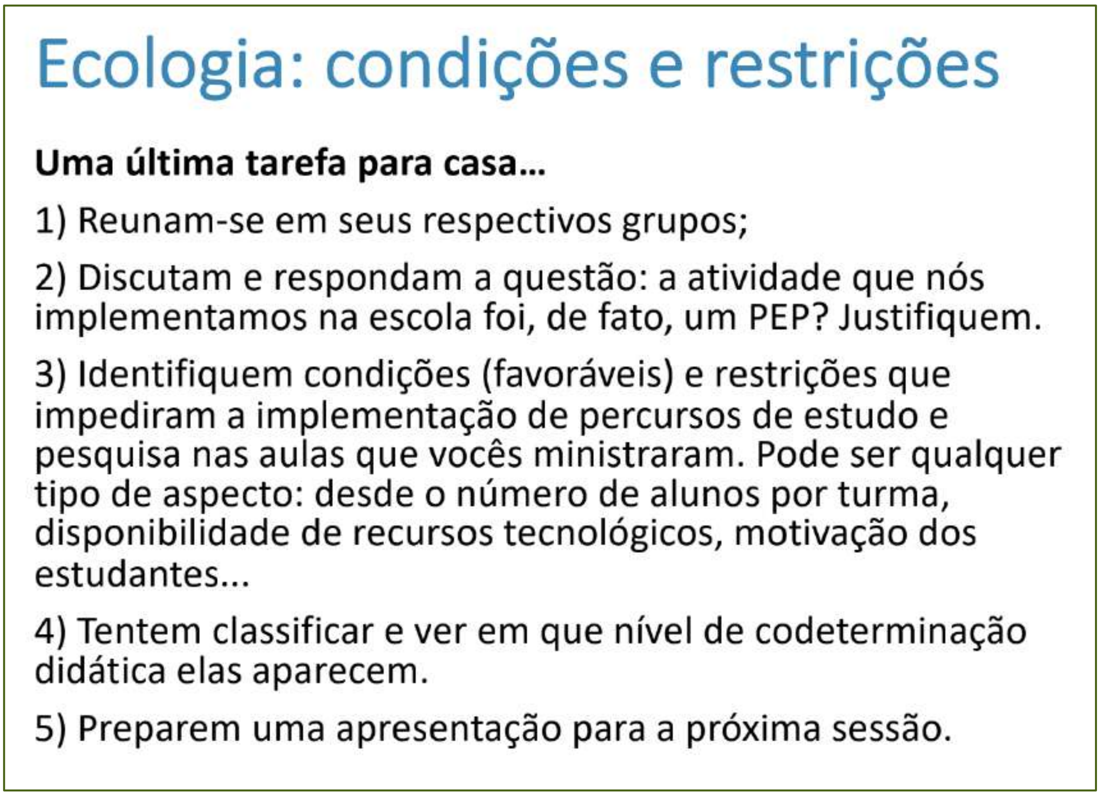
15) Você já **praticou bullying?**
☐ **Sim** ☐ **Não**

16) Você já **presenciou alguém praticando bullying?**
☐ **Sim** ☐ **Não**
<u>Caso sim, você:</u>
☐ **Agiu na intenção de acabar com o bullying**, ou procurou alguém responsável para que essa pessoa **agisse perante aquela situação.**
☐ **Levou na brincadeira**, ou ainda, participou da situação.
☐ **Não fez nada perante aquela situação.**

17) Você já se mutilou? *(Já se cortou, bateu seu corpo com força na parede propositalmente, já se arranhou ou qualquer ato contra seu próprio corpo.)*
☐ **Sim** ☐ **Não**
Caso sim, teve algum motivo específico? *(Relate em poucas palavras, caso se sinta confortável)* _______________________________________

18) Você se sente confortável em sua sala de aula?
☐ **Sim** ☐ **Não**
A) Caso **não**, qual o motivo? _______________________________
B) Caso sim, você se sente confortável em participar das aulas fazendo perguntas e respondendo o professor?
☐ **Sim** ☐ **Não**
Caso **não**, qual o motivo? _______________________________

Módulo IV – análises finais



2) Analisando *a posteriori*, o trabalho realizado, de fato, não foi um PEP, visto que, para ser um PEP, é necessário que haja um trabalho de investigação por parte dos alunos. Tentamos fazer um trabalho de investigação sobre a realidade socioeconômica e emocional dos alunos. Porém, a pesquisa foi elaborada por nós, residentes, aplicada por nós, os dados foram coletados com os alunos, sempre com nossos direcionamentos de como deveria prosseguir o projeto.

3)
Condições Favoráveis:
<u>A pesquisa já havia sido elaborada e aplicada.</u> *(Pedagogia / Escola)*
<u>Várias turmas participaram.</u> *(Escola)*

Restrições:
<u>Acesso à internet</u>. *(Sociedade / Escolas)*
<u>Acesso a computadores ou *tablets*</u>. *(Escolas)*
<u>Tempo</u>: o professor nos deu apenas 4 aulas para finalizar o projeto, pois ele precisava passar revisão para a prova bimestral. *(Escola/Sociedade)*
<u>Conteúdo e Conhecimentos necessários:</u> os alunos tinham dificuldade em calcular porcentagem e construir a tabela. *(Escola/Disciplina/Domínio/Temas)*

Relato

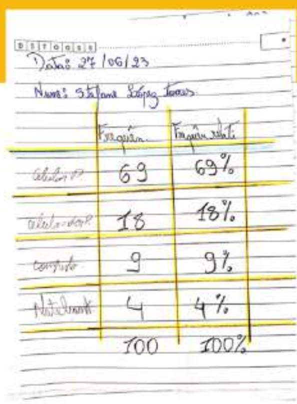
DE EXPERIÊNCIA

dia 27/06

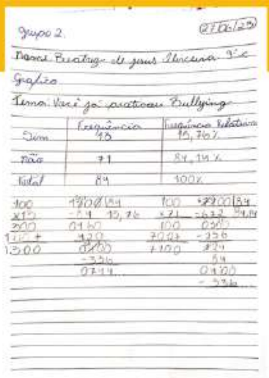
Rodrigo e Giulliano

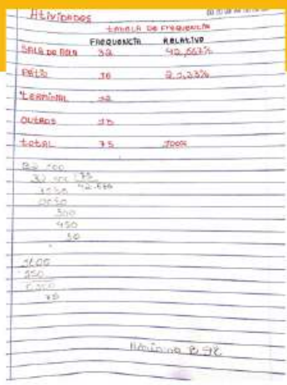
Grupos
Pesquisas
Folhas com dados brutos
Entrega Individual
Analise e organização dos dados para construção da tabela de frequência.
Atendimento aos grupos

As atividades individuais, não foram completadas em um só dia.
8º D

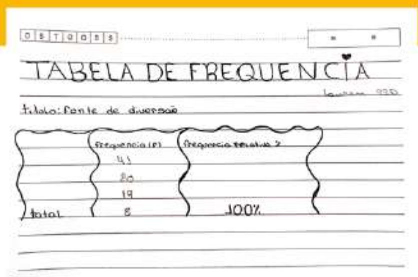


As atividades individuais, não foram completadas em um só dia.
9º C

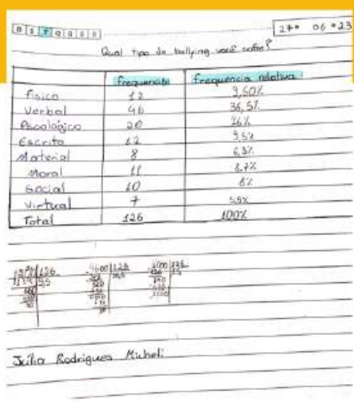




As atividades individuais, não foram completadas em um só dia.
9º D



As atividades individuais, não foram completadas em um só dia.
9º E



dia 28/06

Giovane e Ana Julia

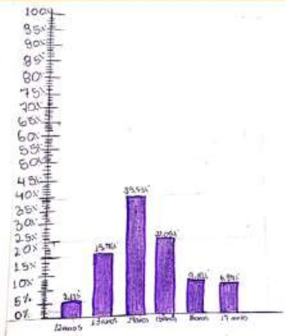
- Grupos
- Continuação da atividade do dia 27
- Pesquisas
- Entrega final da atividade
- Gráficos
- Avaliação
- Tempo
- Operações básicas

Entregas finais por grupo 9º E

GRÁFICO

IDADES DOS ALUNOS DO 8º e 9º ano

O grafico apresenta a quantidade de alunos em porcentagem, tendo o total de 86 alunos, sendo 2 com 12 anos, 17 com 13 anos, 34 com 14 anos, 19 com 15 anos, 8 com 16 anos e 6 com 17 anos.
A frequencia relativa é: 12 anos: 2,12%, 13 anos: 19,76%, 14 anos: 39,53%, 15 anos: 22,09%, 16 anos: 9,30% e 17 anos: 6,97%.
Segue a baixo a representação:



Entregas finais por grupo 9º E

TABELA

Idades dos alunos do 8º e 9º ano

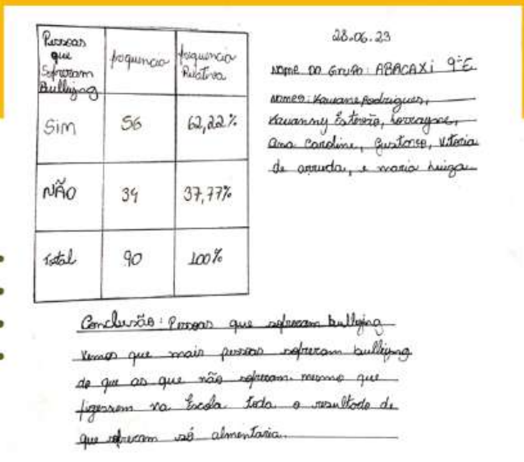
Grupo 1
Júlia Zape
Thaynná
Victoria M.
Alisson
Higor
Diogo

IDADES	Frequencia	Frequencia relativa
12 anos	2	2,12%
13 anos	17	19,76%
14 anos	34	39,53%
15 anos	19	22,09%
16 anos	8	9,30%
17 anos	6	6,97%
total	86	100%

CONCLUSÃO

Observando o gráfico e a tabela, podemos saber a quantidade de alunos que tem de 12 a 17 anos, ao total 86 alunos, avaliados.
A maior quantidade de alunos indicado é de 14anos, tendo 34 alunos e 39,53%. A menor é de 12 anos, tendo 2 alunos e 2,12%

Entregas finais por grupo 9º E

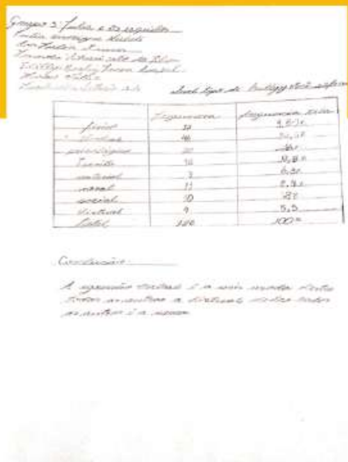


Pessoas que sofreram Bullying	frequência	frequência Relativa
Sim	56	62,22%
NÃO	34	37,77%
total	90	100%

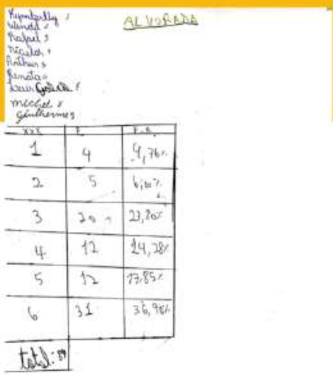
28.06.23

Nome do Grupo: ABACAXI 9ºE

Conclusão: Pessoas que sofreram bullying. Vemos que mais pessoas sofreram bullying do que as que não sofreram. mesmo que fizeram na Escola toda, o resultado de que sofreram só alimentaria.

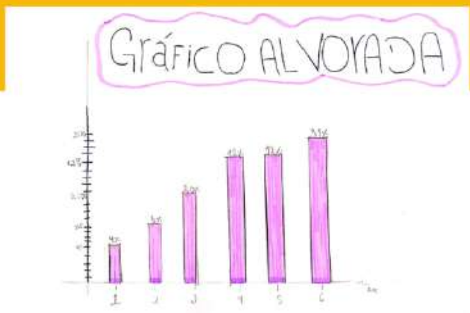


Entregas finais por grupo 8ºD



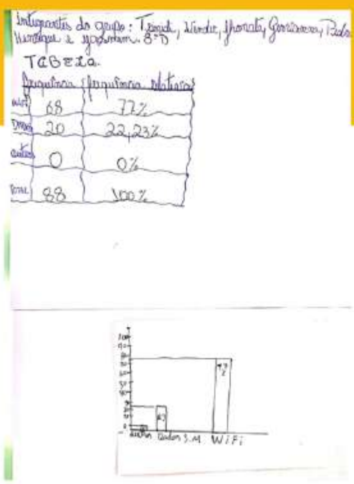
ALVORADA

	F.	F.R.
1	4	4,76%
2	5	6,[illegible]%
3	20	23,80%
4	12	14,28%
5	12	13,85%
6	31	36,98%
total: 84		



Entregas finais por grupo 8ºD

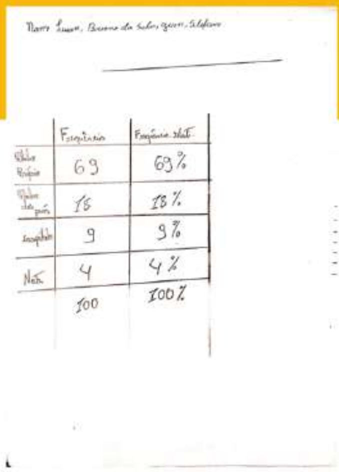
Acesso à Internet



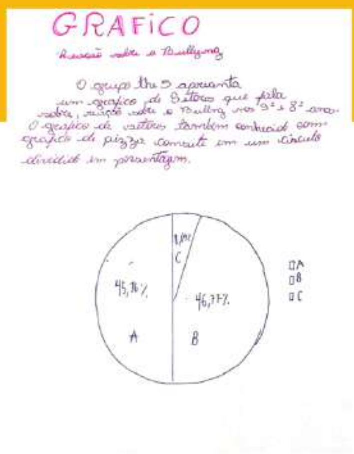
TABELA

	frequência	frequência relativa
wifi	68	77%
Dados	20	22,23%
outros	0	0%
total	88	100%

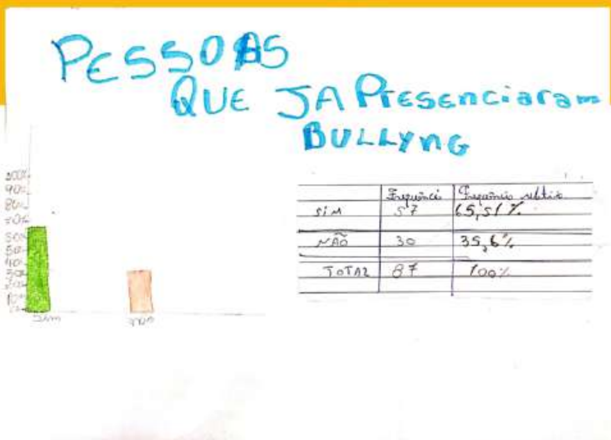
Entregas finais por grupo 8°D



Entregas finais por grupo 9° C

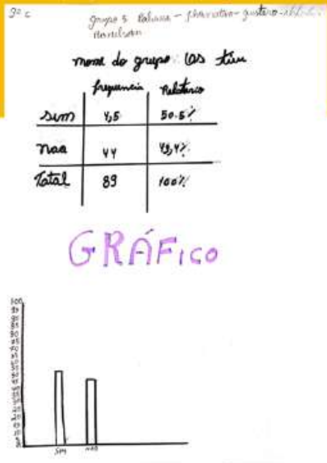


Entregas finais por grupo 9° C

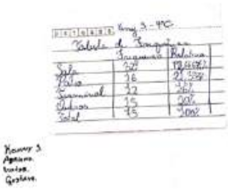


Entregas finais por grupo 9° C

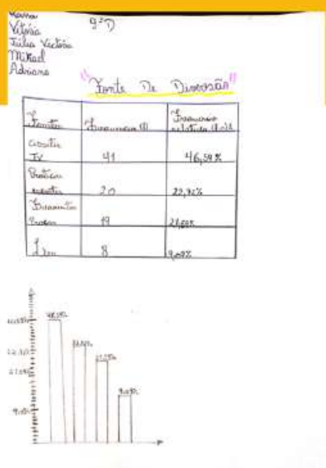
Alunos que já se feriram propositalmente



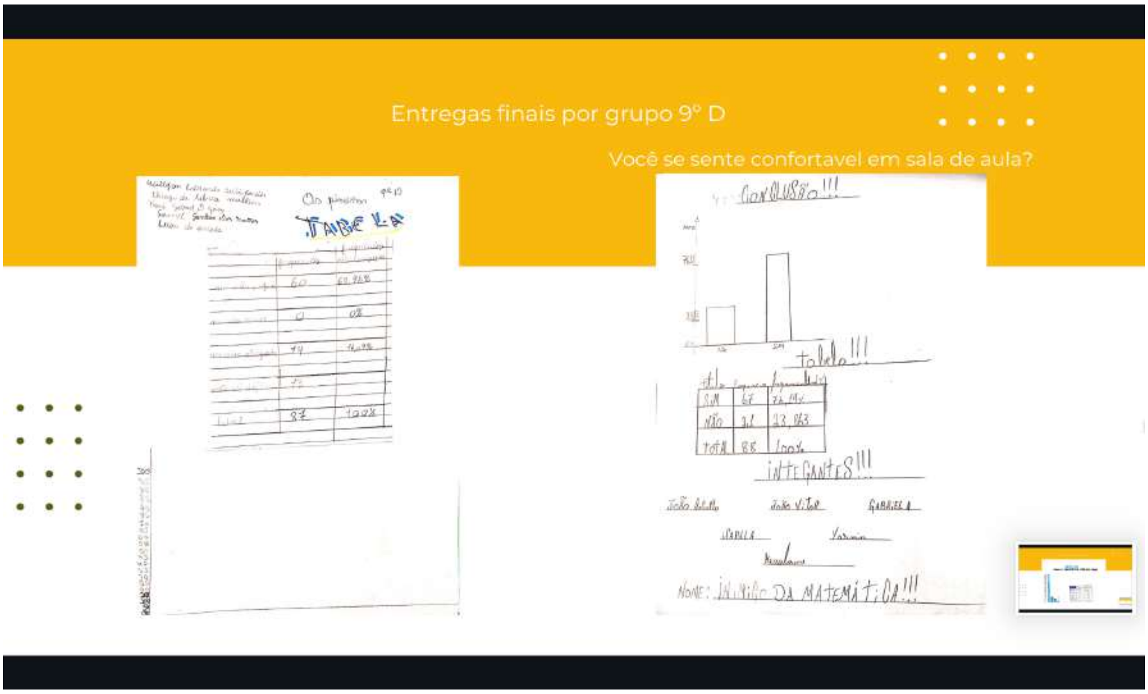
Entregas finais por grupo 9° C

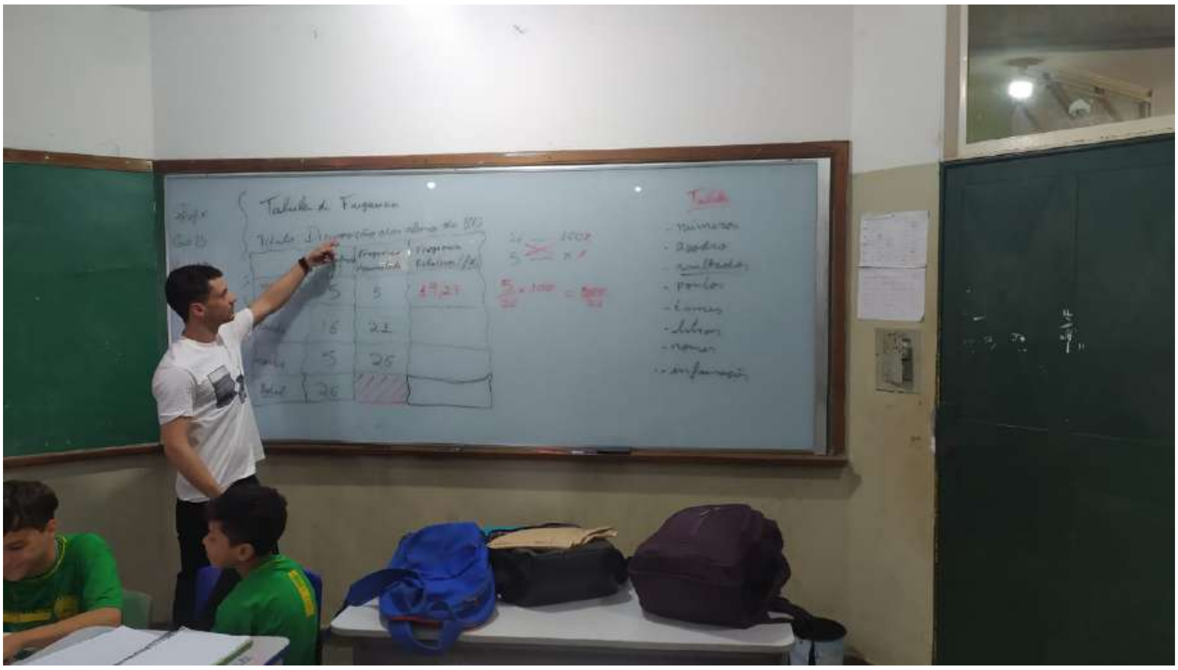


Entregas finais por grupo 9° D




Entregas finais por grupo 9º D
GRÁFICO
Meios de TRANSPORTES PARA IR Á ESCOLA
ÔNIBUS
OUTROS
(F) Frequência
Frequencia Relativa
Ônibus 80 82%
Carro 10 11%
Bicicleta 0 0%
A PÉ 6 6%
OUTROS 1 1%
TOTAL 97 100%


Entregas finais por grupo 9º D
Você se sente confortavel em sala de aula?
TABELA
CONCLUSÃO!!!
Tabela!!!
SIM
NÃO
TOTAL
100%
INTEGRANTES!!!
NOME: INICIÃO DA MATEMÁTICA!!!



Relatório final do Grupo 2

Parte I – PEP

- Questões formuladas Q_i

1. *Como é administrada e gerenciada a distribuição dos recursos hídricos em cada região?*
2. *A distribuição dos recursos hídricos possui alguma interferência política?*
3. *A escassez, em algumas regiões, é reflexo da má gestão do governo?*
4. *Qual o impacto do plano diretor na distribuição dos recursos hídricos na região?*
5. *A falta da Educação Ambiental é um fator a contribuir com a escassez em algumas regiões?*
6. *Em quais bases de dados encontramos sensos atualizados?*
7. *No tempo de pandemia, os recursos hídricos foram suficientes para atender toda a população que ficou em casa?*
8. *A pandemia teve alguma interferência na distribuição hídrica do Brasil?*

- Mídias

Fontes de informação consultadas. Por exemplo, páginas web, documentos etc.

EOS, Organização e sistemas. **Gestão de recursos hídricos no Brasil: Como funciona?** Disponível em: https://www.eosconsultores.com.br/como-funciona- gestao-de-recursos-hidricos-no-brasil/ . Acesso em 31 de abril de 2023.

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OBSERVATÓRIO NACIONAL DOS DIREITOS À ÁGUA E AO SANEAMENTO. **A pandemia de covid-19 e o direito humano à água. Onda, São Paulo, 24 mar. 2020.** Disponível em: https://onda.org.br/a-pandemia-de-covid-19-e-o-direito- humano-a-agua/. Acesso em: 13 de abril de 2023.

RIBEIRO, Lorrany; MACHADO, Isabella. **Música, sustentabilidade e pandemia: entrevista com Chico César.** Mundo Livre, Niterói, v. 13, n. 1, p. 222-231, jan./jun. 2018. Disponível em: https://periodicos.uff.br/mundolivre/article/view/48551/29815. Acesso em: 13 de abril de 2023.

- Respostas preexistentes *R*◊

Identificar as respostas preexistentes encontradas nas mídias, as R◊. Por exemplo, banco de dados etc. Pode ser feito apenas uma descrição do que foi utilizado no estudo.

Questão 1. Como é administrado e gerenciado a distribuição dos recursos hídricos em cada região?

R: O gerenciamento dos recursos hídricos é baseado na Constituição, no Código de Águas, na Lei das Águas e na Lei n.º 9.984, de 17 de Julho de 2000, e a aplicação dessas leis é dada na estrutura que comporta a distribuição de recursos hídricos. Dessa forma, a gestão é composta pelos organismos de âmbito nacional e estadual, os colegiados, a administração direta, o poder outorgante e a entidade da bacia.

Questão 2. A distribuição dos recursos hídricos possui alguma interferência política?
R: Questão inconclusiva.

Questão 3. A escassez em algumas regiões, é reflexo da má gestão do governo?
R: Questão inconclusiva.

Questão 4. Qual o impacto do plano diretor na distribuição dos recursos hídricos na região?

R: Observamos que o plano diretor não tem autonomia gerencial, mas contribui no planejamento urbano e nas questões ambientais da cidade, a fim de proporcionar uma melhor distribuição. Para afirmarmos isso, analisamos as conclusões trazidas por Pizella (2015) no que diz respeito à relação entre Planos Diretores Municipais e Planos de Bacias Hidrográficas na gestão hídrica.

Questão 5. A falta da Educação Ambiental é um fator a contribuir com a escassez em algumas regiões?

R: Em partes. A falta do ensino de Educação Ambiental nas escolas é um fator que afeta negativamente o consumo e preservação de água das regiões, visto que a desinformação pode simplesmente fazer com que as pessoas não pensem (e repensem) sobre o gasto da água (BRK, 2020).

Entretanto, a escassez de água não é causada única e exclusivamente pela população que têm uma má gestão desse recurso. Os que têm uma responsabilidade maior acerca da utilização da água é o agronegócio, visto que é um setor que necessita muito do recurso hídrico (Walbert, 2013).

Questão 6. Em quais bases de dados encontramos sensos atualizados?

R: Em sites como o da Agência Nacional de Águas e Saneamento Básico (ANA), Dados Abertos da ANA, Ministério do Meio Ambiente e Mudança do Clima e outros.

Questão 7. No tempo de pandemia, os recursos hídricos foram suficientes para atender toda a população que ficou em casa?

R: Em tempos "normais", na maior parte do território brasileiro, a garantia da segurança hídrica é vista como um desafio, isso porque o aumento da população urbana, desenvolvimento de setores como indústria e agropecuária, associados aos eventos climáticos extremos, têm pressionado a infraestrutura existente que não dá conta de atender à crescente demanda. E, na pandemia, o cenário foi ainda pior; no Rio de Janeiro, por exemplo, as manchetes encontradas da época foram as seguintes: "As favelas do Rio sofrem com falta d'água e população fica mais vulnerável ao coronavírus." Então, não. Pois não chegava a todas as residências regularmente.

Questão 8. A pandemia teve alguma interferência na distribuição hídrica do Brasil?

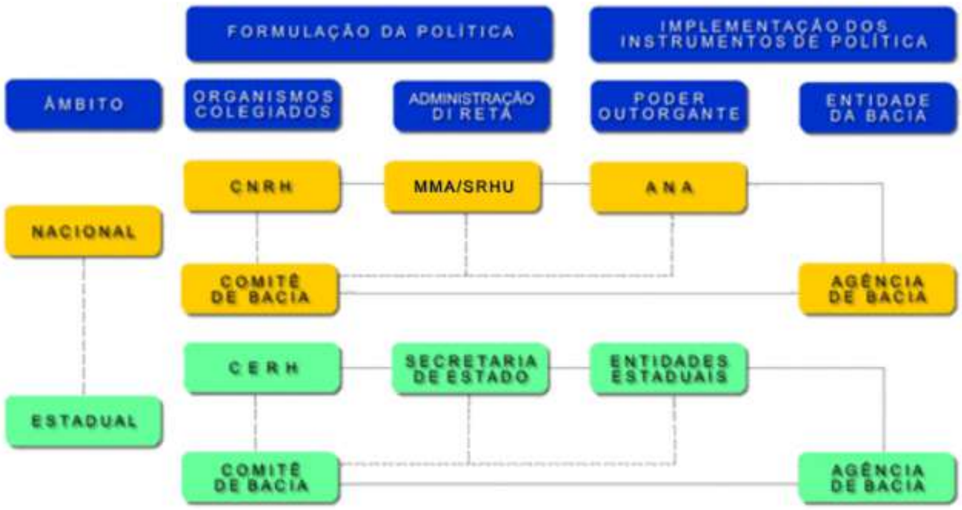
R: A pandemia de Covid-19 não teve uma interferência direta nos recursos hídricos no Brasil, mas a crise sanitária pode ter impactado indiretamente o setor. Como as medidas de distanciamento social e o fechamento de empresas e indústrias reduziram a atividade econômica em algumas áreas, a demanda por água em certos setores também pode ter diminuído.

- **Respostas R_i**

Apresentar as respostas construídas pelo grupo.

Questão 1.

O gerenciamento dos recursos hídricos é baseado na Constituição, no Código de Águas, na Lei das Águas e na Lei n.º 9.984, de 17 de julho de 2000, e a aplicação dessas leis é dada na estrutura que comporta a distribuição de recursos hídricos. Com a figura abaixo podemos observar como é organizada a gestão desses recursos.



De acordo com a figura podemos observar os organismos de âmbito nacional e estadual, os colegiados, a administração direta, o poder outorgante e a entidade da bacia. Assim, temos:

- O Conselho Nacional de Recursos Hídricos (CNRH), instância mais elevada do Singerh, no qual compete analisar propostas, deliberar projetos, arbitrar conflitos e articular para a promoção dos recursos hídricos em todos os âmbitos federais.
- A Secretaria de Recursos Hídricos e Ambiente Urbano (SRHU), órgão do Ministério do Meio Ambiente (MMA), que gerencia os recursos hídricos conforme a estrutura regimental estabelecida pelo Decreto n.º 6.101, de 26 de abril de 2007.
- Agência Nacional das Águas (ANA). Ela é a entidade responsável pela implementação da Política Nacional dos Recursos Hídricos e pelo gerenciamento do Singerh. À ANA compete atuar na elaboração e implementação de planos de recursos hídricos em bacias hidrográficas de domínio federal e oferecer apoio técnico para elaboração desses planos em outras esferas.
- Da mesma forma, acontece no âmbito Estadual.

Questão 2.

De acordo com as pesquisas na internet, não obtivemos uma resposta conclusiva, contudo, já explicitamos como funciona o gerenciamento dos recursos hídricos. Dessa forma, a distribuição é feita pela gestão explicitada na questão 1.

Questão 3.

As respostas são inconclusivas visto que o tema abrange o cenário político amplo. **Obs.**: *Pesquisando no Google por: "Má administração dos recursos hídricos no Brasil"; "A escassez em algumas* *regiões, é reflexo da má gestão do governo?"; "Gestão hídrica e gestão de recursos", temos respostas não tão específicas e desatualizadas.*

Questão 4.

O que é plano diretor?

> O Plano Diretor Estratégico é uma lei municipal que orienta o crescimento e o desenvolvimento urbano de todo o município. Elaborado com a participação da sociedade, é um pacto social que define os instrumentos de planejamento urbano para reorganizar os espaços da cidade e garantir a melhoria da qualidade de vida da população. (*Disponível em: https://planodiretorsp.prefeitura.sp.gov.br/o-que-e-o-plano-diretor/. Acessado em 14.04.2023.)*

De acordo com Pizella (2015), os municípios possuem atribuições legais e de fiscalização das políticas ambientais; porém, no tocante às águas, tal autonomia não é possibilitada. Pizella (2015) afirma que:

> No tocante às águas, entretanto, tal autonomia não é possibilitada, visto que os Planos de Recursos Hídricos, a outorga, cobrança pelo uso dos recursos hídricos e o enquadramento dos cursos d'água são instrumentos da Política Nacional dos Recursos Hídricos a serem implementados pelo SINGRH.

Pizella (2015) ressalta que *"a potencialidade de o Plano Diretor Municipal contribuir para uma gestão hídrica local que seja ambientalmente sustentável e influencie na melhoria progressiva da qualidade e quantidade hídricas no âmbito das bacias hidrográficas."*

- Resposta *R*♥

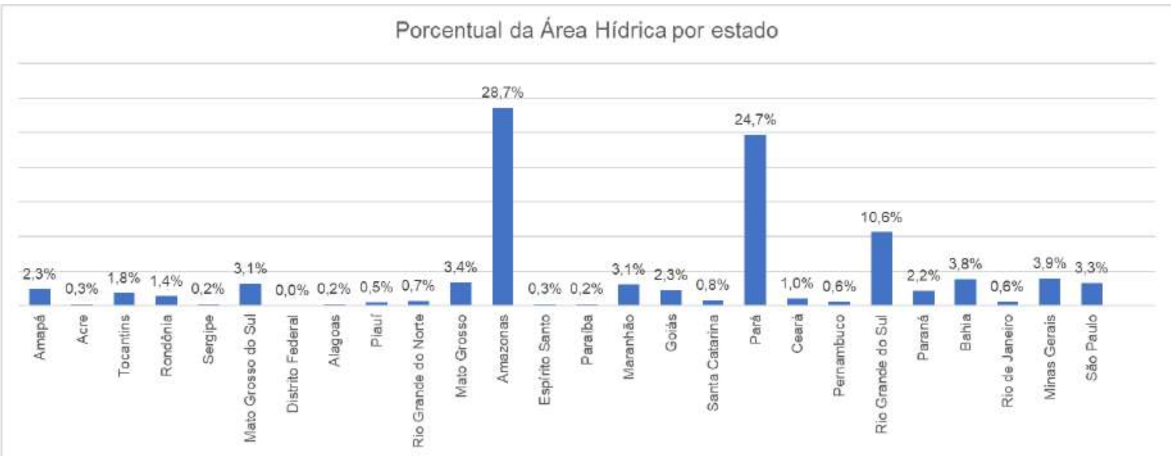
Apresentar a discussão da resposta final para o estudo realizado: análises, tabelas e gráficos construídos etc.

Questão 9. Olhando as distribuições de recursos hídricos e população por estados brasileiros (26 estados + DF), como se comporta esta distribuição segundo dados mais atuais (2022)?

Gráfico 1

Elaborada pelos autores utilizando recursos do Geogebra e base de dados do IBGE. (2023)

Gráfico 2



Elaborada pelos autores utilizando recursos do Geogebra e base de dados do MapBiomas. (2023)

Analisando o gráfico de caixas (Gráfico 1), observamos que os estados Mato Grosso do Sul, Distrito Federal, Alagoas, Piauí, Rio Grande do Norte, Mato Grosso, Amazonas, Espírito Santo, Paraíba, Maranhão, Goiás, Santa Catarina, Pará, Ceará e Pernambuco, compõem 50% da população Brasileira. Pelo gráfico do Percentual da Área Hídrica por Estado (Gráfico 2), vemos que esses estados possuem maior quantidade de água em termos de área, porém em uma distribuição não simétrica, ou seja, claramente vemos que o estado do Amazonas e Pará possuem maior área Hídrica, mas com populações próximos a mediana.

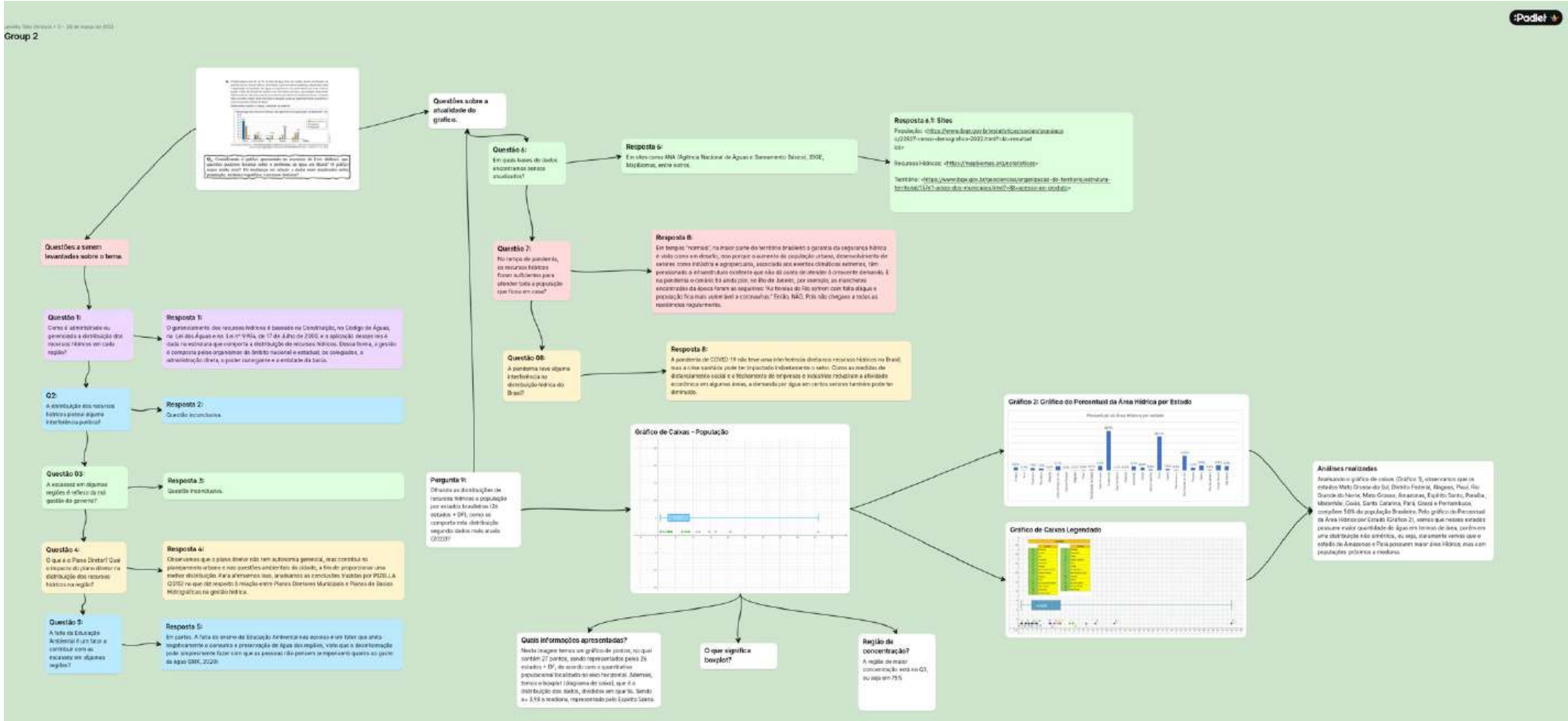
- Questões em aberto

Apresentar as questões que o grupo não conseguiu responder ou as questões que o grupo acredita que poderiam ser estudadas mais profundamente.

9. *No tempo de pandemia, os recursos hídricos foram suficientes para atender toda a população que ficou em casa?*
10. *A pandemia teve alguma interferência na distribuição hídrica do Brasil?*

Youtube: https://www.youtube.com/watch?v=1gfXbdn15ec&ab_channel=TomCooper

Mapa de questões-respostas do Padlet



Parte II – Análise do PEP

<u>Dialética Questões-Respostas:</u> comentem sobre a utilidade desta ferramenta durante o PEP

- **Pontos positivos:** explorou novas áreas, a interdisciplinaridade, explorar outros aplicativos, como o *padlet*.

- **Pontos negativos:** devido à amplitude dos questionamentos, tivemos algumas dispersões.

- **Dificuldades encontradas?**
  A dificuldade surgiu no momento de elaborar as respostas do relatório, pois foi confuso entender o que era para ser feito.

<u>Dialética mídia-meio:</u>

- **Que mídias foram utilizadas durante o PEP?**

  Na elaboração das questões, em determinado momento, utilizamos o nosso próprio grupo como mídia. Os dados foram retirados da internet, por meio de *sites* de busca, *sites* de notícias, *sites* oficiais do governo e artigos relacionados ao tema.

- **Que objetos foram utilizados durante o PEP?**
  Excel, Padlet, Docs, GeoGebra.

- **Que conhecimentos estatísticos foram utilizados durante o PEP?**
  Amostra: populacional, territorial e de recursos hídricos.
  Mediana: utilizamos na análise do gráfico de caixa.
  Porcentagem: na interpretação do gráfico da questão inicial proposta.
  Boxplot e o dotplot: na construção dos gráficos.

- **Que dados foram utilizados durante o PEP?**
  Dados de estimativa populacional, de recursos hídricos, de área territorial, política, ambiental e da saúde.

- **Que estudos poderiam ou precisam ser aprofundados?**
  Educação ambiental e a pegada hídrica.

- **Dificuldades encontradas?**
  Encontrar alguns dados oficiais e confiáveis.

<u>Dialética Indivíduo-Coletivo:</u> **Comentem sobre a organização do grupo durante o PEP**

- **Discussões coletivas?**

Ocorreu na elaboração das questões, de forma produtiva.

- **Divisão de tarefas?**

A elaboração das perguntas foi feita por meio da discussão do grupo; após isso, nos dividimos para encontrar os dados necessários para responder às perguntas e, por fim, devido aos compromissos diários, escolhemos uma forma prática de responder às questões. Dessa forma, dividimos as questões entre o grupo para que cada um fosse responsável por responder, em média, duas questões. Posteriormente, analisamos todas as respostas obtidas.

- **Dificuldades encontradas?**

A dificuldade encontrada foi em responder as perguntas sem uma discussão do grupo, o que poderia ter tornado as respostas mais bem elaboradas.

- **O trabalho em grupo poderia ser melhorado? Em quais aspectos?**

Acreditamos que poderíamos não ter nos dividido para responder às questões individualmente, mas que poderíamos analisar os dados e respondê-las juntos.

Módulo 3 – planejamento de atividade de investigação

Turma:
Escola:

- Questão inicial da atividade de investigação.
- Como se coloca a questão, o que se propõe fazer e que ferramentas estão disponíveis?
- Como o estudo está planejado para ser administrado em classe: horário aproximado
- Como se espera que a atividade seja concluída
- Qual/quais conteúdo(s) curricular(es) (em matemática e outros assuntos) pode(m) ser abordado(s) pela atividade proposta.

UNIVERSIDADE FEDERAL DE MATO GROSSO DO SUL

Residência Pedagógica 2023/01

Escola: Escola Estadual Prof.º Emygdio Campos Widal
Coordenadora: Prof.ª Sonia Maria Monteiro da Silva Burigado
Investigadora: Prof.ª Janielly Taila dos Santos Verbisck
Professor participante: Marilena Bittar
Professor(a) da Escola: xxx
Residentes: yyyy

PLANO DE AULA

1. IDENTIFICAÇÃO

Componente Curricular: Matemática	**Duração de cada aula:** 50 min/Aula
Turma(s): UC- Quanto vale um real.	**Data da aula:** a definir
Quantidade de Aulas: 4 h/a	
Unidade Temática: Matemática Financeira	

2. ÁREA DE CONHECIMENTO

Matemática e suas Tecnologias.

3. CONTEÚDO

Taxa de Câmbio e Paridade do poder de compra através de recursos gráficos e estatísticos.

4. OBJETIVO/HABILIDADES/COMPETÊNCIAS

4.1. ***Objetivo Geral:***	*Propor e desenvolver, juntamente com os estudantes, um projeto investigativo envolvendo coleta e tratamento de dados, com o tema "Taxa de Compra e Paridade do Poder de Compra."*

4.2. *Objetivos específicos:*	*Desenvolver o estudo da taxa de compra através de uma questão inicial: Quanto custa 1 real? Coletar e tratar dados; introduzir os tipos de gráficos (barra, setor, linha e coluna); elaboração de gráficos.*
4.3. *Habilidades BNCC:*	*(MS - EMIFCG01) - Identificar, selecionar, processar e analisar dados, fatos e evidências com curiosidade, atenção, criticidade e ética, inclusive utilizando o apoio de tecnologias digitais.* *(MS - EM13MAT101) Interpretar criticamente situações econômicas, sociais e fatos relativos às Ciências da Natureza que envolvam a variação de grandezas, pela análise dos gráficos das funções representadas e das taxas de variação, com ou sem apoio de tecnologias digitais.* *(MS - EM13MAT102) Analisar tabelas, gráficos e amostras de pesquisas estatísticas apresentadas em relatórios divulgados por diferentes meios de comunicação, identificando, quando for o caso, inadequações que possam induzir a erros de interpretação, como escalas e amostras não apropriadas.*
4.4. *Competências*	*Competências Gerais BNCC: 2 e 10.* *Competências Específicas: 1, 2, 3, 4 e 5.*

5. DURAÇÃO

4 aulas com duração de 50 minutos.

6. RECURSOS DIDÁTICOS

Datashow, lousa, giz/ canetão, apagador, celular, *notebook,* caderno, lápis, borracha, caneta, cartolina, régua, compasso, canetinha, lápis de cor.

7. ENCAMINHAMENTOS METODOLÓGICOS

7.1 Ações didáticas: O presente Plano de Aula tem a finalidade de esquematizar as etapas das atividades que serão desenvolvidas dentro de sala. Ele orientará a sequência didática que será aplicada, com o objetivo de proporcionar um ambiente de aprendizagem escolar. Assim, a sequência metodológica será estruturada baseada no planejamento do professor preceptor ou regente da turma, para maior organização e sequenciamento do conteúdo que vem sendo abordado em sala.

1º Encontro: ***Em um primeiro momento,*** será realizada uma rápida apresentação pessoal dos professores residentes; logo em seguida, será apresentada a proposta de atividade da turma. ***Em um segundo momento,*** será realizada uma rápida discussão de quanto vale 1 real? Como podemos mensurar? O que influencia no preço das coisas? O que podemos comprar com 1 real? ***Em um terceiro momento***, iremos idealizar uma dinâmica com os alunos através da ferramenta *online* "Mentimeter". A proposta é solicitar que cada aluno pegue o seu celular (os que não tiverem o celular, fará a atividade com algum colega), onde, através do *link* que iremos disponibilizar no data show, responderão às perguntas: 1 - Relacione o conceito de "Estatística" com três palavras; 2 - Você sabe o que é um gráfico de Coluna? 3 - Você sabe o que é um gráfico de setor? 3 - Você sabe o que é um gráfico de linhas? 4 - Você sabe o que é um gráfico de Barras?

Esse questionário tem o objetivo de fomentar a curiosidade e serve para termos uma estimativa numérica de quantos alunos sabem os conceitos mencionados acima. Logo em seguida, continuaremos o questionário, porém mais voltado ao tema da aula (Quanto vale 1 real?). Os alunos serão convidados a responderem às seguintes perguntas: 5 - O que é inflação? 6 - Quais fatores você acha que influenciam no valor da nossa moeda (do Real)? 7 - Cite três moedas que você conhece. A proposta aqui, é idealizar um ambiente dinâmico e prático, no qual eles iriam interagindo conosco de forma virtual.

Em um quarto momento, iremos realizar a explicação no quadro de como se calcula a taxa de câmbio real, através da expressão abaixo:

$$E = e \cdot \frac{p_1}{p_2}$$

$Sendo:$
$E: taxa\ de\ câmbio\ real;$
$e: Taxa\ de\ câmbio\ nominal$
$p_1: valor\ do\ produto\ no\ país\ 1$
$p_2: valor\ do\ produto\ no\ país\ 2$

Essa dinâmica segue com 1 questão exemplificadora que será resolvida no quadro, e 3 questões que a turma irá desenvolver:

Exemplo 1 - Um produto custa no Brasil R$28,00 e nos Estados Unidos US$7,00. A taxa de câmbio do dia no mercado é de R$4,93 (05/06/23). Qual a taxa de câmbio real deste produto? É possível calcular a valorização ou desvalorização do Real em relação ao dólar para este produto?
Atividade 1 - Um tênis esportivo da Nike custa R$370,00, enquanto na França é 85,00€. A taxa de câmbio nominal do euro é R$5,28 (05/06/23). Qual a taxa de câmbio real desse produto? A nossa moeda valorizou ou desvalorizou?
Atividade 2 - Um vestido no Brasil custa R$210,00, enquanto no Japão custa 4200,00¥ (Iene). A taxa de câmbio de mercado do Iene em 05/06/23 é de R$0,035. Qual a taxa de câmbio real desse produto? A nossa moeda valorizou ou desvalorizou?
Atividade 3 - Uma refeição do McDonalds custa em média aqui no Brasil R$32,50, enquanto no Egito custa 120,00LE. A taxa de câmbio de mercado da Libra Egípcia em 05/06/23 é de R$0,16. Qual a

taxa de câmbio real deste produto? A nossa moeda valorizou ou desvalorizou?

Em um quinto momento, será realizada uma divisão dos alunos em 5 grupos. Aqui o objetivo é que cada grupo fique responsável por um continente para que possa ser dada a continuidade a proposta de atividade.

2º Encontro: ***Em um primeiro momento,*** faremos o Contato com a sala, chamada, organização dos alunos para o início da aula. ***Em um segundo momento,*** faremos a explicação sobre alguns tipos de gráficos utilizados na estatística, bem como sobre elementos importantes para a confecção, manipulação e interpretação de dados estatísticos presentes nos gráficos. ***Em um terceiro momento,*** retornaremos à formação dos grupos realizada no primeiro encontro, bem como dividindo-os em continentes. Realizada essa divisão, solicitaremos que escolham três países (com moedas diferentes se possível). Após as escolhas, faremos uma votação na sala para escolherem um tipo de produto que gostariam de estudar. A proposta aqui é que eles realizem uma pesquisa durante a aula, orçando o valor do produto escolhido, nos respectivos países escolhidos. O objetivo, é calcular a valorização da moeda em relação ao dólar e transcrever as informações em um gráfico. Os produtos que iremos propor são: refeição MCDonald, Coca-Cola, banana, ovos, tomate, batata, cebola, passagem de transporte público, gasolina, internet, cinema.

Em um quarto momento, os alunos terão o tempo para realizar a pesquisa, os cálculos e plotar os gráficos.

7.2 Programação:

DIA	PROCEDIMENTOS EM SALA
1º Encontro **Data:** a definir **Horário Tempo:** 2 tempos de 50 minutos cada.	**Primeiro momento (5~10 min):** *- Contato com a sala, chamada, organização dos alunos para o início da aula e apresentação de proposta de atividade.* **Segundo momento (10~15 min):** *- Discussão sobre a questão problema: Quanto vale 1 real? participação da turma.* **Terceiro momento (15~20 min):** *- Separação dos alunos em 4 a 5 grupos.* *- Atividade Mentimeter através do questionamento: Como o valor da nossa moeda é calculado? participação da turma.* *- Exposição das respostas no data show;* **Quarto momento (15~20 min):** *- Discussão sobre o tópico "Paridade do poder de Compra e Taxa de Câmbio"; data show + participação da turma.* *- Entrega de uma questão problema por grupo; participação da turma.* **Quinto momento (15~20 min):** *- Início ao levantamento de dados da taxa de câmbio de diferentes moedas; participação da turma.*

2º Encontro **Data:** a definir **Horário Tempo:** 2 tempos de 50 minutos cada.	**Primeiro momento (5~10 min):** - *Contato com a sala, chamada, organização dos grupos.* **Segundo momento (10~15 min): Se necessário** - *Término da coleta de dados; participação da turma.* **Terceiro momento (20~30 min):** - *Explicação teórica sobre os tipos de gráficos (barra, setor, linha e coluna);* **Quarto momento (15~20 min):** - *Plotagem dos gráficos; participação da turma.* - *Discussão dos resultados.*

REPLANEJADO

DIA	***PROCEDIMENTOS EM SALA***
1º Encontro ***Data:** a definir* ***Horário Tempo:*** *2 tempos de 50 minutos cada.*	***Primeiro momento (5~10 min):*** *- Contato com a sala, chamada, organização dos alunos para o início da aula e apresentação de proposta de atividade.* ***Segundo momento (10~15 min):*** *- Discussão sobre a questão problema: Quanto vale 1 real? Participação da turma.* ***Terceiro momento (15~20 min):*** *- Atividade Mentimeter através do questionamento: Como o valor da nossa moeda é calculada? Participação da turma.* *- Exposição das respostas no data show;* ***Quarto momento (15~20 min):*** *- Discussão sobre o tópico "Paridade do poder de Compra e Taxa de Câmbio"; data show + participação da turma.* *- Realização dos exercícios; participação da turma.* ***Quinto momento (15~20 min):*** *- Divisão dos grupos; participação da turma.*
2º Encontro ***Data:** a definir* ***Horário Tempo:*** *2 tempos de 50 minutos cada.*	***Primeiro momento (5~10 min):*** *- Contato com a sala, chamada, organização dos grupos.* ***Segundo momento (10~15 min): Se necessário*** *- Explicação teórica sobre gráficos; participação da turma.* ***Terceiro momento (10~15 min):*** *- Retomar a divisão dos grupos por continente; escolha dos países; Escolha do produto. Participação da turma.* ***Quarto momento (15~20 min):*** *- Cálculos de valorização e desvalorização; participação da turma.* *- Plotagem dos gráficos; participação da turma.* *- Discussão dos resultados. participação da turma.*

6. AVALIAÇÃO

A avaliação será processual, isto é, ela ocorrerá durante todo o processo de aprendizagem, levando em conta o entendimento do aluno sobre estatística, no sentido de tabulação dos dados, fomentação de questões, etc.

7. BIBLIOGRAFIA

BRASIL. Ministério da Educação. ***Base Nacional Comum Curricular.*** *Brasília, 2018.*

Rascunho para atividade

1 - Quanto vale 1 real?
"Você já parou para pensar quanto o real brasileiro vale em relação a outras moedas do mundo?"

1. América do Norte:
    - Dólar dos Estados Unidos (USD)
    - Dólar canadense (CAD)
    - Peso mexicano (MXN)
2. América do Sul:
    - Real brasileiro (BRL) - Guarani
    - Peso argentino (ARS)
    - Peso colombiano (COP)
3. Europa:
    - Euro (EUR)
    - Libra Esterlina (GBP)
    - Franco suíço (CHF)
4. África:
    - Rand sul-africano (ZAR)
    - Naira nigeriana (NGN)
    - Dólar de Zimbabwe (ZWL)
5. Ásia:
    - Iene japonês (JPY)
    - Yuan chinês (CNY)
    - Rúpia indiana (INR)
6. Oceania:
    - Dólar australiano (AUD)
    - Dólar neozelandês (NZD)
    - Kina Papua-Nova Guiné (PGK)

- Contato com a sala, chamada, organização dos alunos para o início da aula e apresentação de proposta de atividade.
- Discussão sobre a questão problema: Quanto vale 1 real?
- Atividade Mentimeter através do questionamento:

Módulo IV – análises finais

Ecologia: condições e restrições

Uma última tarefa para casa...

1) Reunam-se em seus respectivos grupos;

2) Discutam e respondam a questão: a atividade que nós implementamos na escola foi, de fato, um PEP? Justifiquem.

3) Identifiquem condições (favoráveis) e restrições que impediram a implementação de percursos de estudo e pesquisa nas aulas que vocês ministraram. Pode ser qualquer tipo de aspecto: desde o número de alunos por turma, disponibilidade de recursos tecnológicos, motivação dos estudantes...

4) Tentem classificar e ver em que nível de codeterminação didática elas aparecem.

5) Preparem uma apresentação para a próxima sessão.

PERGUNTAS

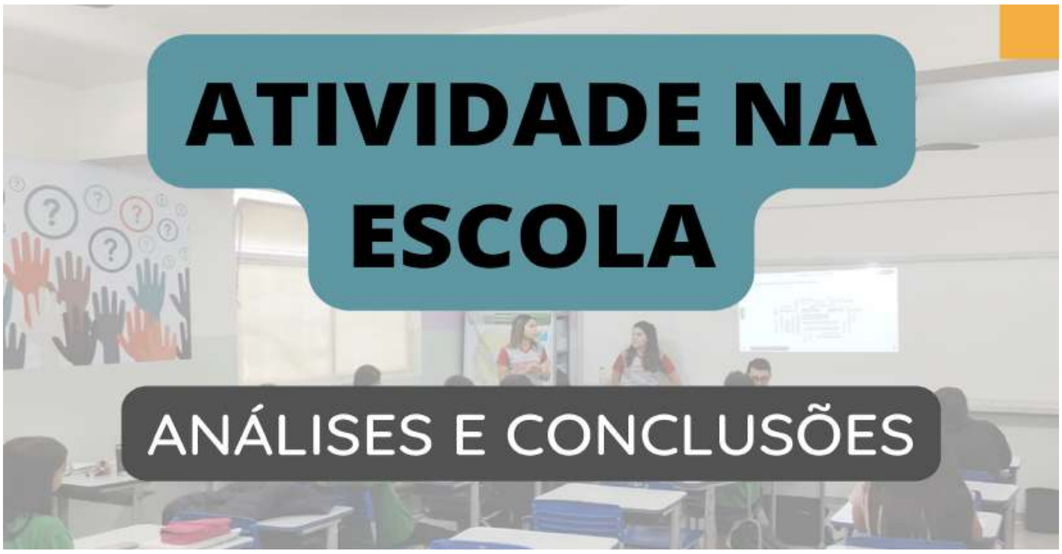
1. **A atividade que nós implementamos na escola foi, de fato, um PEP? Justifique:** Faremos diferente, iremos primeiro analisar alguns encaminhamentos e depois responderemos se foi ou não um PEP.
Durante o processo de elaboração da atividade, tivemos que nos alinhar com o professor da escola para realizar uma proposta de atividade com a turma. Um desafio inicial, foi adaptar o trabalho de estatística com o tema da aula "Quanto vale 1 real?" Tivemos que analisar o plano de aula do professor, **estudar** uma melhor estratégia de forma a atender o planejamento e a proposta do projeto. Após realizar escolhas e analisarmos as possibilidades, conseguimos alinhar as ideias. Feito isso, tivemos que **elaborar** a dinâmica da apresentação, **implementar** as ideias com o auxílio de ferramentas tecnológicas, estudar como iríamos realizar as abordagens, **estudar** o que era taxa de câmbio, **estudar** como poderíamos fazer com que os alunos aprendessem. O tópico não tem ideias intuitivas, pois lidar com taxa de câmbio, por vezes é confuso. Então, precisamos nos **"desconfundir"** primeiramente, para não os confundir. Nos adaptamos, estudamos, nos atualizamos, e dessa forma, aplicamos as atividades. Deu certo e a professora da turma gostou.
Dessa forma, sim! A atividade foi de fato uma PEP.
2. **Identifiquem condições (favoráveis) e restrições que impediram a implementação de percursos de estudo e pesquisa nas aulas que vocês ministraram. Pode ser qualquer tipo de aspecto: desde o número de alunos por turma, disponibilidade de recursos tecnológicos, motivação dos estudantes...**
Favoráveis:
    1. Todos os alunos tinham celular e a maioria tinha acesso à internet;

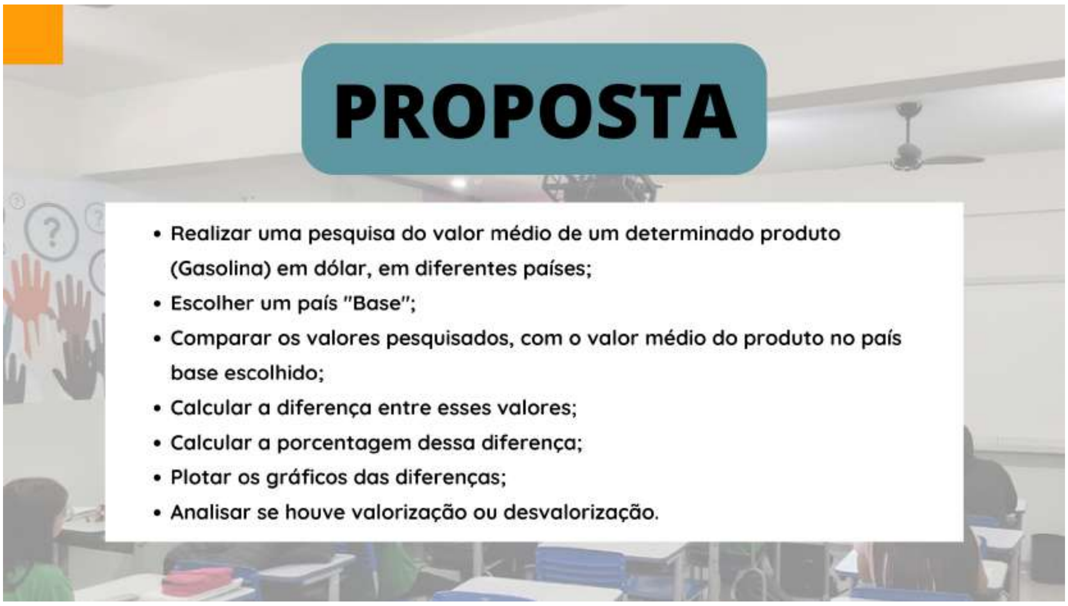
2. A sala de aula tinha data show;
3. A professora da turma é muito flexível e extremamente simpática, dando-nos liberdade e autonomia para fazer o que quiséssemos desde que estivesse no tema da aula.
4. A escola possui um padrão de qualidade diferente de outras escolas, uma vez que os alunos têm mais empenho.

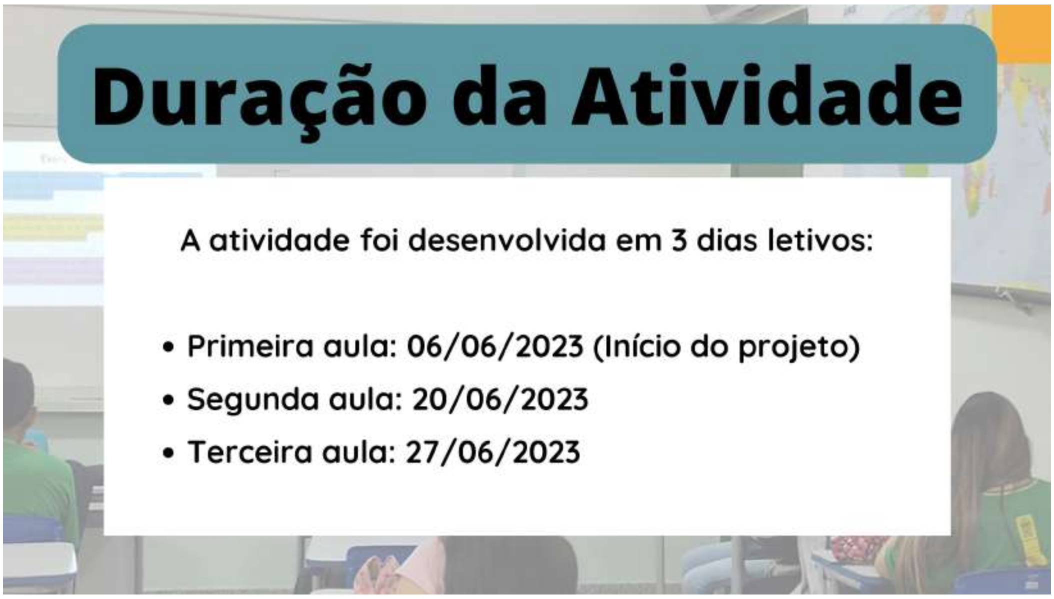
Restrições:

1. Rigidez do plano de aula da disciplina.
2. Sala de tecnologia não disponível nos dias que fizemos as atividades.
3. Falta de habilidade dos alunos, com cálculos matemáticos básicos.
4. Dificuldade em manusear a calculadora.
5. Dificuldade de diferenciar ponto e vírgula.
6. Complexidade do conteúdo.
7. O produto que os alunos escolheram é comercializado em diferentes unidades de medida (Galão e litro).
8. Informações não confiáveis na internet; resultados não válidos.
9. Engajamento dos alunos (alunos desmotivados, alunos dormindo).
10. Tempo das aulas, que percebemos que foi pouco.
11. Feriados no meio do percurso.
12. Interrupções de alguns professores na aula.

Slides – Apresentação 30/06/2023

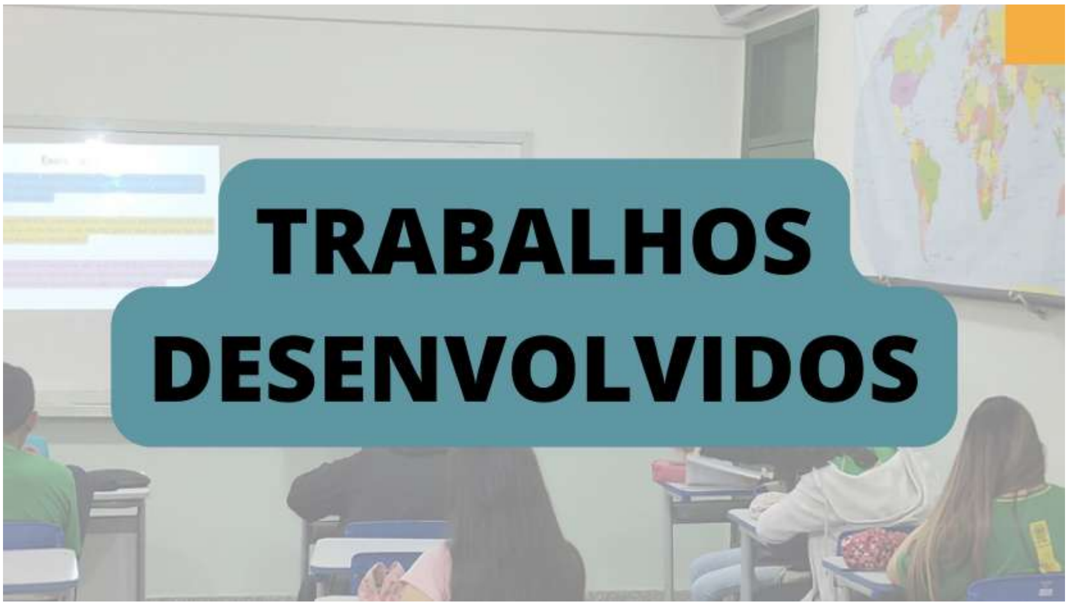


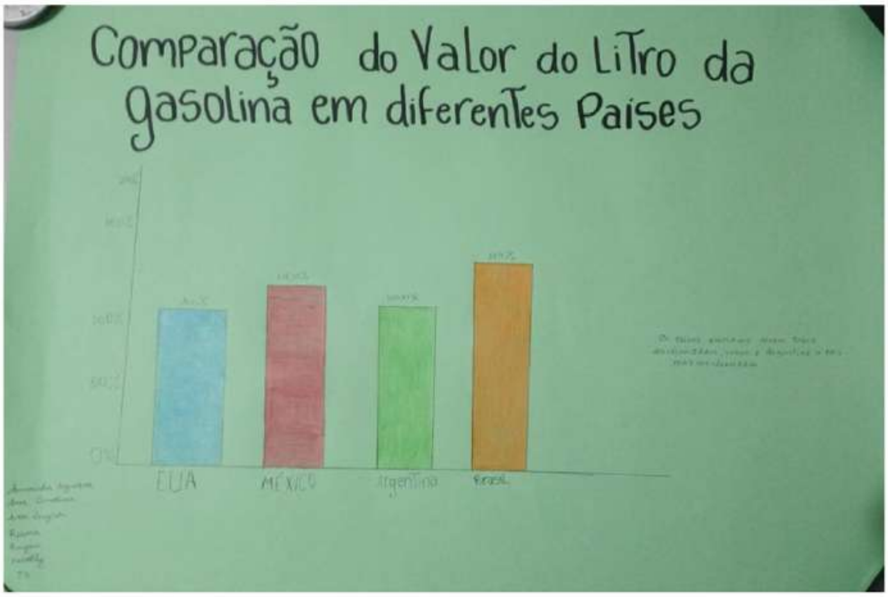

PROPOSTA
• Realizar uma pesquisa do valor médio de um determinado produto (Gasolina) em dólar, em diferentes países;
• Escolher um país "Base";
• Comparar os valores pesquisados, com o valor médio do produto no país base escolhido;
• Calcular a diferença entre esses valores;
• Calcular a porcentagem dessa diferença;
• Plotar os gráficos das diferenças;
• Analisar se houve valorização ou desvalorização.


Duração da Atividade
A atividade foi desenvolvida em 3 dias letivos:
• Primeira aula: 06/06/2023 (Início do projeto)
• Segunda aula: 20/06/2023
• Terceira aula: 27/06/2023


FOTOS


Beautiful Teacher


TRABALHOS
DESENVOLVIDOS


Comparação do Valor do Litro da gasolina em diferentes Paises
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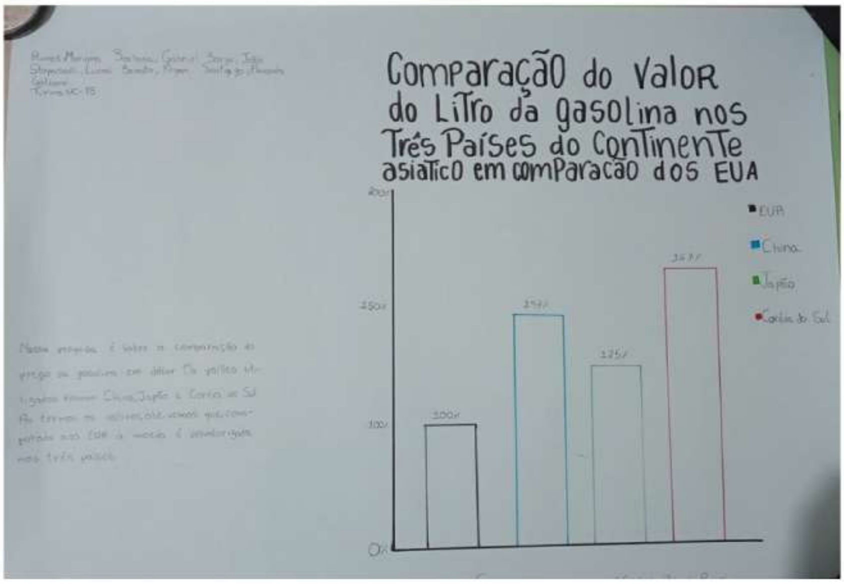
Comparação do Valor do Litro da gasolina nos Três Países do Continente asiatico em comparação dos EUA
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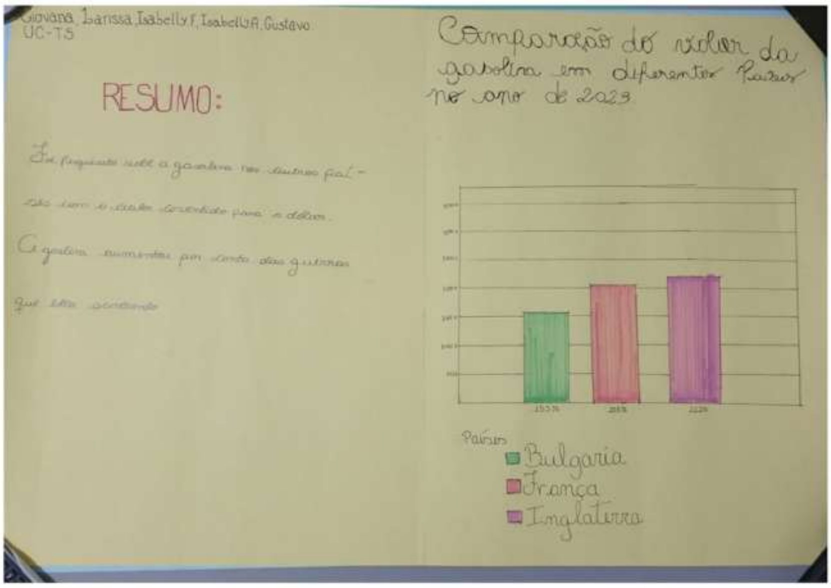
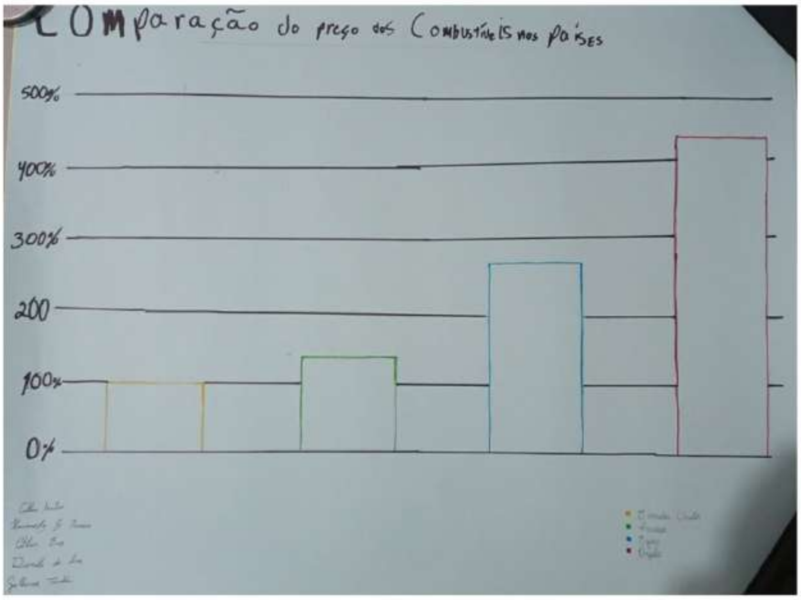

Giovana, Larissa, Isabelly F, Isabella A, Gustavo
UC-T5
RESUMO:
Comparação do valor da gasolina em diferentes Países no ano de 2023
Países
Bulgaria
França
Inglaterra


COMparação do preço dos Combustíveis nos países
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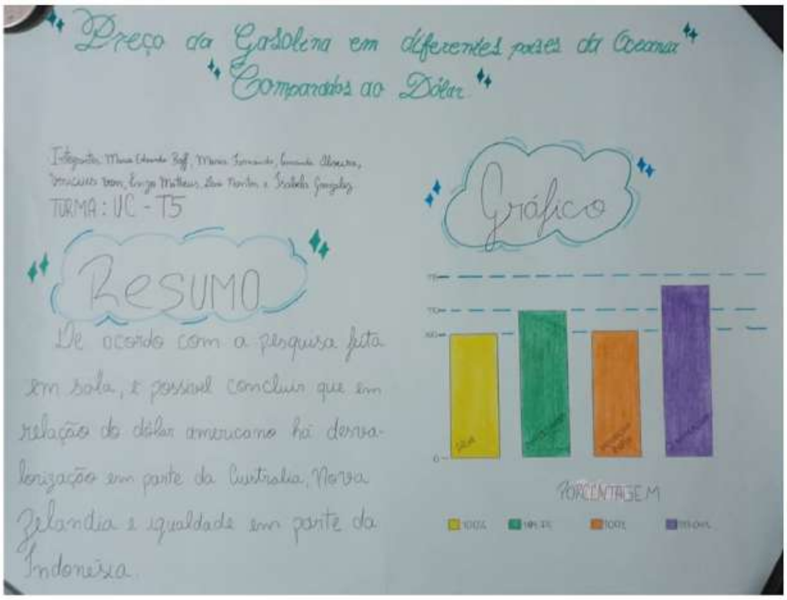
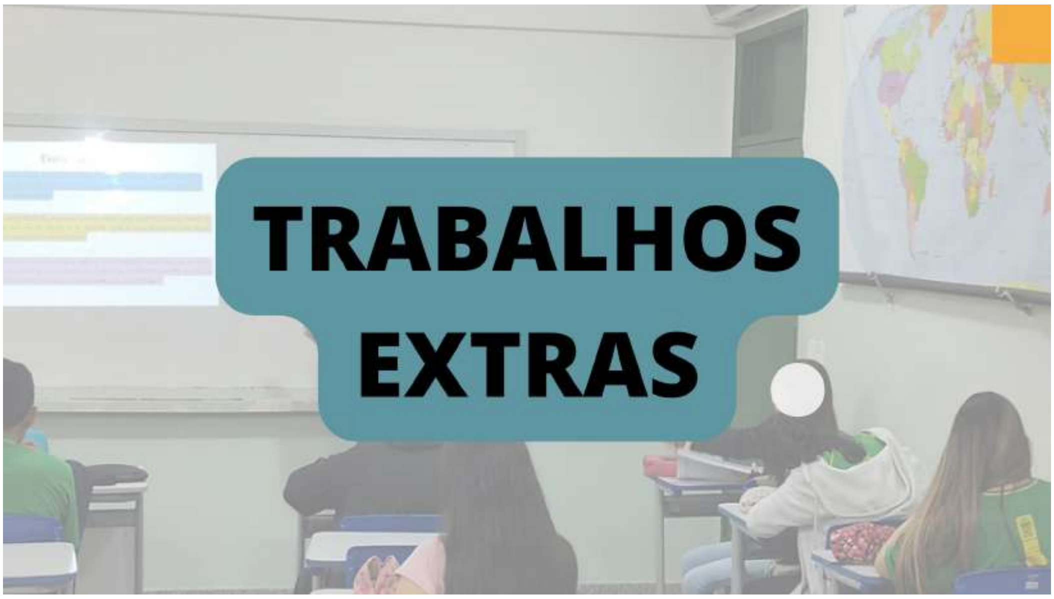
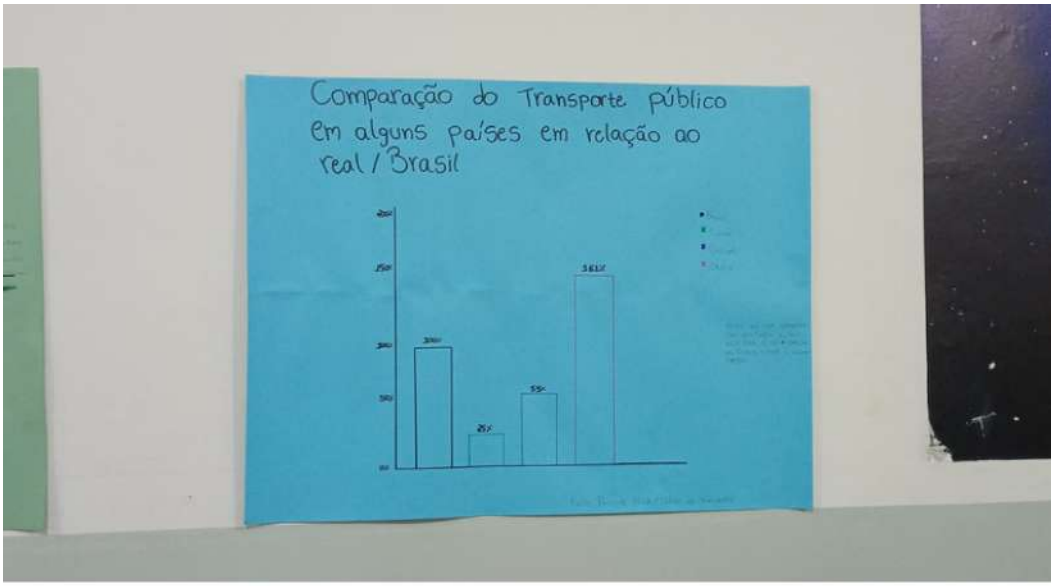
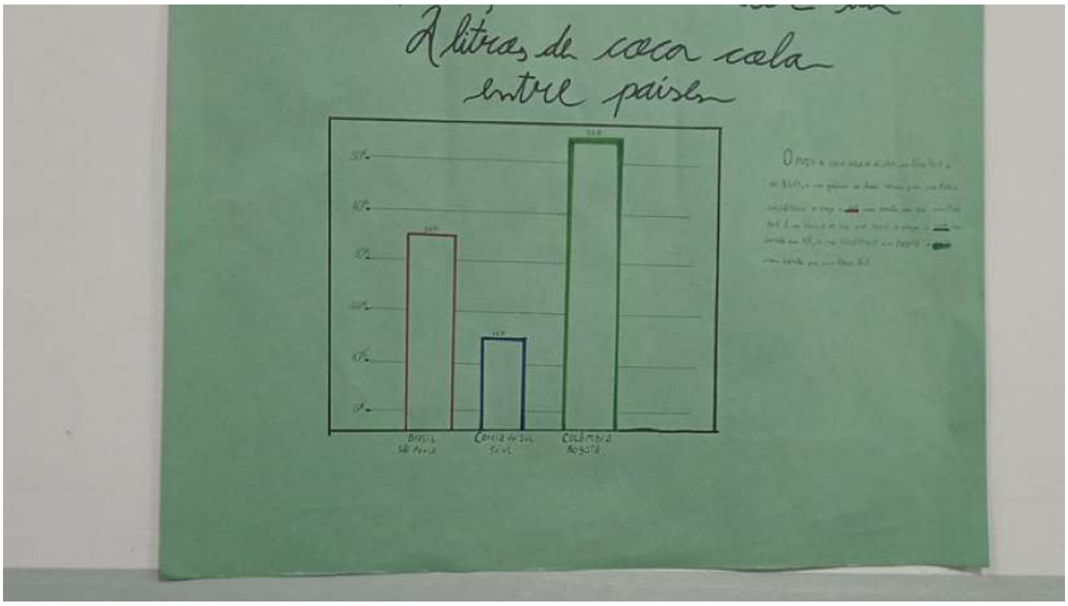
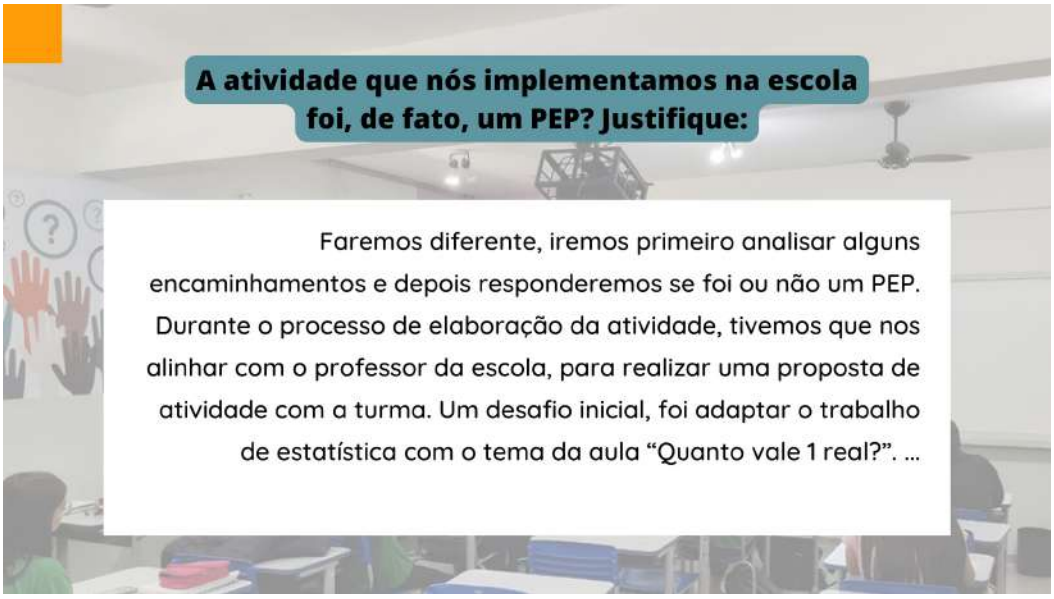

Preço da Gasolina em diferentes países da Oceania Comparados ao Dólar
TURMA: UC - T5
Resumo
De acordo com a pesquisa feita em sala, é possível concluir que em relação do dólar americano há desvalorização em parte da Australia, Nova Zelandia e igualdade em parte da Indonesia.
Gráfico
PORCENTAGEM


TRABALHOS
EXTRAS


Comparação do Transporte público em alguns países em relação ao real / Brasil


2 litros de coca cola entre países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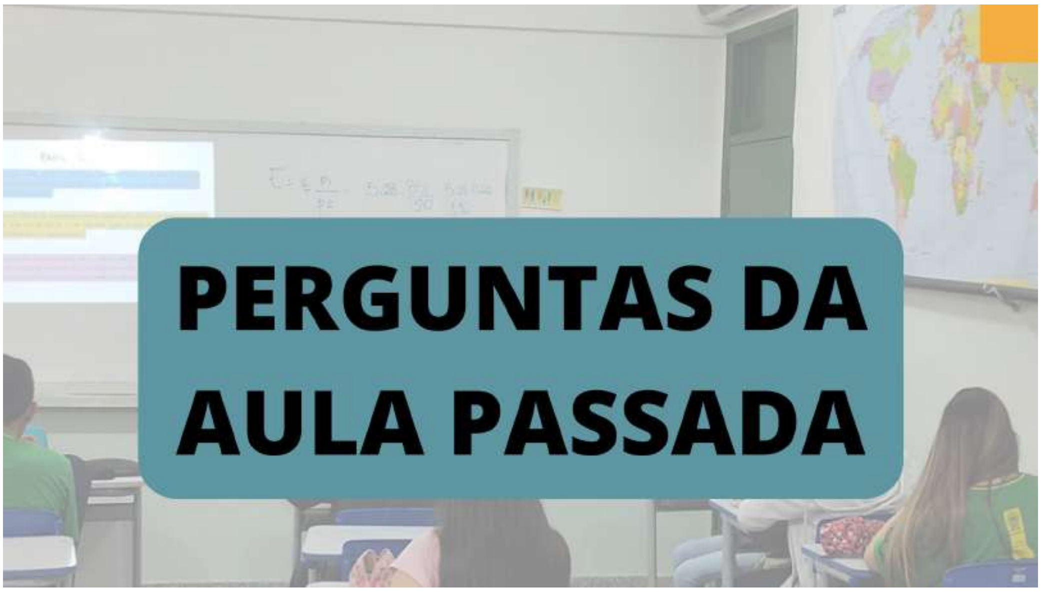
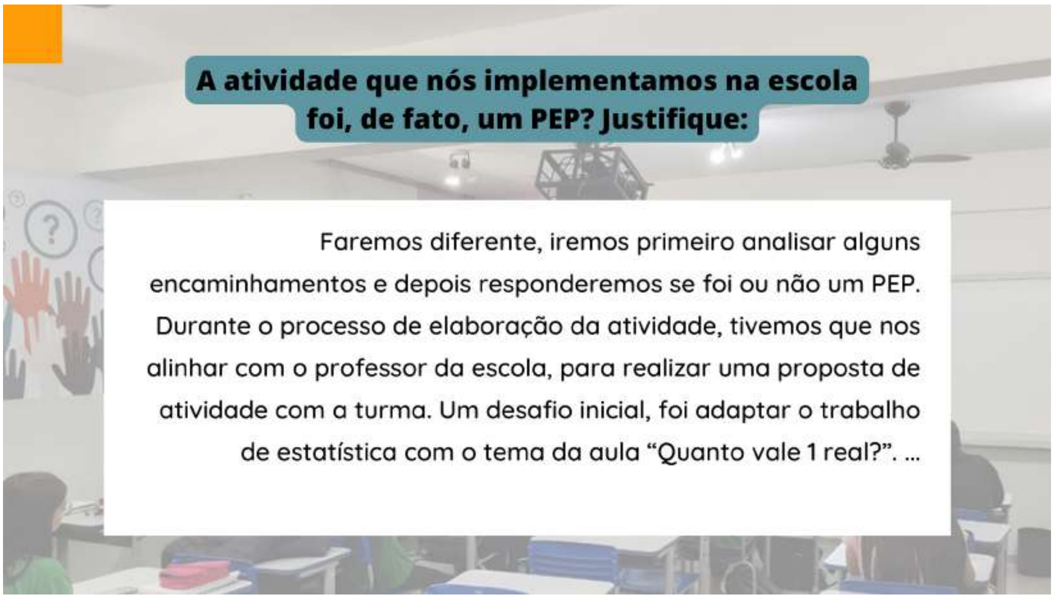

PERGUNTAS DA AULA PASSADA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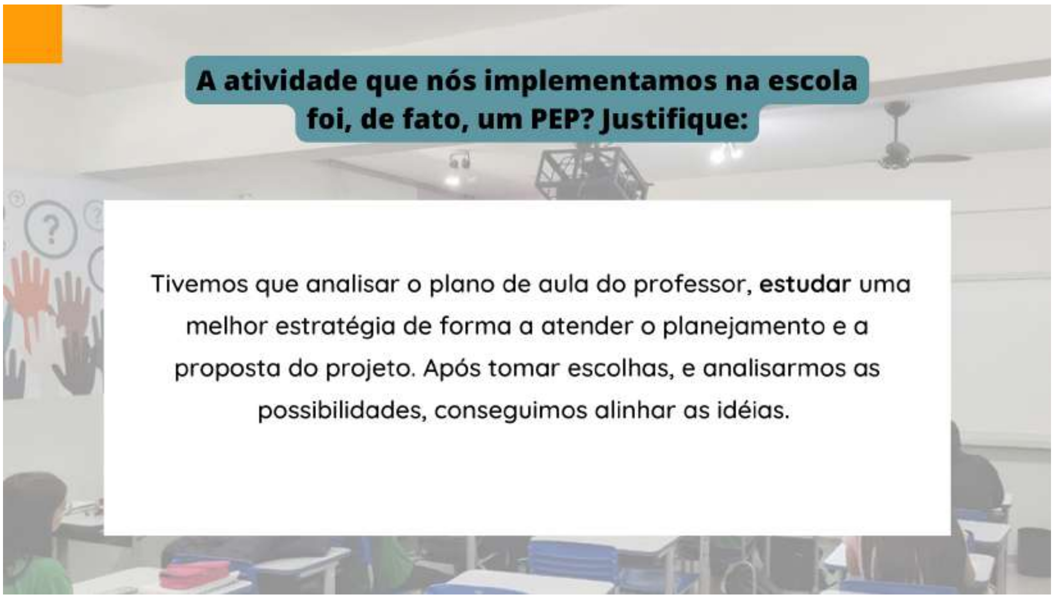
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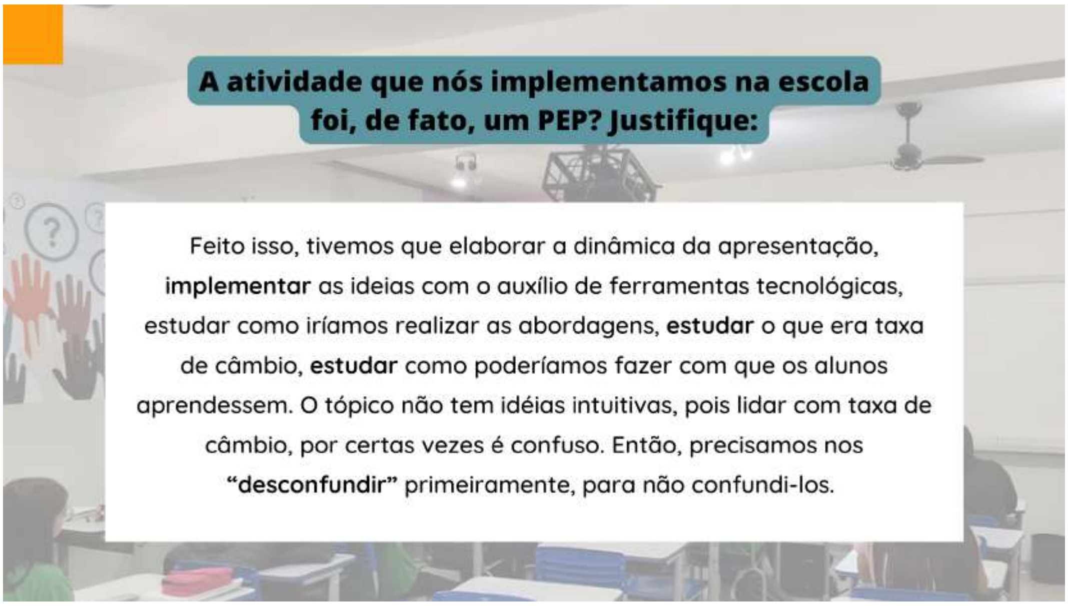
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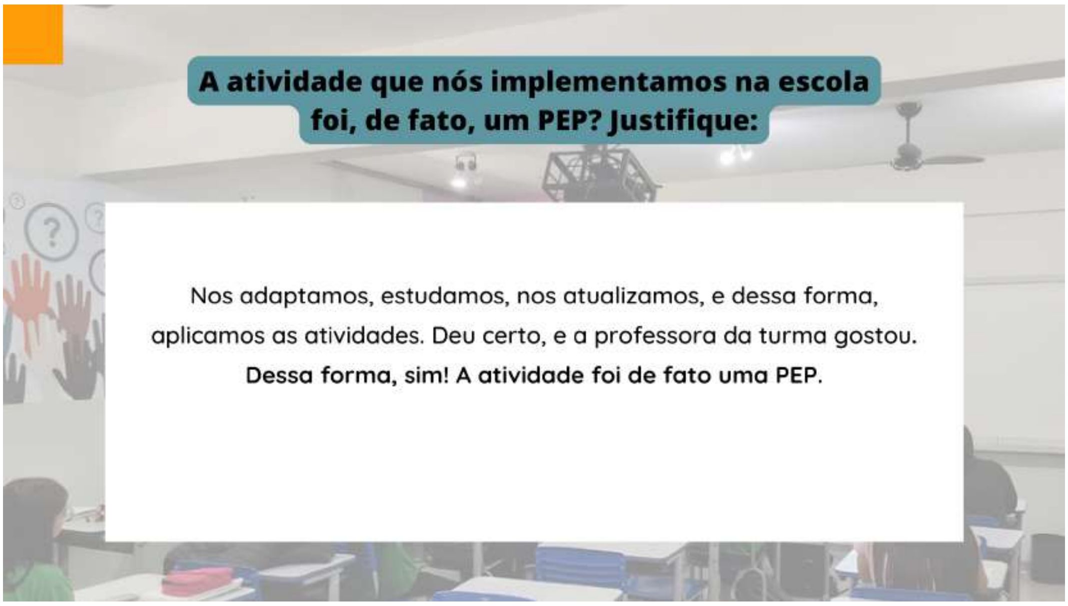
Feito isso, tivemos que elaborar a dinâmica da apresentação, **implementar** as ideias com o auxílio de ferramentas tecnológicas, estudar como iríamos realizar as abordagens, **estudar** o que era taxa de câmbio, **estudar** como poderíamos fazer com que os alunos aprendessem. O tópico não tem idéias intuitivas, pois lidar com taxa de câmbio, por certas vezes é confuso. Então, precisamos nos **"desconfundir"** primeiramente, para não confundi-los.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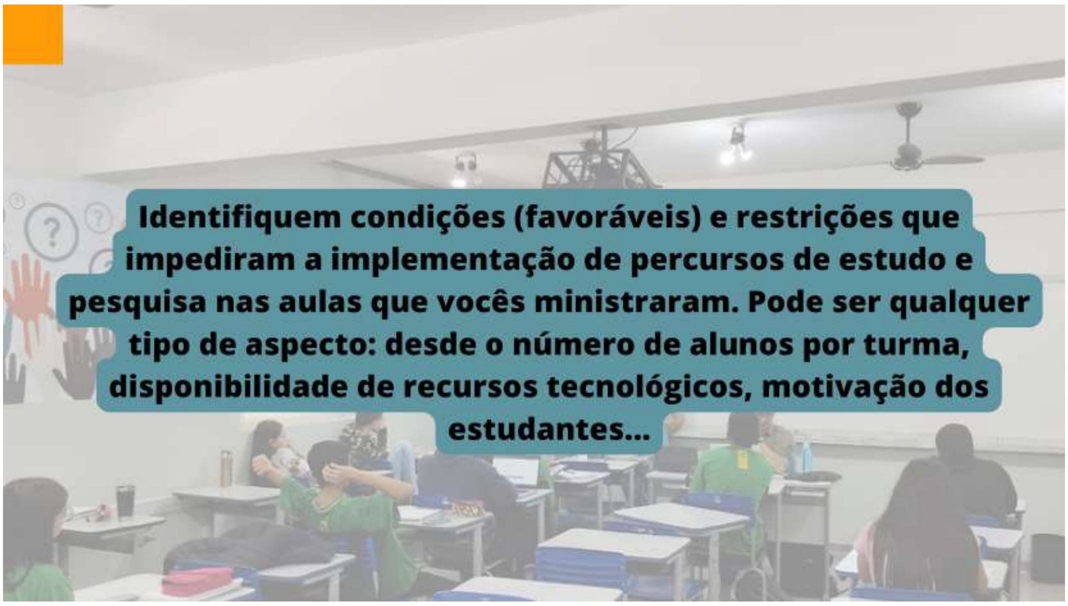


FAVORÁVEIS

Condição Favorável	Classificação
Todos os alunos tinham celular e a amaioria tinha acesso a internet;	Sociedade.
A sala de aula tinha data show;	Escolas.
A professora da turma é muito flexível e extremamente simpática, dando-nos liberdade e autonomia para fazer o que quisesse desde que estivesse no tema da aula.	Escola, pedagogia, disciplina.
A escola possui um padrão de qualidade diferente de outras escolas, uma vez que os alunos possuem mais empenho.	Escola.

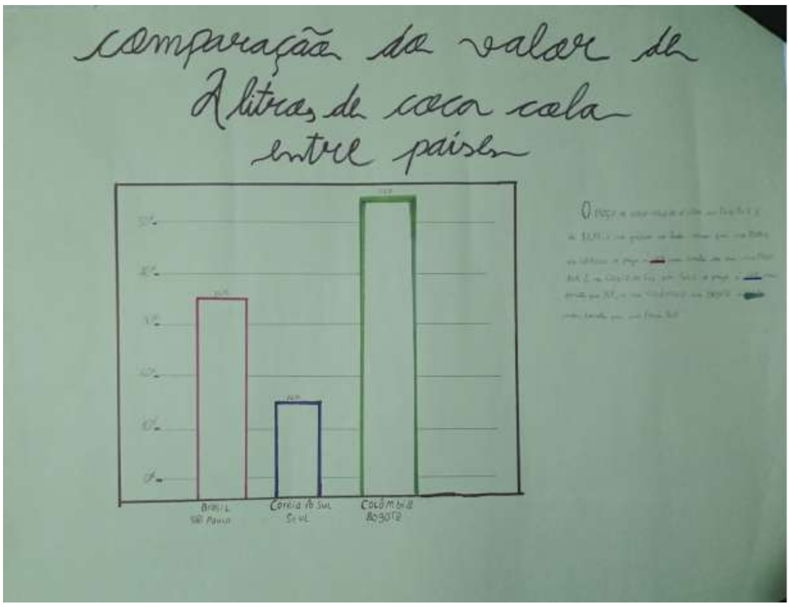
RESTRIÇÕES

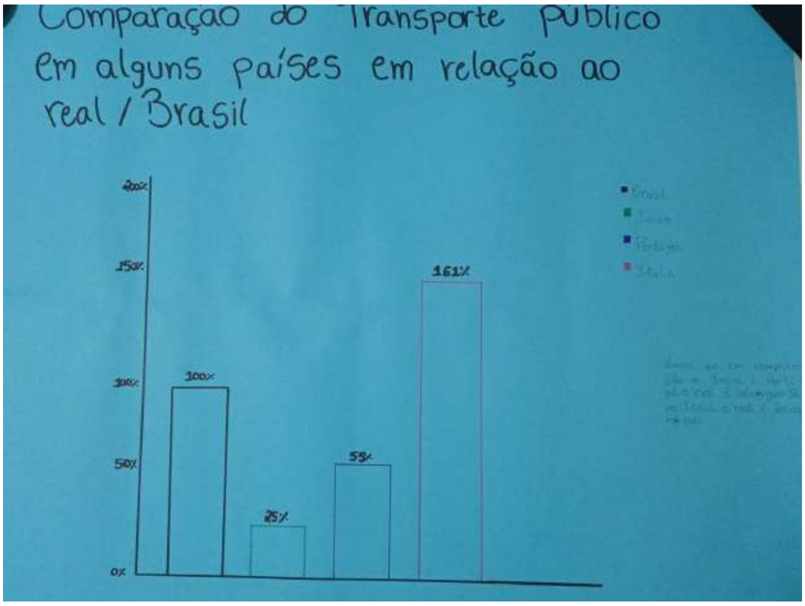
Condição Favorável	Classificação
Rigidez do plano de aula da disciplina;	Escola; Pedagogias; Disciplina
Sala de tecnologia não disponível nos dias que fizemos as atividades;	Escola;
Falta de habilidade dos alunos, com cálculos matemáticos básico;	Domínio;
Dificuldade em manusear a calculadora;	Domínio;
Dificuldade de diferenciar ponto e vírgula;	Domínio

RESTRIÇÕES

Condição de Restrição	Classificação
Complexidade do conteúdo;	Tema;
O produto que os alunos escolheram é comercializado em diferentes unidade de medida (Galão e litro);	Civilização; Sociedade.
Informações não confiáveis na internet; Resultados não válidos	Civilização; Setores; temas.
Engajamento dos alunos (alunos desmotivados, alunos dormindo);	Sociedade; Escola; Disciplina.
Tempo das Aulas, que percebemos que foi pouco	Escola;
Feriados no meio do percurso	Sociedade
Interrupção de alguns professores na aula.	Escola.

Fotos das produções dos estudantes




Comparação do Transporte público em alguns países em relação ao real / Brasil
200%
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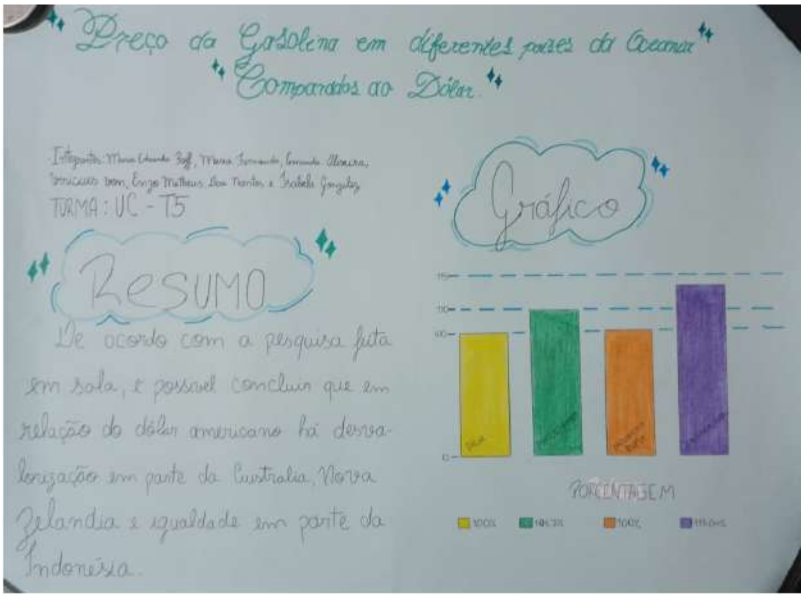
Preço da Gasolina em diferentes países da Oceania Comparados ao Dólar
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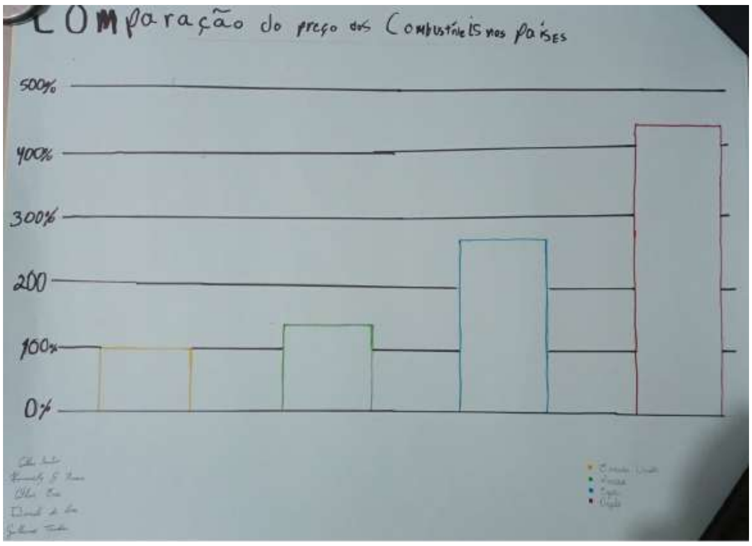
Gráfico
PORCENTAGEM


Comparação do preço dos Combustíveis nos países
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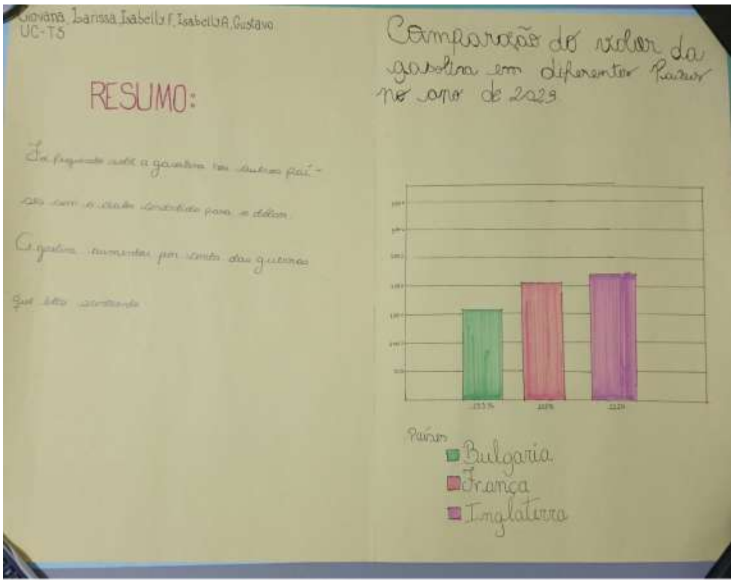
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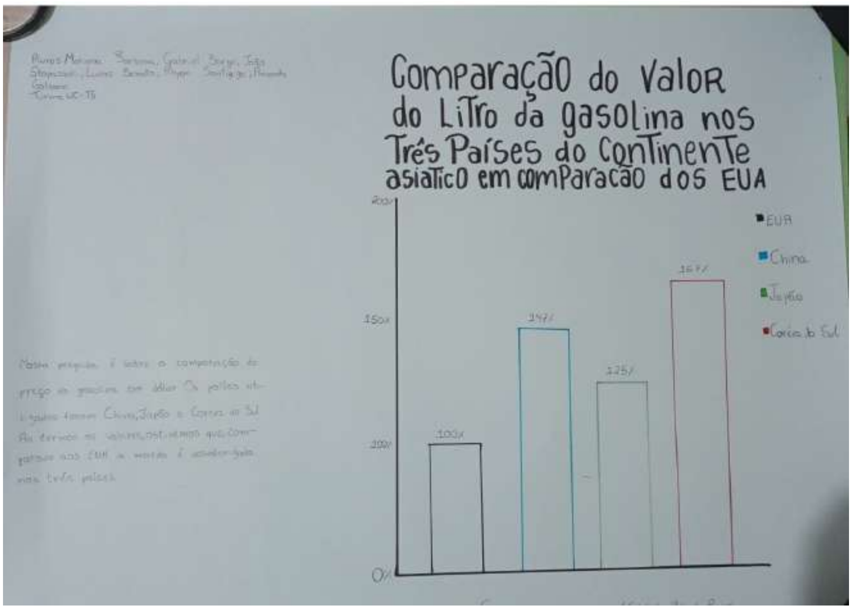
Comparação do valor da gasolina em diferentes Países no ano de 2023
Países
Bulgaria
França
Inglaterra


Comparação do Valor do Litro da gasolina nos Três Países do Continente asiatico em comparacão dos EUA
200%
150%
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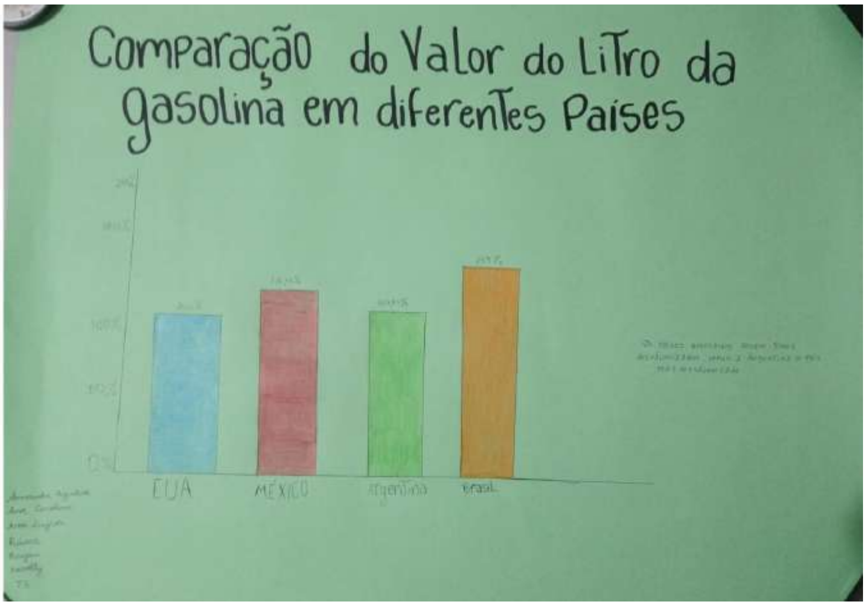
167%
EUA
China
Japão
Coréia do Sul


Comparação do Valor do Litro da gasolina em diferentes Países
EUA
MÉXICO
Argentina
Brasil

Relatório final do Grupo 3

Módulo I – PEP

- Questões formuladas *Q*

Pergunta 1: Existe algum sistema de controle e/ou monitoramento da qualidade da água?

Pergunta 2: Como as mudanças climáticas podem afetar a disponibilidade e a qualidade de água no Brasil?

Pergunta 3: Por que, principalmente nas áreas urbanas, ocorre uma menor administração dos recursos hídricos?

Pergunta 4: Existem alternativas para melhorar a gestão e/ou distribuição da água?

Pergunta 5: O problema das Regiões Norte e Nordeste seria a escassez natural ou a má distribuição da água?

Pergunta 6: Como está a qualidade da água nas diferentes regiões do país?

- **Mídias**

http://sisagua.saude.gov.br/sisagua/paginaExterna.jsf

https://www.gov.br/ana/pt-br/assuntos/gestao-das-aguas/panorama-das-aguas/mudancas-climaticas-recursos-hidricos#:~:text=O%20ciclo%20da%20%C3%A1gua%20est%C3%A1,de%20recursos%20h%C3%ADdricos%20para%20todos

https://www.scielo.br/j/ea/a/SfqYWrhrtvkxybFsjYQtx7v/#:~:text=As%20%C3%A1guas%20urbanas%20geralmente%20incluem,das%20%C3%A1guas%20urbanas%20da%20cidade

https://www.ecycle.com.br/11-solucoes-da-arquitetura-para-gestao-domestica-de-recursos-hidricos/

https://jornalcomunicacao.ufpr.br/crise-hidrica-afeta-as-cinco-regioes-do-brasil/

- Respostas preexistentes $R^{\lozenge}$

:Padlet

Janielly Taila Verbisck + 3 • 1d

Grupo 3 - PEP-FP UFMS

Criado com um caloroso abraço

Pergunta 1

Existe algum sistema de controle e/ou monitoramento da qualidade da água?

Resposta 1:

sisagua.saude.go...

paginaExterna

O Programa Nacional de Vigilância da Qualidade da Água para Consumo Humano (Vigiagua) consiste no conjunto de ações adotadas continuamente pelas autoridades de saúde pública para garantir à população o acesso à água em quantidade suficiente e qualidade compatível com o padrão de potabilidade, estabelecido na legislação vigente: Anexo XX da Portaria de Consolidação nº 5/2017 do Ministério da Saúde (PRC nº 05/2017, Anexo XX), como parte integrante das ações de prevenção dos agravos transmitidos pela água e de promoção da saúde, previstas no Sistema Único de Saúde (SUS). As ações do Vigiagua são desenvolvidas pelas Secretarias de Saúde Municipais, Estaduais, e do Distrito Federal e pelo Ministério da Saúde, por meio da Coordenação Geral de Vigilância em Saúde Ambiental.

Fonte: http://sisagua.saude.gov.br/sisagua/paginaExterna.jsf

Pergunta 2

Como as mudanças climáticas podem afetar a disponibilidade e a qualidade de água no Brasil?

Resposta 2:

gov.br

gov.br

Mudanças climáticas e recursos hídricos

O ciclo da água está diretamente ligado ao clima. Assim, mudanças no clima que alterem o regime de chuvas podem provocar o aumento da ocorrência de eventos hidrológicos extremos, como inundações e longos períodos de seca. Esses eventos afetam a oferta de água, ameaçando o suprimento de recursos hídricos para todos.

Fonte: https://www.gov.br/ana/pt-br/assuntos/gestao-das-aguas/panorama-das-aguas/mudancas-climaticas-recursos-hidricos#:~:text=O ciclo da água está,de recursos hídricos para todos.

Pergunta 3

Por que principalmente nas áreas urbanas ocorre uma menor administração dos recursos hídricos?

Resposta 3:

scielo.br

Águas urbanas

As águas urbanas geralmente incluem abastecimento de água e saneamento. Nessa perspectiva, saneamento envolve a coleta de tratamento de efluentes domésticos e industriais, não inclui drenagem urbana, gestão dos resíduos sólidos, porque ainda perdura uma visão desatualizada da gestão das águas urbanas da cidade.

fonte: https://www.scielo.br/j/ea/a/SfqYWrhrtvkxybFsjYQtx7v/#:~:text=As águas urbanas geralmente incluem,das águas urbanas da cidade.

Pergunta 4

Existem alternativas para melhorar a gestão e/ou distribuição da água?

Resposta 4

ecycle.com.br

11 soluções estruturais para gestão doméstica de recursos hídricos - eCycle

Soluções estruturais para gestão doméstica de recursos hídricos:

1. Instalar sistemas de monitoramento
2. Buscar limitação no uso da água potável
3. Aproveitar as águas pluviais
4. Utilizar equipamentos de baixo consumo de água
5. Instalar biodigestores residenciais para tratamento de esgoto

Fonte: https://www.ecycle.com.br/11-solucoes-da-arquitetura-para-gestao-domestica-de-recursos-hidricos/

Pergunta 5

O problema das regiões norte e nordeste seria a escassez natural ou a má distribuição?

Pergunta 6

Como está a qualidade da água nas diferentes regiões do país?

Resposta 6

jornalcomunicaca...

Crise hídrica afeta as cinco regiões do Brasil

No nordeste entre 2017 e 2020 houve 80% das secas do território brasileiro .
o índice de uso de água por pessoa no sudeste é o mais alto do país.
no norte do país, tem muita água e pouca gente, região acumula 60% da disponibilidade hídrica do Brasil
no sul, por sua vez, enfrenta como principais desafios problemas relacionados às cheias, abastecimento de hidrelétricas, qualidade de água e irrigação

Fonte: https://jornalcomunicacao.ufpr.br/crise-hidrica-afeta-as-cinco-regioes-do-brasil/

- Questões em aberto

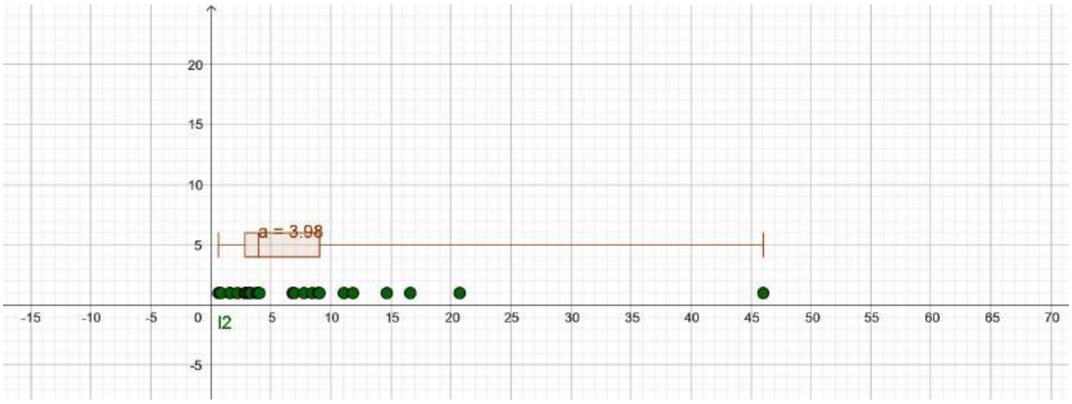
O problema das Regiões Norte e Nordeste seria a escassez natural ou a má distribuição da água? (Estudo que requer mais aprofundamento).

- **Respostas** ***R***♥

Olhando as distribuições de recursos hídricos e a população por estados brasileiros (26 estados + DF), como se comporta esta distribuição segundo dados mais atuais (2022)?

No gráfico abaixo, na caixa, são apresentados os estados que possuem a população com valores aproximados. A linha vermelha dentro da caixa representa a mediana, sendo ela o Espírito Santo. Os dados que se encontram do lado de fora da caixa apresentam os estados que têm sua população em uma quantia muito maior do que os estados semelhantes que estão dentro da caixa, ou próxima de zero.

Através das discussões realizadas no estudo hídrico e sistêmico das regiões do Brasil.



Qual a população (números de habitantes) brasileira? Qual a população (números de habitantes) por regiões? Onde encontramos dados mais atualizados sobre isso (fonte dos dados)
De acordo com o Instituto Brasileiro de Geografia e Estatística (IBGE), a população brasileira em 2022 era de aproximadamente 207 milhões de pessoas.

Aqui está a população brasileira por regiões, segundo o IBGE em 2022:

Região Norte: 17.834.762 milhões de pessoas

Região Nordeste: 55.389.382 milhões de pessoas

Região Centro-Oeste: 16.492.326 milhões de pessoas

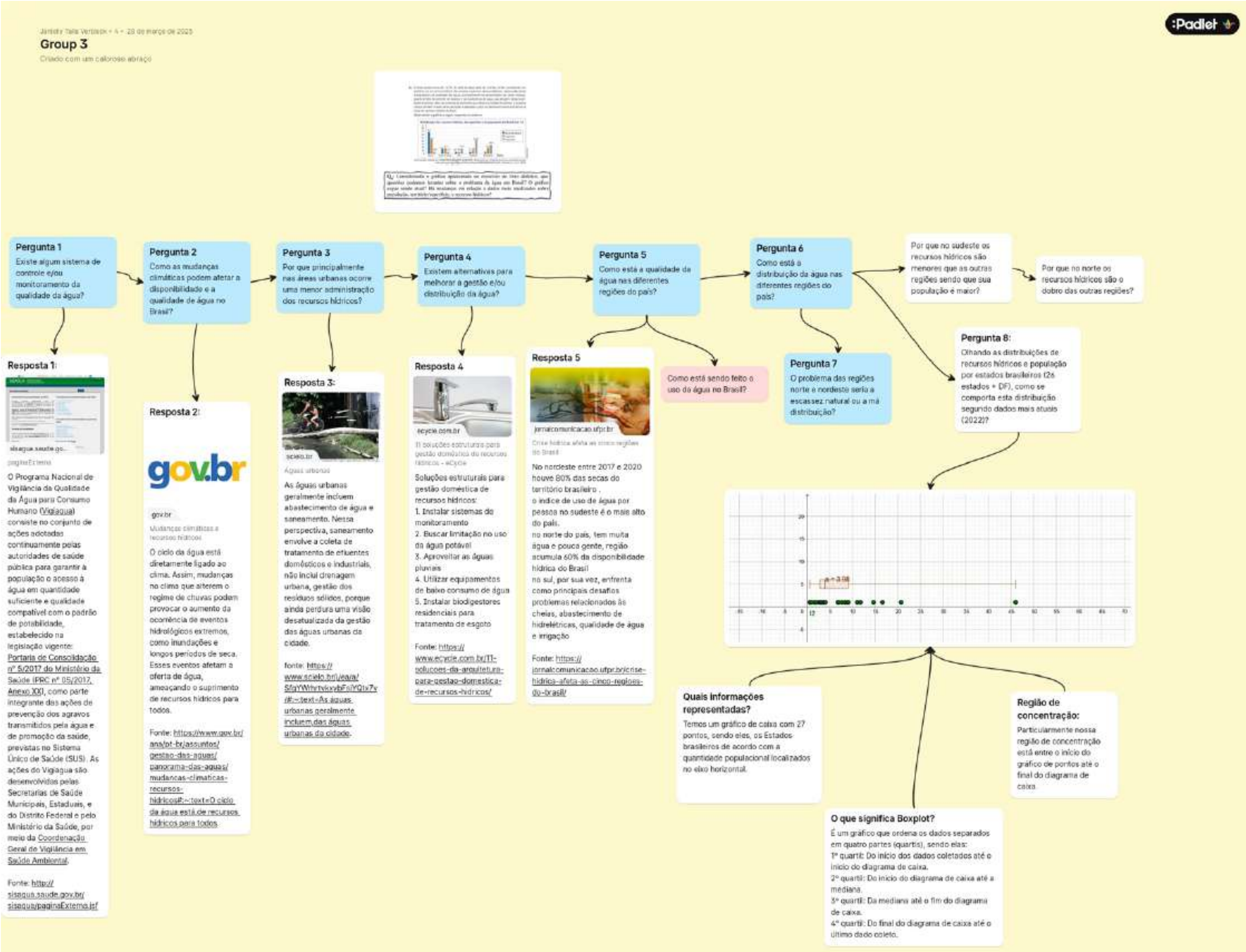
Região Sudeste: 87.348.223 milhões de pessoas

Região Sul: 30.685.598 milhões de pessoas

A unidade de medida para a população é o número de habitantes.

Youtube: https://www.youtube.com/watch?v=1gfXbdn15ec&ab_channel=TomCooper

Mapa de questões-respostas



Módulo II – Análise do PEP

Dialética Questões-Respostas: Comentem sobre a utilidade desta ferramenta durante o PEP

- pontos positivos:
  Ter a possibilidade de relacionar várias perguntas com o mesmo tema; melhor visualização das questões criadas.

- pontos negativos:
  Ter deixado questões em aberto.

- Dificuldades encontradas?
  Encontrar fontes confiáveis, encontrar respostas coerentes às perguntas e fazer a construção do gráfico.

Dialética mídia-meio:

- Que mídia foram utilizadas durante o PEP?
  Conhecimentos pessoais e uso de tecnologias.

- Que objetos foram utilizados durante o PEP?
  Computador.

- Que conhecimentos estatísticos foram utilizados durante o PEP?
  Mediana utilizada no diagrama de caixa, amostra, população, média e variáveis.

- Que dados foram utilizados durante o PEP?
  Dados da população do Estado encontrados pelo IBGE.

- Que estudos poderiam ou precisam ser aprofundados?
  Estudos de estatística, instituições responsáveis pela distribuição de água.

- Dificuldades encontradas?

- Encontrar fontes confiáveis.
- Através de uma pesquisa feita em uma plataforma de IA (Inteligência Artificial) com o objetivo de encontrar dados atuais, nos foram apresentados dados desatualizados.

Dialética Indivíduo-Coletivo: Comentem sobre a organização do grupo durante o PEP

- Discussões coletivas?
  Sobre todas as questões feitas e respondidas.
  Debatemos sobre a discussão do gráfico feito e o significado dele.
  Discutimos sobre dados incorretos durante a pesquisa.

- Divisão de tarefas?
  As perguntas foram feitas de maneira individual e, após isso, em discussão com o grupo, foi realizada a coerência da pergunta.
  As respostas foram realizadas de maneira individual e cada um respondia sua própria pergunta.
  O gráfico foi realizado de maneira coletiva.

- Dificuldades encontradas?
  Reunir o grupo para organização das apresentações, devido aos horários não flexíveis.

- O trabalho em grupo poderia ser melhorado? Em quais aspectos?
  Sim, no aspecto de participação de pesquisa, apresentação e responsabilidade com o grupo formado por 4 indivíduos.

Módulo III – Planejamento/desenho de atividade de investigação

Turma:
Escola:

- Questão inicial da atividade de investigação.
- Como se coloca a questão, o que se propõe fazer e que ferramentas estão disponíveis?
- Como o estudo está planejado para ser administrado em classe: horário aproximado
- Como se espera que a atividade seja concluída
- Qual/quais conteúdo(s) curricular(es) (em matemática e outros assuntos) pode(m) ser abordado(s) pela atividade proposta.

PLANEJAMENTO DE AULA 1

TURMA: 6º ano.

Conteúdo: Estatística.

OBJETIVO DE APRENDIZAGEM:

Coleta de dados

Construção de tabela

RECURSOS:

Lápis, papel, borracha, quadro negro, giz, régua.

METODOLOGIA:

1º momento: introduzir o tema "Estatística" com os alunos, com seus significados e como trabalharemos nesta aula.

2º momento: buscar junto com os alunos informações sobre o tema.

3º momento: levantar as questões que serão respondidas no final do processo. Pedir que a turma opine sobre os possíveis resultados e registrar todas as hipóteses para, mais tarde, compará-las com as conclusões.

4º momento: neste momento, faremos a coleta dos dados junto com a turma e registraremos na lousa para acesso de todos os alunos.

5º momento: separar a sala em grupos com média de quatro alunos.

6º momento: pedir para os alunos organizarem os dados de maneira que fique mais simplificado com a intenção deles construírem uma tabela com os dados.

7º momento: iremos fazer a conferência do que os alunos construíram.

8º momento: Será feita a análise do que eles construíram através de perguntas. Por exemplo: Qual o time que tem maior torcida dentro da sala de aula?

Qual o time que tem menor torcida dentro da sala de aula?

Quantas pessoas torcem para um determinado time?

Quantas pessoas não torcem para nenhum time? (Caso houver)

AVALIAÇÃO: A avaliação dos alunos será realizada de forma continuada, levando em consideração a participação dos discentes nas questões propostas e nas discussões originadas ao longo da aula**.**

PLANEJAMENTO DE AULA 2

Turma: 6º ano. **Duração:** entre 4 e 6 aulas.

Conteúdo: Estatística.

Objetos de conhecimento (BNCC): coleta de dados, organização e registro; construção de diferentes tipos de gráficos para representá-los e interpretação das informações.

Habilidade (BNCC): planejar e coletar dados de pesquisa referente a práticas sociais escolhidas pelos alunos e fazer uso de planilhas eletrônicas para registro, representação e interpretação das informações em tabelas, vários tipos de gráficos e textos.

Recursos:

Lousa, Lápis, papel, papel quadriculado, cartolina, borracha, giz, régua.

Metodologia:

<u>Aulas 1 e 2 - 01/06/2023</u>

A aula será iniciada com uma breve apresentação dos professores regentes e, em seguida, vamos propor alguns minutos de discussão em que perguntaremos aos estudantes se eles já realizaram ou se eles sabem como se realiza uma pesquisa estatística. Após este pequeno debate, diremos aos estudantes que a proposta de aula é justamente que eles realizem uma pequena pesquisa estatística em que nós professores seremos os responsáveis por guiá-los e orientá-los durante a pesquisa.

Temos como principal objetivo possibilitar que os estudantes vivam um pequeno processo de investigação estatística cujas principais etapas são:

Elaboração de uma questão de investigação → coleta de dados → organização dos dados → representação dos dados em tabelas e/ou gráficos → análise dos dados → comunicação dos resultados encontrados.

Para isso, dividiremos os estudantes em cinco grupos (entre 5 a 7 estudantes por grupo) e diremos a cada grupo que o primeiro passo é elaborar uma questão que os interesse para ser investigada. Por exemplo: “Qual o seu esporte favorito?” “Quantos animais de estimação você possui?” “O que você gostaria de ser quando crescer?” dentre outras. Estes são apenas exemplos de questões que talvez os estudantes elaborem.

Após cada grupo já ter definido sua questão de investigação (e as possíveis respostas, ou se será uma questão “aberta” etc.), guiaremos os grupos para o momento de coleta dos dados. Cada grupo irá definir um integrante para realizar a coleta de dados. Esta coleta será feita dentro da própria sala de aula, isto é, a amostra será a própria turma. Teremos, então, um estudante de cada grupo colhendo respostas dos colegas e dele mesmo. Para esta coleta, será utilizada uma folha sulfite e lápis ou caneta. Acreditamos que até aqui terá transcorrido as duas aulas. Assim, finalizaremos estas primeiras duas aulas guardando as respostas coletadas e dizendo aos estudantes que seguiremos com esta atividade nas próximas aulas.

Aulas 3 e 4 - 07/06/2023

Nestas duas aulas, daremos continuidade à atividade de investigação. Os mesmos grupos se reunirão para organizar os dados coletados. Acreditamos que aqui surgirá a ideia de organizar os dados em uma tabela de dupla entrada e vamos orientá-los sobre como se faz esta construção.

Em seguida, também apresentaremos o gráfico de barras como um possível registro de representação destes dados. Vamos distribuir aos grupos folhas quadriculadas para que os ajudem no momento de construir os gráficos. Auxiliaremos todos que precisarem e tiverem dificuldades. Após estas construções, os grupos tentarão analisar os dados (nas representações tabular e gráfica) e discutirão sobre "conclusões". Os gráficos construídos serão colados em uma cartolina e nesta cartolina os grupos escreverão as conclusões que tiverem. Pensamos em algumas questões que possam auxiliá-los, por exemplo:

Se algum grupo pensou na questão "Qual seu esporte favorito?", poderíamos questionar:

→ Qual esporte grande parte da turma gosta?
→ Qual esporte que poucos gostam?
→ Que resposta mais surpreendeu vocês?
→ Que esporte vocês imaginavam que apareceria nas respostas, mas que não apareceu?
→ Que conclusões vocês tiram sobre as respostas coletadas e analisadas?
→ Que dificuldades vocês tiveram ao longo da investigação?

Acreditamos que talvez estas duas aulas não sejam suficientes para finalizar com uma discussão coletiva sobre as conclusões, então pensamos que esta discussão coletiva possa ser realizada na próxima aula, no dia 14/06/2023.

Aulas 5 e 6 - 14/06/2023

Nesta aula, finalizaremos a atividade de investigação com a exposição dos cartazes dos grupos e com uma discussão coletiva sobre as pesquisas realizadas, sobre dificuldades dos estudantes, sobre pontos positivos e negativos da atividade que propusemos.

AVALIAÇÃO: a avaliação dos alunos será realizada de forma continuada, levando em consideração a participação dos discentes nas questões propostas e nas discussões originadas ao longo da aula**.**

Anotações de aula

Dia 01/06/2023

A Janielly perguntou para os alunos se eles sabem o que é pesquisa estatística, responderam que sabem o que é pesquisa "perguntar algo que quer saber, pesquisar no Google". Foram falados exemplos de pesquisas estatísticas (pesquisas de eleição, dados da pandemia) e questionado como os pesquisadores chegaram nesses resultados, "chegam através da ciência, porcentagem, perguntando" obtemos essas respostas.

Depois dessa introdução, pedimos para que os alunos criassem questões como objeto de pesquisa.

- Time favorito
- Mês de aniversário
- Idade
- Esporte favorito
- País favorito
- Messi, CR7 ou Neymar
- Aro 29, Cross ou Titan

Tiveram alunos que não quiseram responder as perguntas, demorou um pouco para que eles respondessem, também tiveram os que não queriam fazer grupos, a professora Katiane formou um grupo com esses alunos. Na hora da coleta, cada grupo se dividiu de um jeito, os alunos se revezavam ou somente um fazia a coleta.

Um grupo não colocou os nomes dos alunos na hora da coleta e se perderam para chegar no total (33 respostas). Outro grupo se confundiu e, ao invés de pegarem os dados do que estavam pesquisando, coletou os dados da questão feita pelo outro grupo; assim, tiveram que fazer a coleta dos dados novamente.

Perguntamos o que são dados, disseram "são o que queremos saber". Fizemos uma breve análise de qual time tem mais torcida, qual teve menos e qual não teve nenhum. Falamos para organizar os dados coletados e os alunos já estavam contando e organizando.

Depois que foi explicado como fazer uma tabela, os alunos fizeram e mostraram facilidade. Questionamos se sabiam o que era um gráfico, responderam "gráfico de pizza", dissemos que na próxima aula montaremos gráficos de barras com dados que eles coletaram, dito isso, antes da explicação, já tinha um aluno fazendo a construção.

Dia 07/06/2023

Muitos alunos faltaram por conta do feriado (8 de junho, Corpus Christi) e outros alunos saíram da sala para ensaiar quadrilha, ficando aproximadamente a metade dos alunos na sala.

Alguns alunos quiseram mudar de grupo, mas não deixamos por conta do que eles já tinham produzido. Com a falta dos alunos, um grupo estava com apenas um integrante presente.

Relembramos o que aconteceu na última aula, explicamos como construir um gráfico de barras e pictogramas. A montagem da tabela ocorreu bem, no gráfico os alunos tiveram dificuldade na escala, vários alunos tiveram que refazer, houve também um aluno que estava fazendo as barras em tamanhos desproporcionais, pedimos que arrumasse.

Alguns alunos terminaram os gráficos e tabelas e começaram a montagem do cartaz. O aluno que estava sozinho terminou a conclusão e toda montagem do cartaz, ficou faltando colocar o nome do grupo e dos integrantes.

Dia 21/06/2023

Finalizando o trabalho, lembramos nosso último encontro e foi feita a explicação da montagem dos gráficos novamente; além dos alunos que não estavam presentes montarem os gráficos deles, também era necessário relembrar porque a professora Katiane vai aplicar uma prova com o conteúdo da pesquisa.

Deixamos um tempo para os alunos terminarem suas construções (tabelas e gráficos) e cartazes.

Houve um grupo que o aluno estava sozinho, são três integrantes, um faltou e o outro teve que ir embora, pois se machucou na escola. Esse aluno que ficou sozinho não estava presente na última aula, a Janielly explicou para ele o que os colegas fizeram e o ajudou quando precisava.

A turma estava muito agitada e somente um grupo não conseguiu fazer a conclusão, gastaram muito tempo enfeitando o cartaz.

Fizemos a exposição dos cartazes para podermos fazer a análise. Percebemos que os alunos fizeram as conclusões olhando somente para a barra maior e menor presente nos gráficos.

Na exposição, falamos os nomes dos grupos, o que foi pesquisado e as conclusões a que eles chegaram. Houve um grupo que errou na conclusão, colocou que o mês que tinham menos aniversariantes é dezembro, mas o correto é julho, por não ter nenhuma barra no gráfico teve essa confusão.

Buscamos trazer outras questões para que eles pudessem pensar, se somarmos dois meses quantos aniversariantes vamos ter, quando perguntado se o Flamengo ainda seria a maior torcida em relação ao país, um aluno respondeu "sim, no Google fala" outro falou "não, porque só pesquisamos na nossa sala".

Mudanças no plano de aula

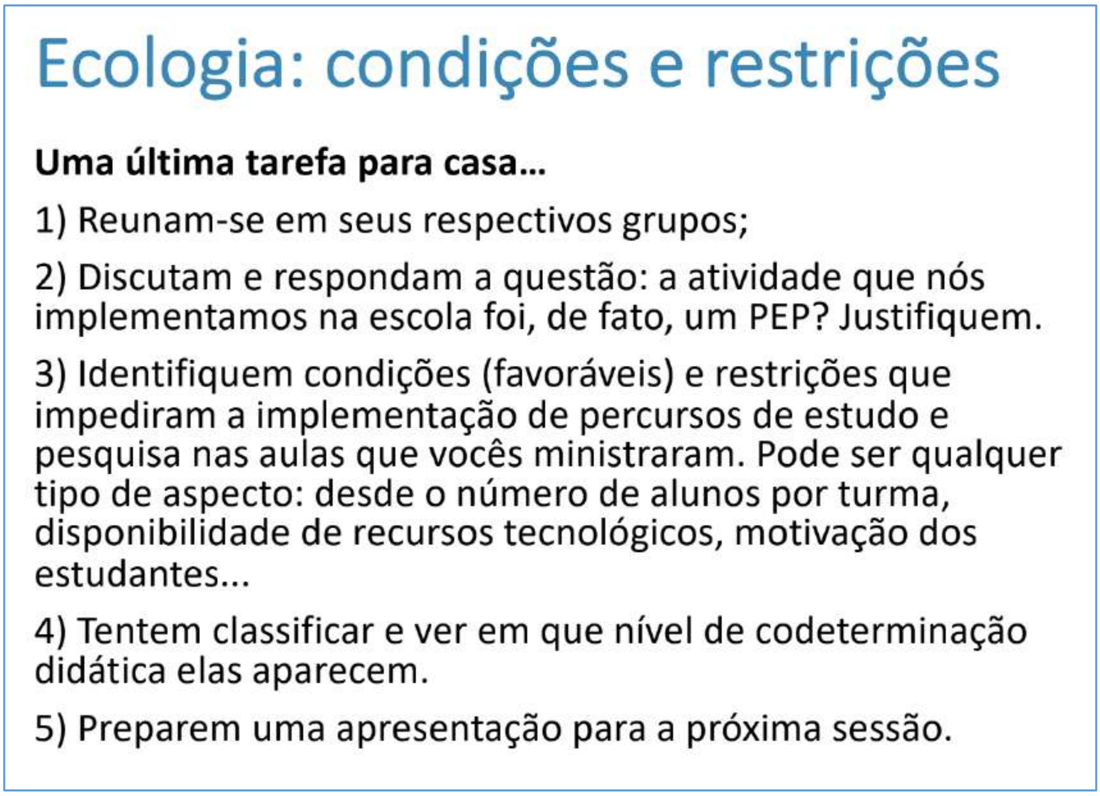
Nas aulas 1 e 2, os alunos já começaram a organizar os dados em tabelas e alguns construíram gráficos de barras.

Nas aulas 3 e 4, retomamos o que foi feito e os alunos continuaram a construir suas tabelas e gráficos. Um grupo conseguiu concluir o cartaz nessas 4 aulas.

Nas aulas 5 e 6, lembramos como foi o projeto e alguns alunos que ainda não tinham feito tabelas e gráficos fizeram uma construção para colocar no cartaz (no segundo dia de aplicação do projeto, muitos faltaram por conta do feriado e outros alunos tiveram que sair da sala para ensaiar quadrilha) .

Na parte de avaliação também houve mudanças, a professora titular da turma passou uma prova bimestral referente ao conteúdo da pesquisa

Módulo IV – análises finais



Tarefa

2) Sim, pois os alunos fizeram a coleta de dados, organizaram em tabelas e gráficos e chegaram a uma conclusão.
ou
Devido à realização da coleta de dados pelos estudantes, seguida da organização sistemática desses dados em tabelas e gráficos, foi possível alcançar uma conclusão embasada.

3) É uma pesquisa fácil de implementar, podendo fazer adaptações para determinada amostra, no nosso caso, no decorrer do projeto, tivemos que "conter" o barulho gerado; porém, isso era algo que já se esperava pela faixa etária dos alunos.
Condições favoráveis: interesse e participação dos alunos.
Restrições: disponibilidade de recursos tecnológicos. Níveis Genéricos: Civilização.
ou
Esta pesquisa apresenta uma alta viabilidade de implementação, permitindo adaptações específicas para uma amostra determinada, como ocorreu no nosso caso durante o desenvolvimento do projeto, quando foi necessário mitigar o ruído gerado, algo já esperado devido à faixa etária dos alunos envolvidos. As condições propícias para o sucesso da pesquisa incluem o interesse e a participação ativa dos alunos. No entanto, é importante ressaltar que a disponibilidade de recursos tecnológicos representa uma restrição relevante.

4) É possível identificar diferentes níveis de codeterminação didática em cada etapa do processo de ensino. A codeterminação didática se refere à forma como o professor organiza as atividades e interações em sala de aula, buscando engajar os alunos e promover a construção de conhecimento de forma participativa. Vamos analisar cada parte do plano de aula em relação ao nível de codeterminação didática:

- **Introdução da pesquisa estatística e proposição da atividade:** Nível de codeterminação didática – alto. Nessa etapa, os professores apresentam o objetivo da aula e propõem a realização de uma pesquisa estatística. Ao perguntar aos alunos sobre suas experiências anteriores e orientar a atividade, os professores estão engajando os alunos de forma ativa na definição do tema e na importância da pesquisa estatística.

- **Elaboração da questão de investigação e coleta de dados:** Nível de codeterminação didática: médio. Ao dividir os alunos em grupos e instruí-los a elaborar suas próprias questões de investigação, os professores estão promovendo a participação ativa dos alunos na definição do objeto de estudo. No entanto, a coleta de dados é atribuída a um único integrante de cada grupo, o que limita a participação de todos os alunos nessa etapa.

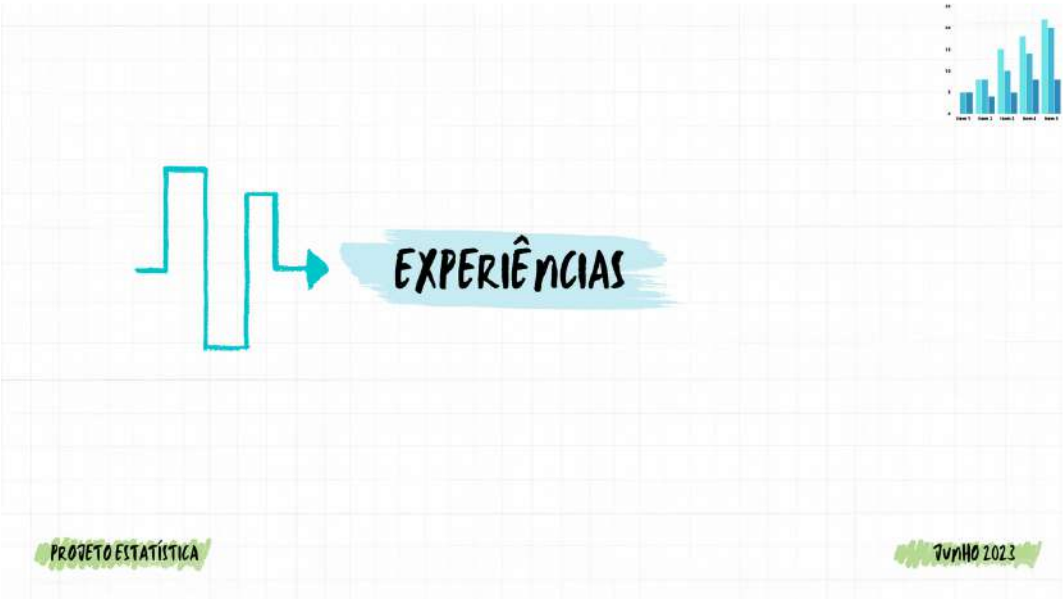
- **Organização dos dados e representação em tabelas e gráficos:** Nível de codeterminação didática: alto. Nessa etapa, os professores orientam os alunos sobre como organizar os dados coletados em uma tabela de dupla entrada e apresentam o gráfico de barras como uma forma de representação. Ao distribuir folhas quadriculadas para auxiliar na construção dos gráficos e oferecer suporte aos grupos, os professores estão incentivando a participação ativa dos alunos na construção das representações.
- **Análise dos dados e discussão das conclusões:** Nível de codeterminação didática: alto. Os professores propõem questões para guiar a análise dos dados e incentivar a discussão das conclusões. Essas perguntas estimulam os alunos a refletir sobre os resultados encontrados, tirar conclusões e compartilhar suas perspectivas. Isso promove a participação ativa dos alunos na interpretação dos dados e na discussão coletiva.

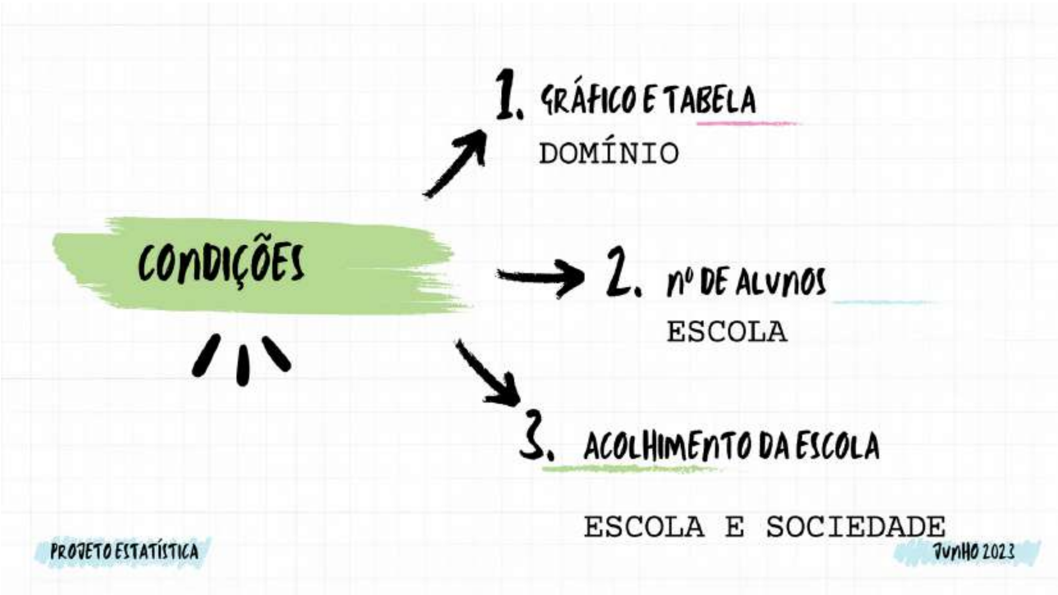
- **Exposição dos cartazes e discussão coletiva:** Nível de codeterminação didática: alto. Nessa etapa final, os grupos apresentam seus cartazes e ocorre uma discussão coletiva sobre as pesquisas realizadas, dificuldades dos alunos e pontos positivos e negativos da atividade. Essa discussão estimula a participação ativa de todos os alunos, permitindo que expressem suas opiniões, compartilhem suas experiências e contribuam para a reflexão coletiva.

Em resumo, o plano de aula apresenta um nível geral de codeterminação didática alto, pois busca envolver ativamente os alunos em todas as etapas do processo de pesquisa estatística, desde a elaboração das questões até a discussão coletiva das conclusões. No entanto, é importante considerar as limitações identificadas, como a atribuição da coleta de dados a um único integrante de cada grupo, porém buscamos consertar este empecilho.

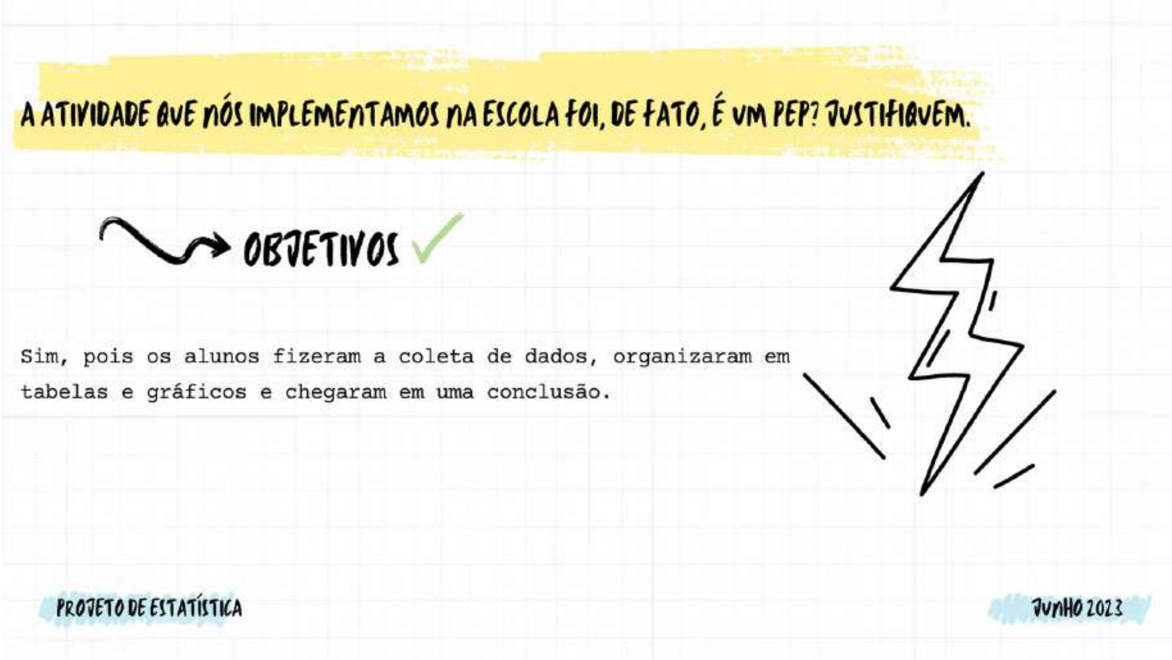
Para os slides:

Em resumo, o plano de aula apresenta um nível geral de codeterminação didática alto, pois busca envolver ativamente os alunos em todas as etapas do processo de pesquisa estatística, desde a elaboração das questões até a discussão coletiva das conclusões. Embora existam algumas limitações, como a atribuição da coleta de dados a um único integrante de cada grupo, o plano promove uma abordagem participativa e colaborativa, permitindo que os alunos construam conhecimento de forma significativa e enriquecedora.

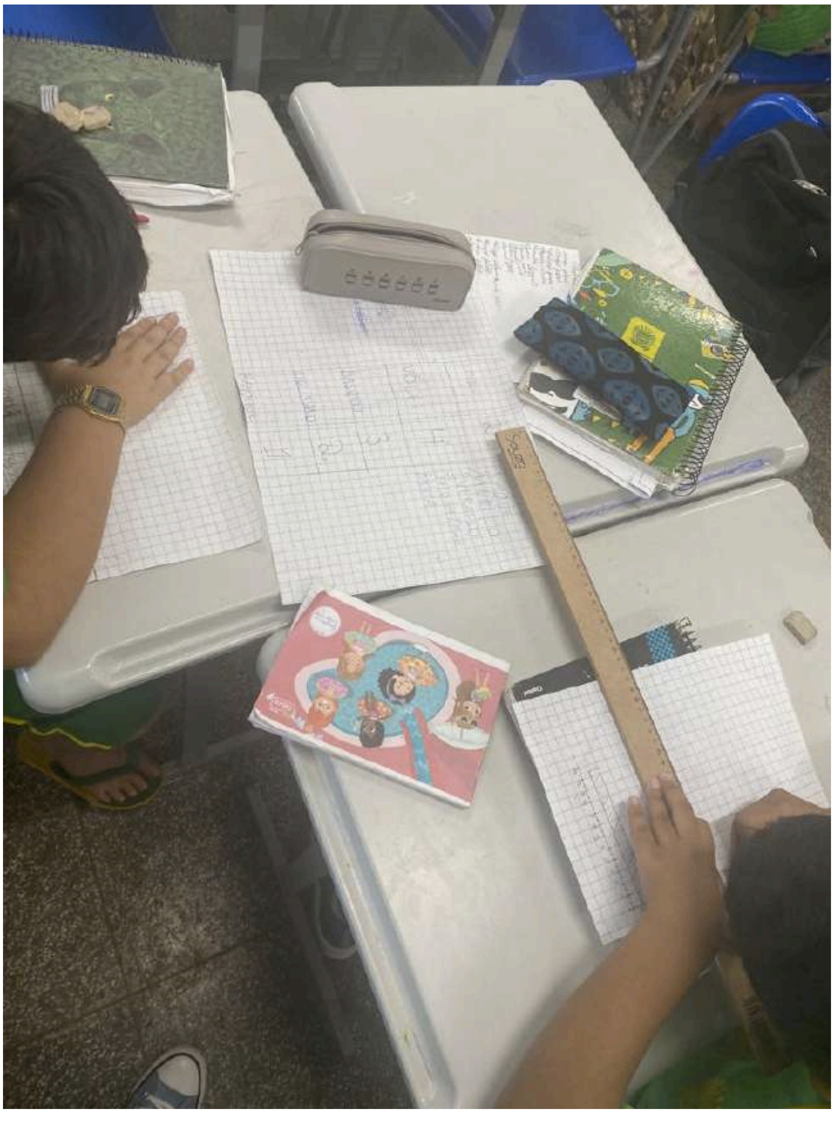

EXPERIÊNCIAS
PROJETO ESTATÍSTICA
JUNHO 2023

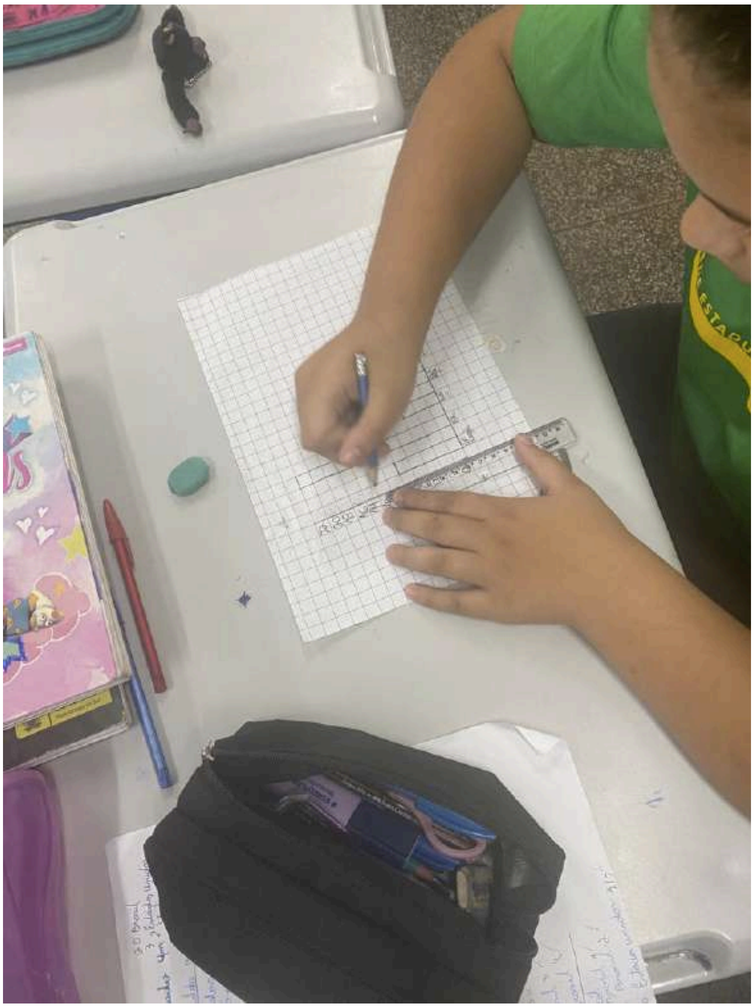

CONDIÇÕES
1. GRÁFICO E TABELA
DOMÍNIO
2. nº DE ALUNOS
ESCOLA
3. ACOLHIMENTO DA ESCOLA
ESCOLA E SOCIEDADE
PROJETO ESTATÍSTICA
JUNHO 2023

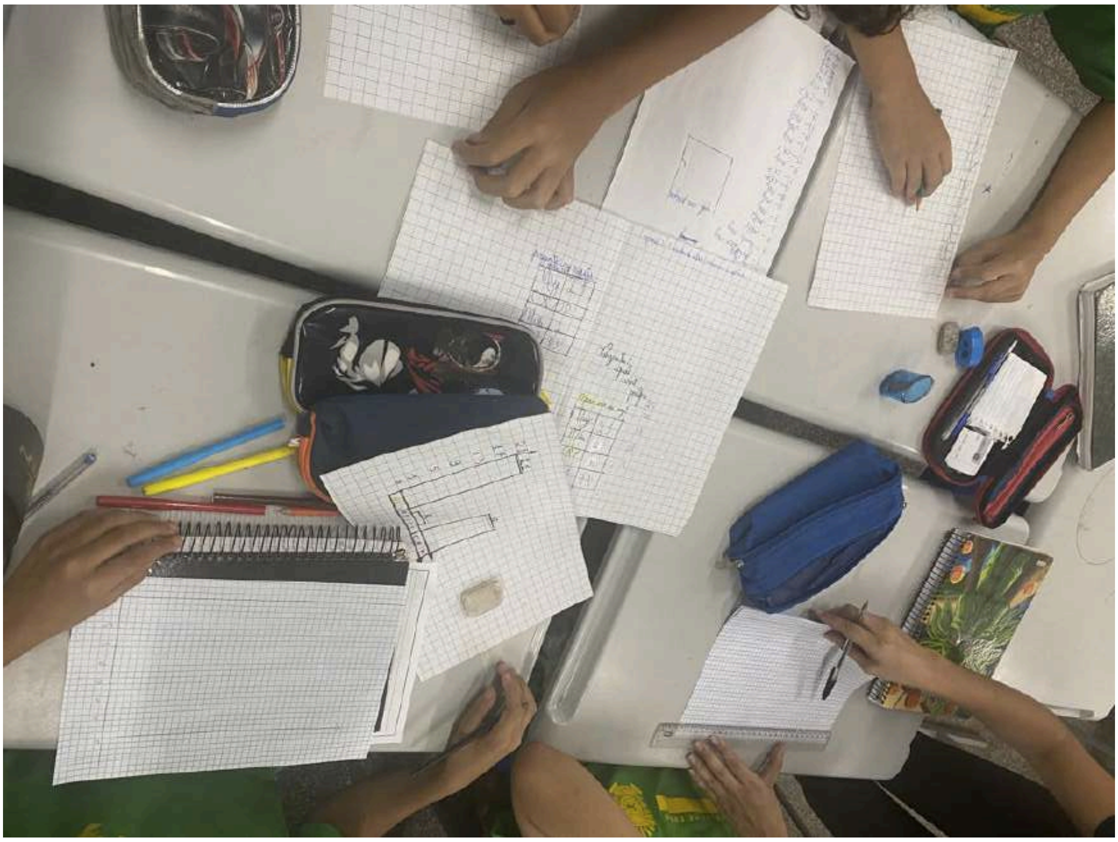

RESTRIÇÕES
1. FALTA DE ACESSO À STE
ESCOLA
2. FALTA DE ALUNOS EM UMA AULA
SOCIEDADE/ESCOLA
PROJETO ESTATÍSTICA
JUNHO 2023

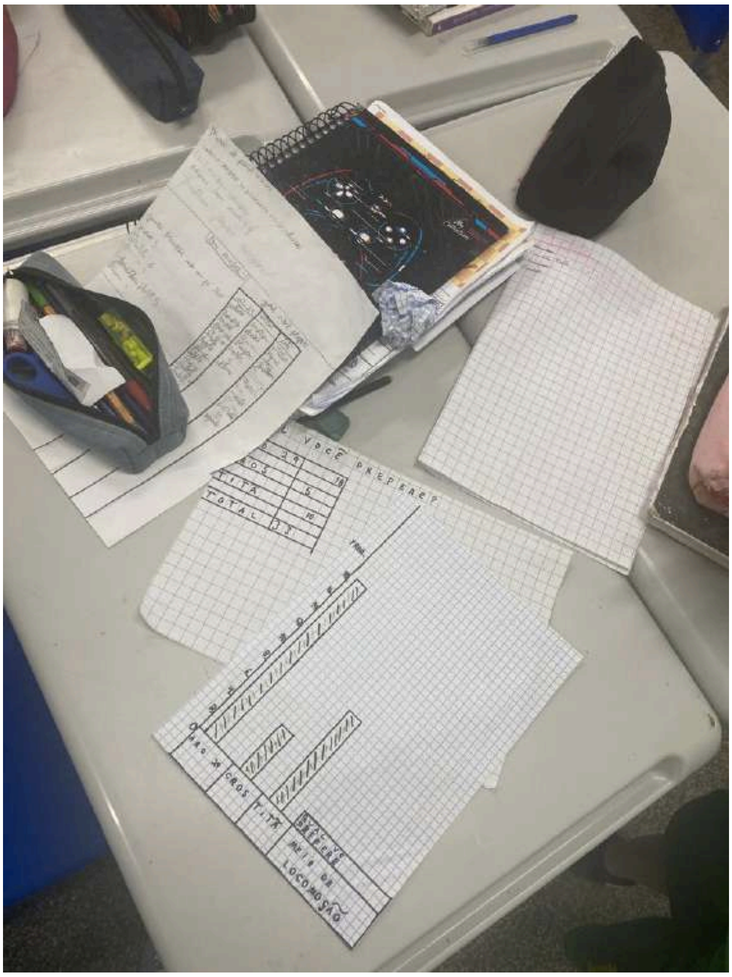


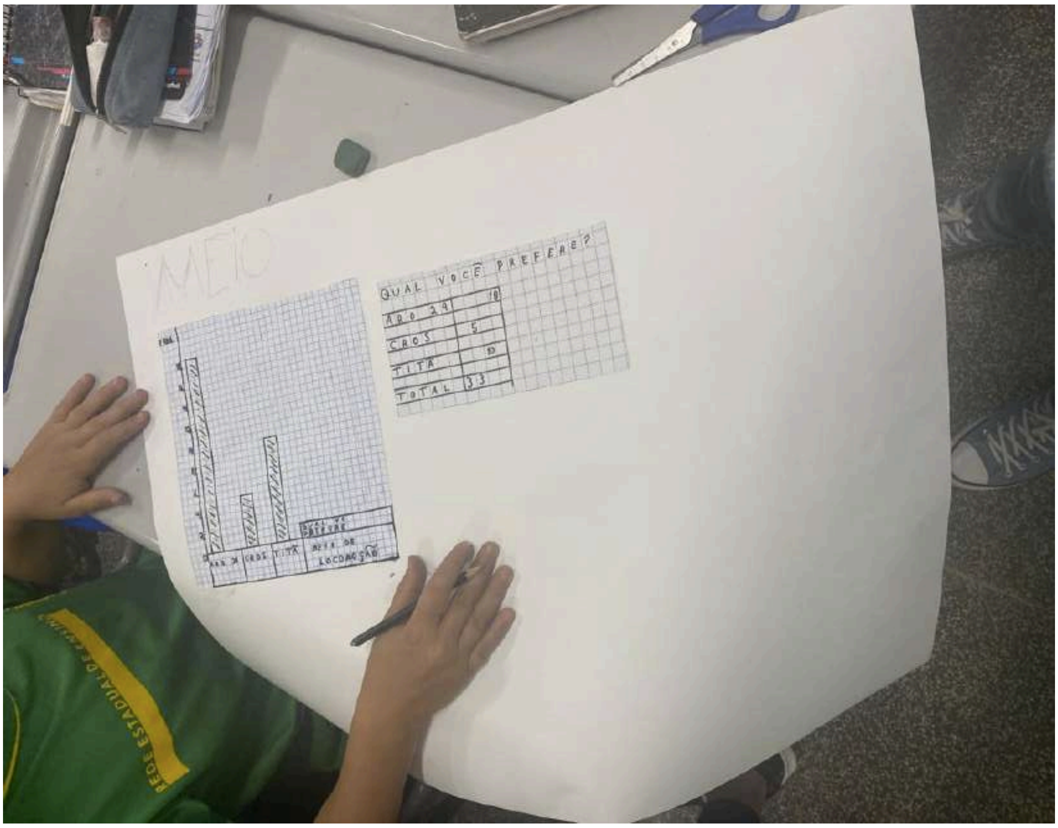
Fotos durante a implementação:










MEIO
QUAL VOCÊ PREFERE?
ADO 24 | 18
CROS | 5
TITA | 10
TOTAL | 33

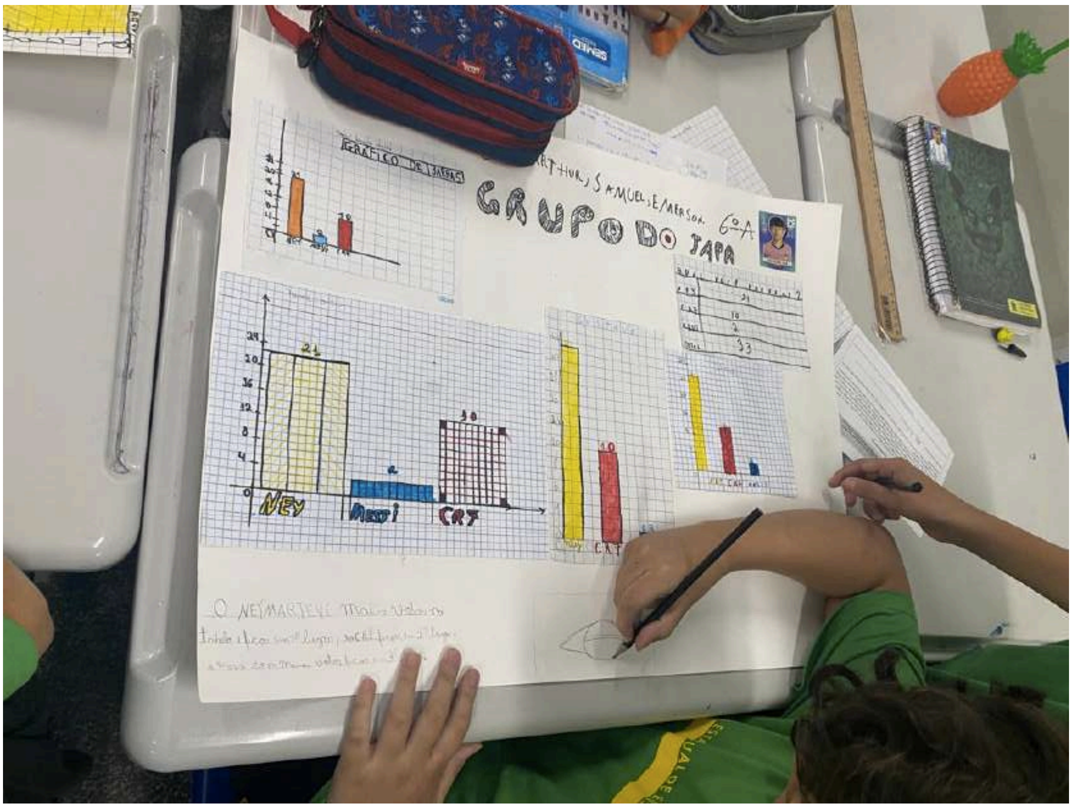
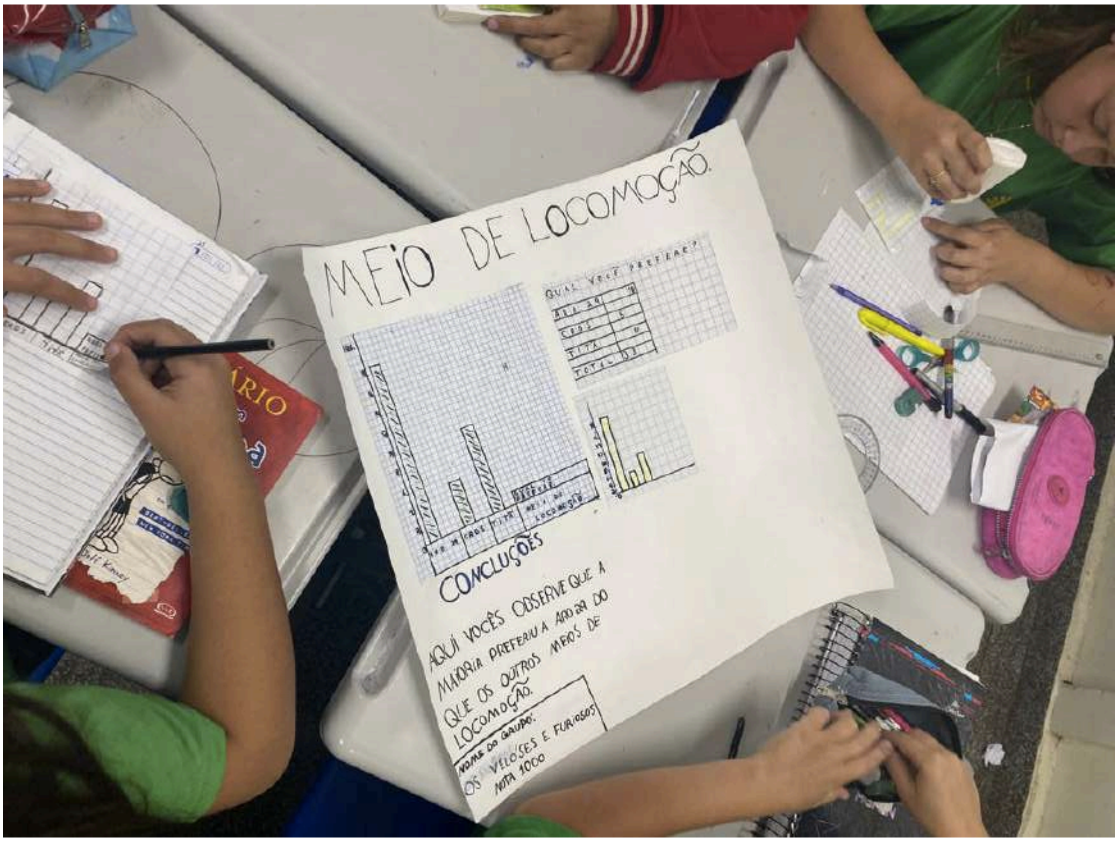
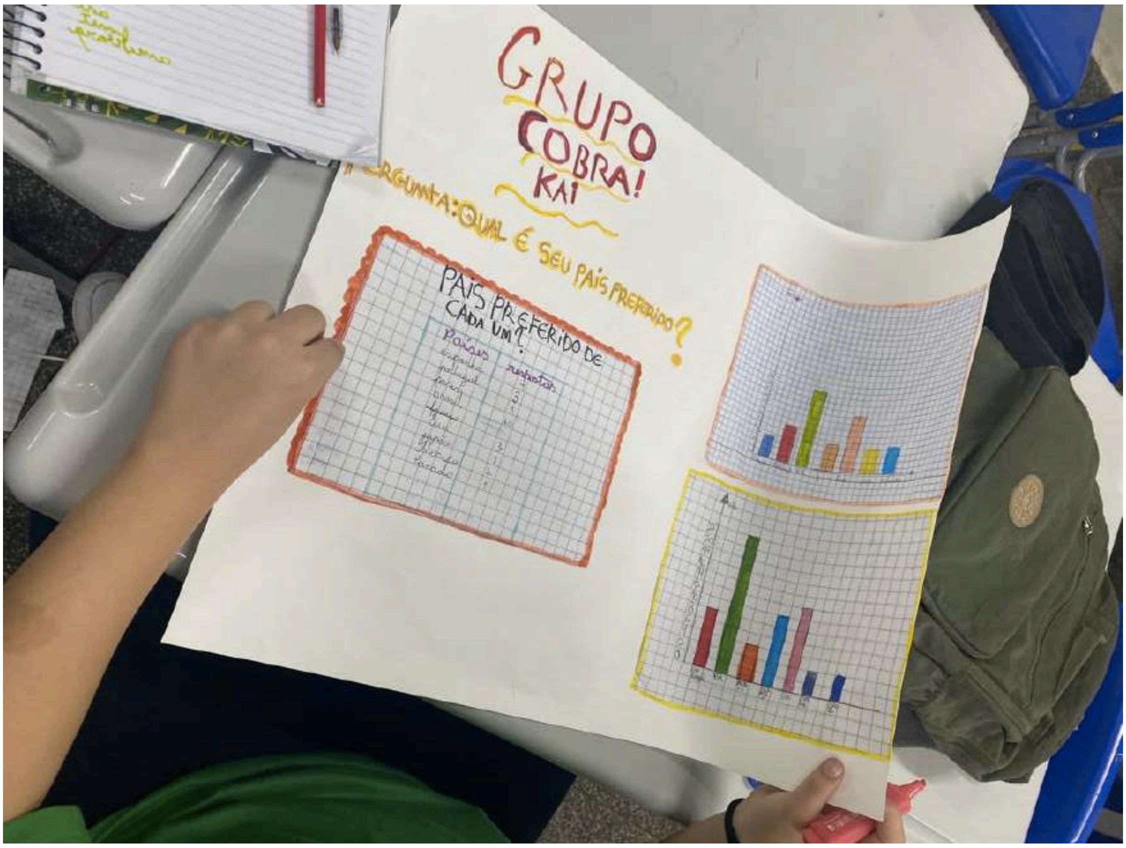
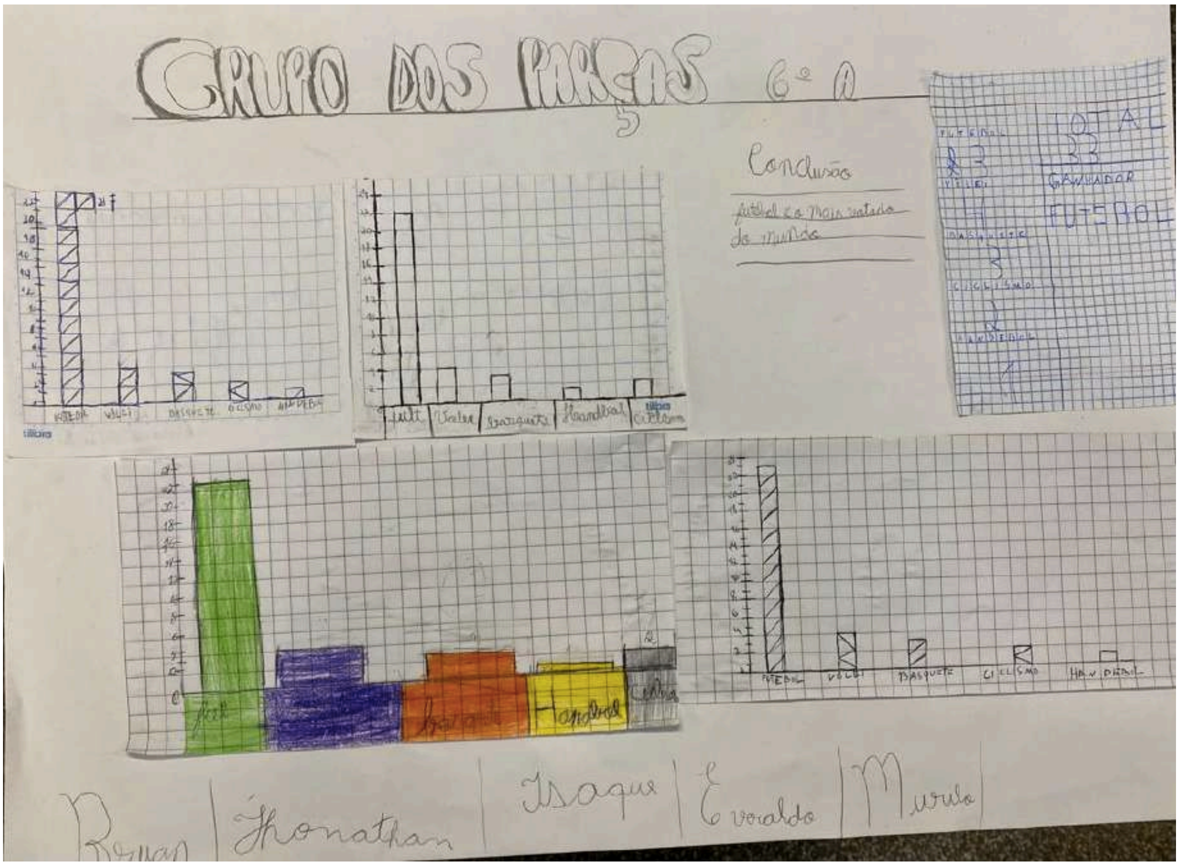


GRÁFICO DE JAPA
GRUPO DO JAPA
6ºA
NEY
CR7


MEIO DE LOCOMOÇÃO
CONCLUSÕES
AQUI VOCÊS OBSERVE QUE A MAIORIA PREFERIU A ANDAR DO QUE OS OUTROS MEIOS DE LOCOMOÇÃO
NOME DO GRUPO:
OS VELOZES E FURIOSOS
NOTA 1000


GRUPO COBRA KAI!
PERGUNTA: QUAL É SEU PAÍS PREFERIDO?
PAÍS PREFERIDO DE CADA UM?


GRUPO DOS PARÇAS 6º A
Conclusão
futebol é o mais votado do mundo
TOTAL 33
GANHADOR FUTEBOL

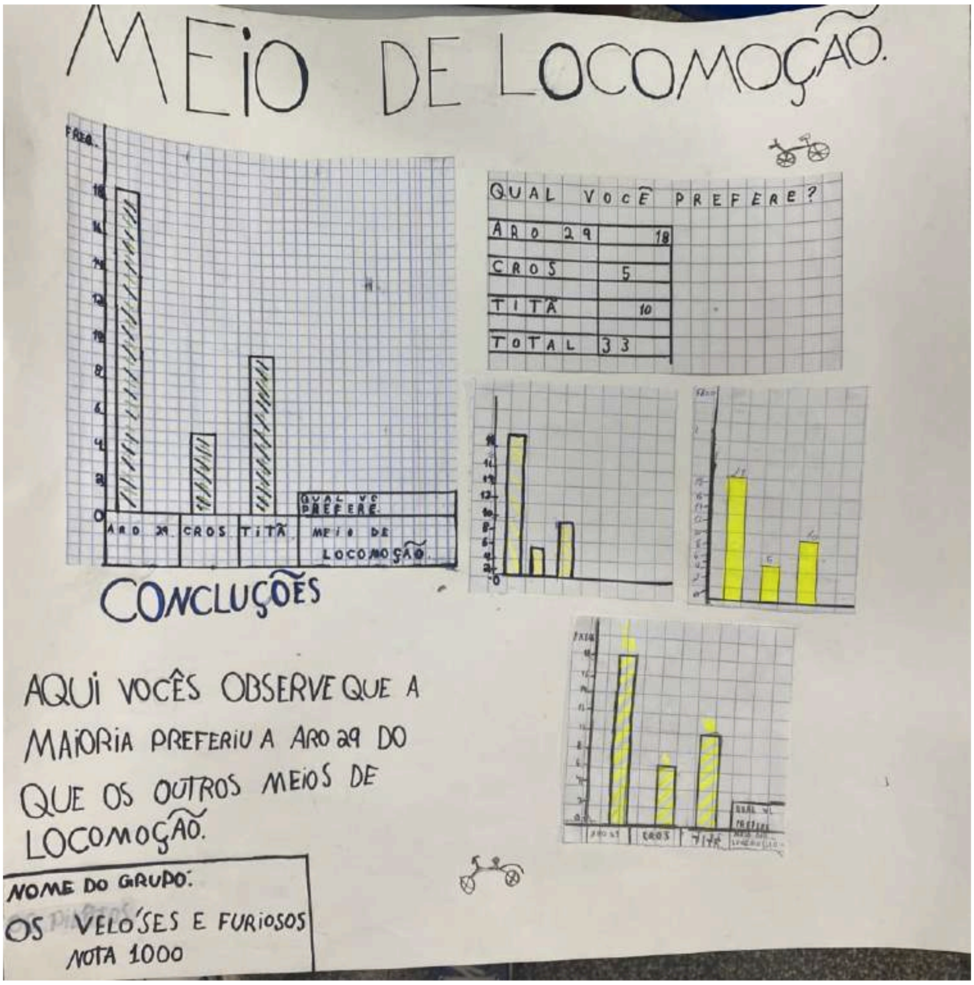
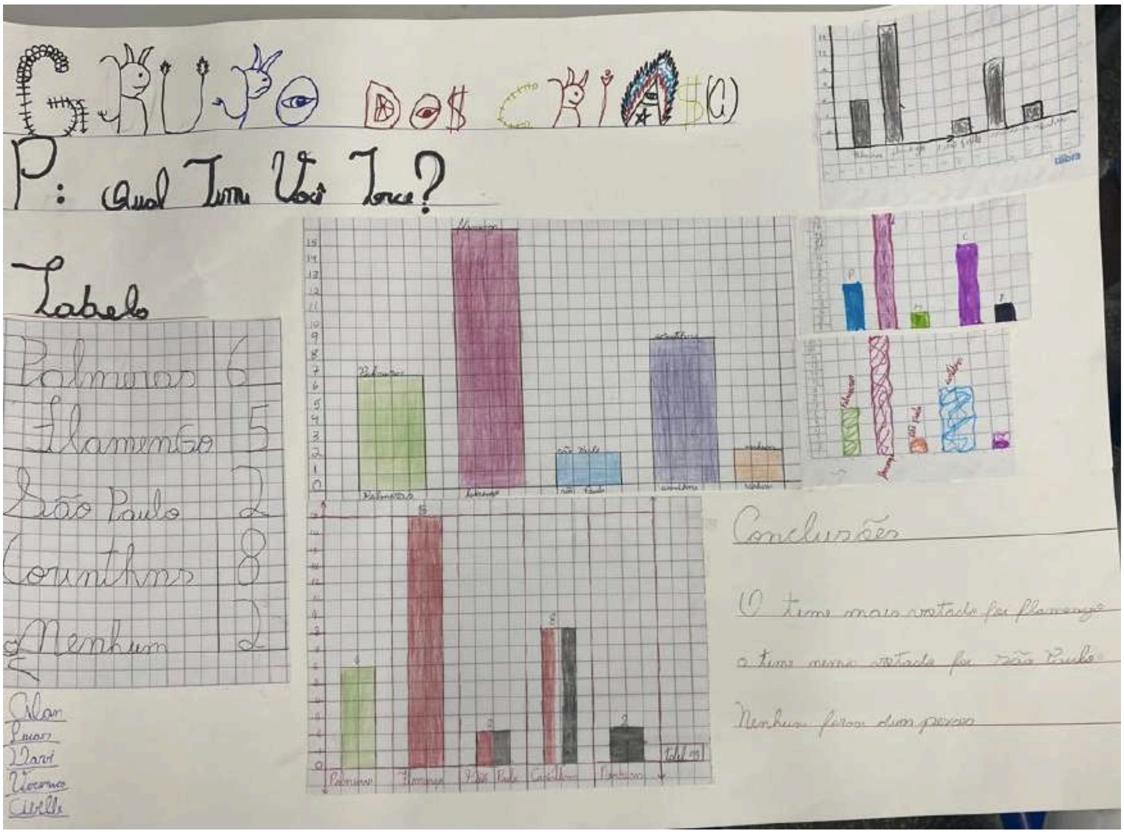
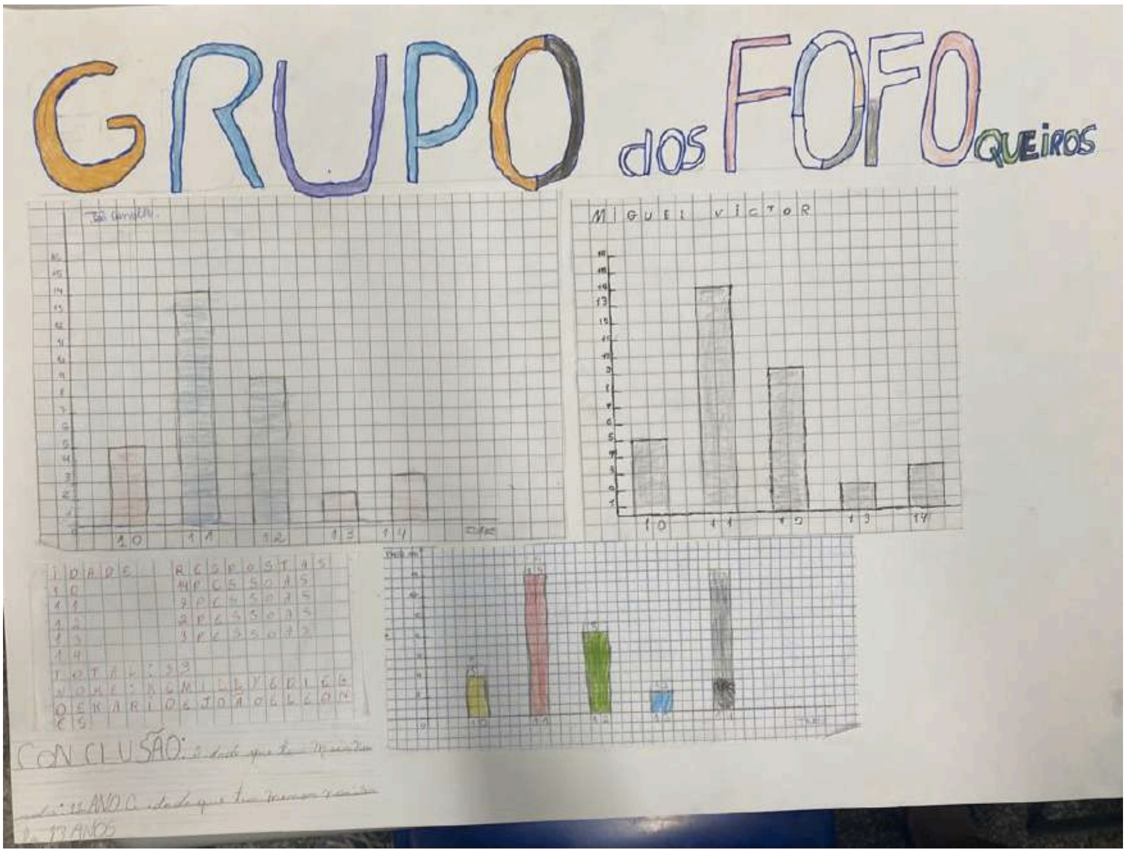
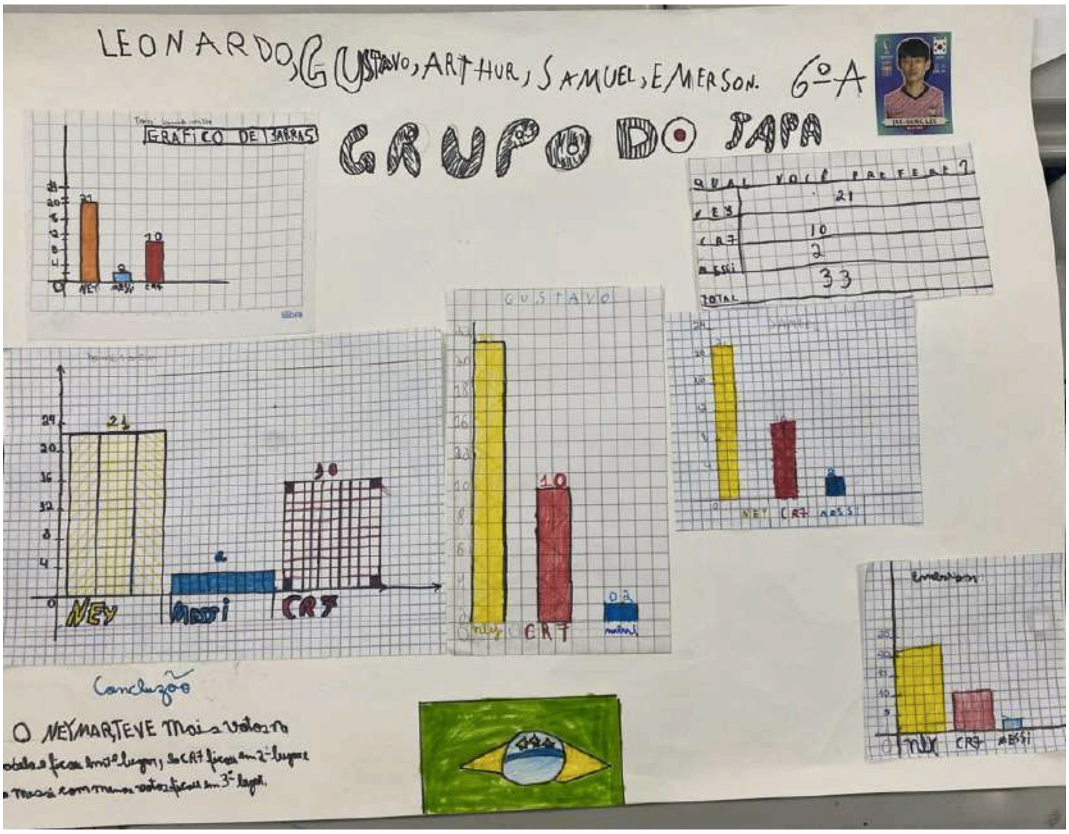
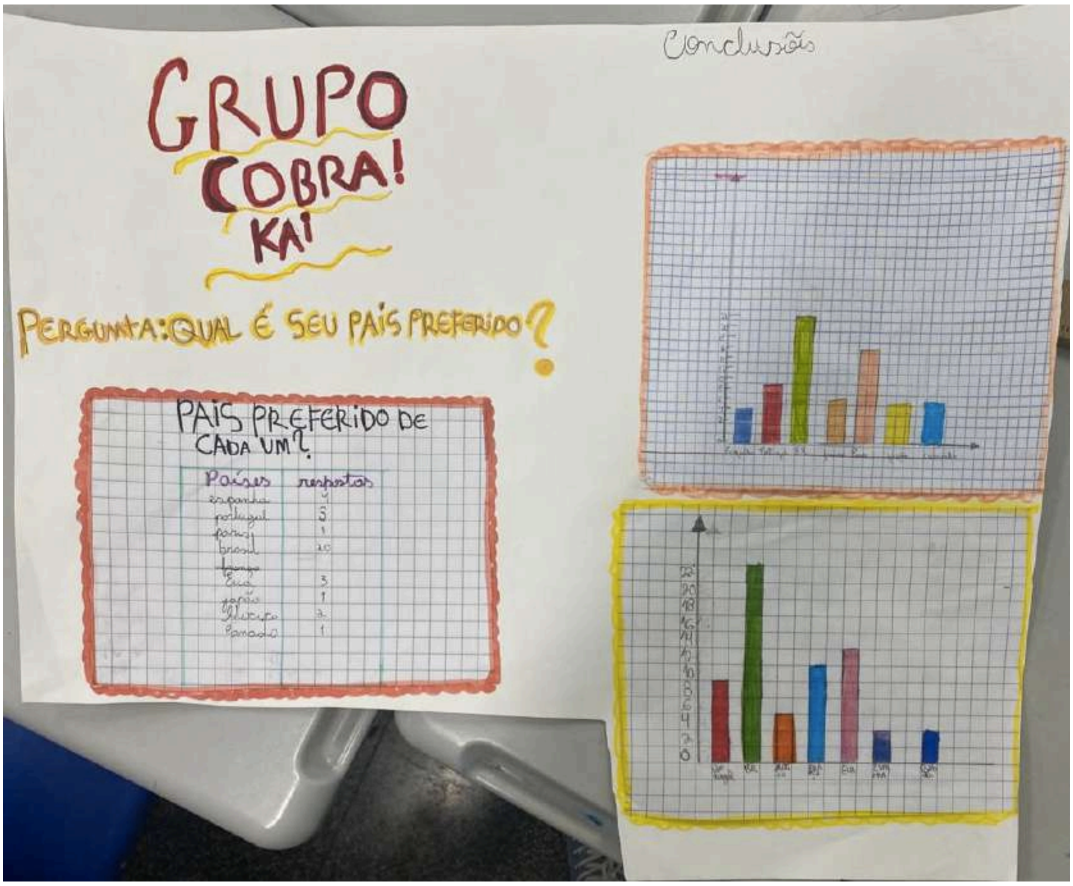

MEIO DE LOCOMOÇÃO.
QUAL VOCÊ PREFERE?
ARO 29 18
CROS 5
TITÃ 10
TOTAL 33
CONCLUÇÕES
AQUI VOCÊS OBSERVE QUE A MAIORIA PREFERIU A ARO 29 DO QUE OS OUTROS MEIOS DE LOCOMOÇÃO.
NOME DO GRUPO:
OS VELO'SES E FURIOSOS
NOTA 1000


Tabela
Gráficos →
CONCLUSÕES:







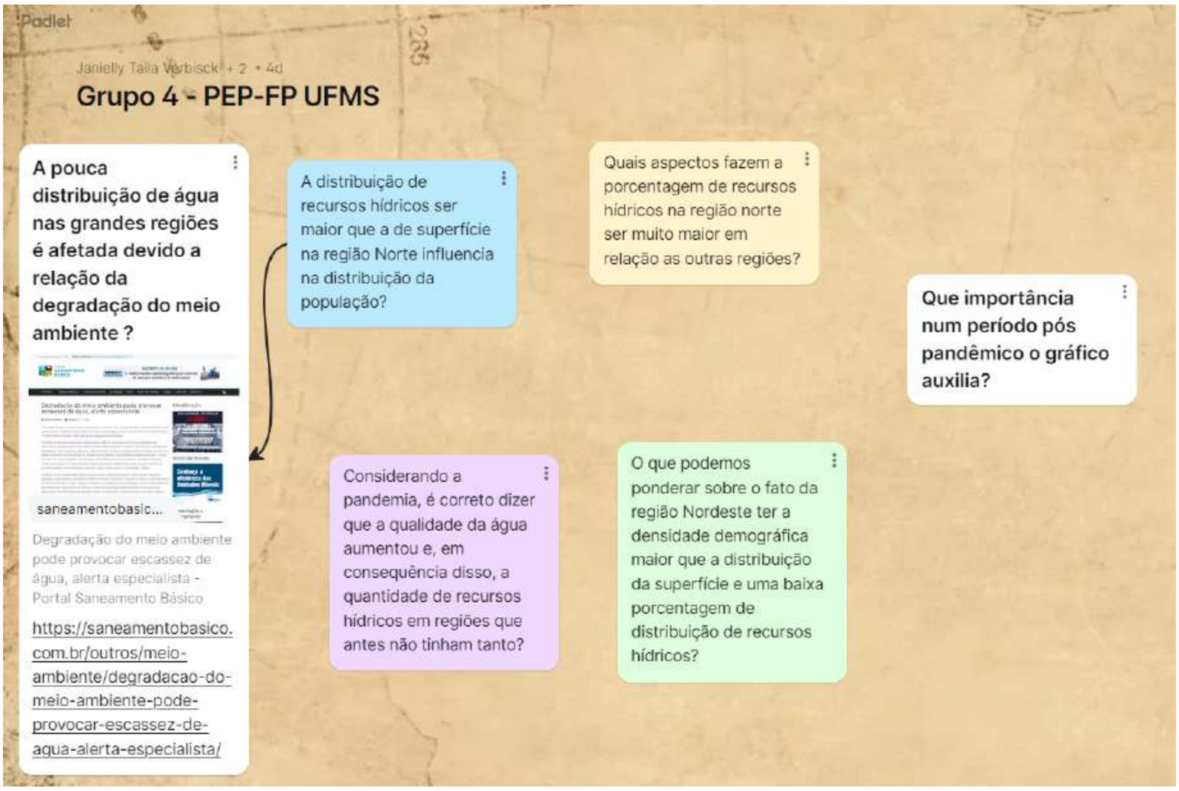


Relatório final do Grupo 4

Parte I – PEP

- Questões formuladas Q_i

Colocar as perguntas/ questões consideradas no estudo. Pode ser o próprio mapa de questões construído no Padlet ou a lista de questões elaboradas.



- Mídias

Fontes de informação consultadas. Por exemplo, páginas web, documentos etc.

https://saneamentobasico.com.br/outros/meio-ambiente/degradacao-do-meio-ambiente-pode-provocar-escassez-de-agua-alerta-especialista/

https://brasil.mapbiomas.org/estatisticas/

chrome-extension://efaidnbmnnnibpcajpcglclefindmkaj/https://www.scielosp.org/article/ssm/content/raw/?resource_ssm_path=/media/assets/rsp/v36n3/10502.pdf

https://www.aguasustentavel.org.br/conteudo/blog/129-o-desmatamento-tem-relacao-com-a-crise-hidrica-entenda

https://www.aguasustentavel.org.br/conteudo/blog/157-um-panorama-geral-sobre-agua-na-regiao-norte https://agenciabrasil.ebc.com.br/radioagencia-nacional/geral/audio/2017-03/muita-agua-e-pouco-acesso-pela-populacao-uma-realidade-na

https://www.ebc.com.br/especiais-agua/agua-no-brasil/#:~:text=Cerca%20de%2012%25%20da%20disponibilidade,dos%20recursos%20h%C3%ADdricos%20do%20pa%C3%ADs

- **Respostas preexistentes** ***R***$^{\Diamond}$

Identificar as respostas preexistentes encontradas nas mídias, as R$^{\Diamond}$*. Por exemplo, banco de dados etc. Pode ser feito apenas uma descrição do que foi utilizado no estudo.*

A pouca distribuição de água nas grandes regiões é afetada devido a relação da degradação do meio ambiente?

A degradação ambiental é um dos motivos da pouca distribuição de água nas grandes regiões. De acordo com Moraes e Jordão (2002):

> Há poucas regiões no mundo ainda livres dos problemas da perda de fontes potenciais de água doce, da degradação na qualidade da água e da poluição das fontes de superfície e subterrâneas. Os problemas mais graves que afetam a qualidade da água de rios e lagos decorrem, em ordem variável de importância, segundo as diferentes situações, de esgotos domésticos tratados de forma inadequada, de controles inadequados dos efluentes industriais, da perda e destruição das bacias de captação, da localização errônea de unidades industriais, do desmatamento, da agricultura migratória sem controle e de práticas agrícolas deficientes. Os ecossistemas aquáticos são perturbados, e as fontes vivas de água doce estão ameaçadas (Moraes; Jordão, 2002, p. 372).

Além disso, são motivos da pouca distribuição de água das grandes regiões: a poluição, o uso inadequado da água, a seca, as queimadas, a super exploração de recursos naturais, o desmatamento etc. Por essa razão, ocorre a redução do nível de água dos rios e pode secar nascentes, ocasionando até o aumento da emissão de gases poluentes causadores do efeito estufa.

A distribuição de recursos hídricos ser maior que a de superfície na Região Norte influencia na distribuição da população?

A distribuição dos recursos hídricos tem relação com a distribuição da população nas regiões. Percebemos isso pelo gráfico da questão e, também, por meio do texto *Um panorama geral sobre água na Região Norte*, que comenta o fato do nosso país possuir “uma vasta riqueza hídrica”, mas que a distribuição não é homogênea, consequentemente, esse desequilíbrio populacional acontece.

Mesmo com essa riqueza hídrica na Região Norte, não é toda a população que tem acesso. O artigo *Muita água e pouco acesso pela população, uma realidade na Região Norte* traz alguns números que embasam essa afirmação feita em 2017 e, além disso, o superintendente de Gestão da Rede Hidrometeorológica da Agência Nacional de Águas, nesse mesmo ano, disse que "a Amazônia é um paradoxo. Você tem uma grande disponibilidade hídrica, vê aquele rio enorme que não enxerga o outro lado dele, mas, ao mesmo tempo, as pessoas têm problemas de ter água em casa".

Quais aspectos fazem a porcentagem de recursos hídricos na região norte ser muito maior em relação às outras regiões?

O site *Onde está a água no Brasil?* diz que na Região Norte sobra água em áreas com baixa densidade demográfica e há pouca demanda para uso.

Acreditamos que o clima também influencia, pois, dentre os fatores que contribuem para o desequilíbrio hídrico na Região Nordeste, por exemplo, estão: baixos índices e irregularidades de chuvas, reduzida disponibilidade de águas subterrâneas, temperaturas elevadas durante todo o ano, forte insolação e altas taxas de evapotranspiração.

O que podemos ponderar sobre o fato de a Região Nordeste ter a densidade demográfica maior que a distribuição da superfície e uma baixa porcentagem de distribuição de recursos hídricos?

De acordo com o site *Onde está a água no Brasil?*, "a baixa capacidade de produção de água subterrânea no Nordeste é decorrência da falta de chuvas". Além disso, na maioria dos rios do Nordeste, só é possível garantir uma oferta contínua de água com o uso de açudes e reservatórios. Isso ocorre uma vez que os rios naturalmente secam durante os meses de estiagem, tanto devido à baixa pluviosidade quanto à baixa espessura de solo.

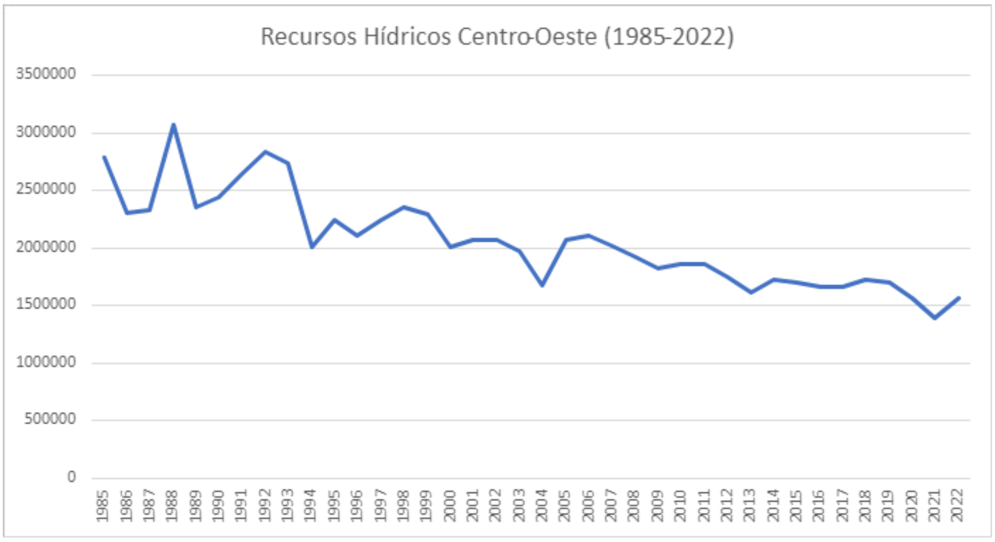
- **Respostas R_i**

Apresentar as respostas construídas pelo grupo.

Dados anteriores/dados atuais: como era a distribuição, no Brasil, em anos anteriores a pesquisa atual?

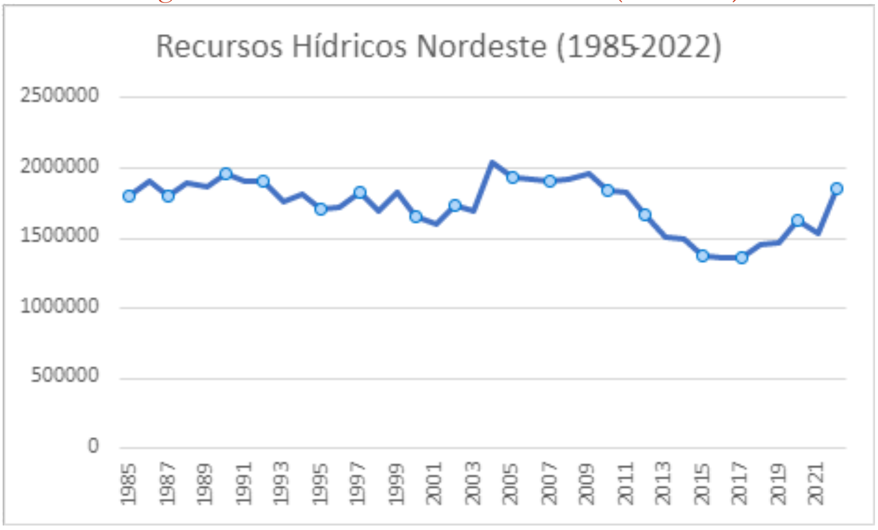
As Figuras 1 a 5 mostram a distribuição de recursos hídricos (área/ha) por região do Brasil entre os anos 1985 e 2022. Os gráficos foram feitos a partir dos dados da tabela no *link* *Dados dos recursos hídricos por região (1985-2022)*, por meio da ferramenta de produção de gráficos do Excel.

Figura 1 - Recursos hídricos Centro-Oeste (área/ano)



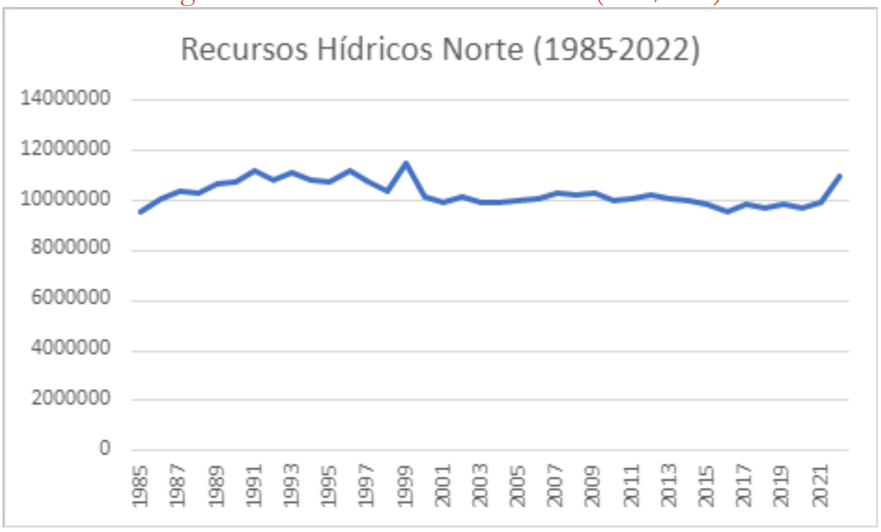
Fonte: produzidos pelos autores.

Figura 2 - Recursos hídricos Nordeste (área/ano)



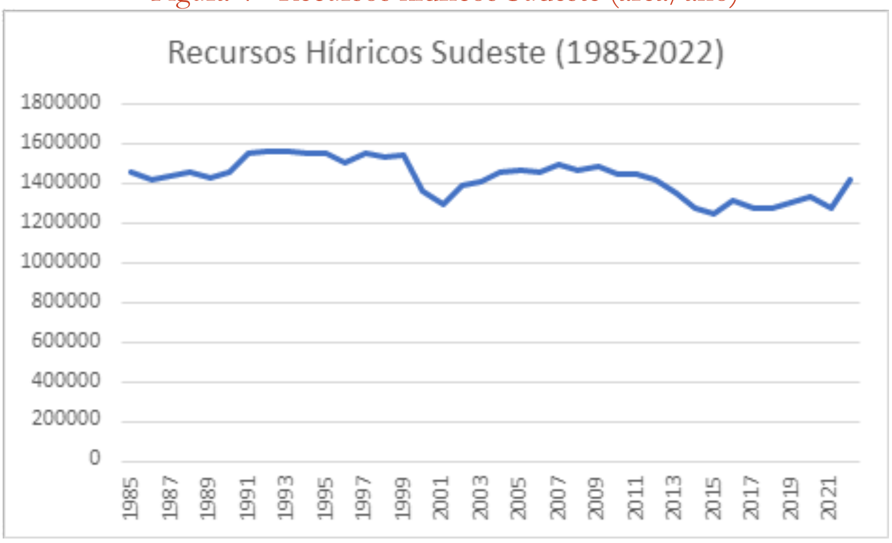
Fonte: produzidos pelos autores.

Figura 3 - Recursos hídricos Norte (área/ano)



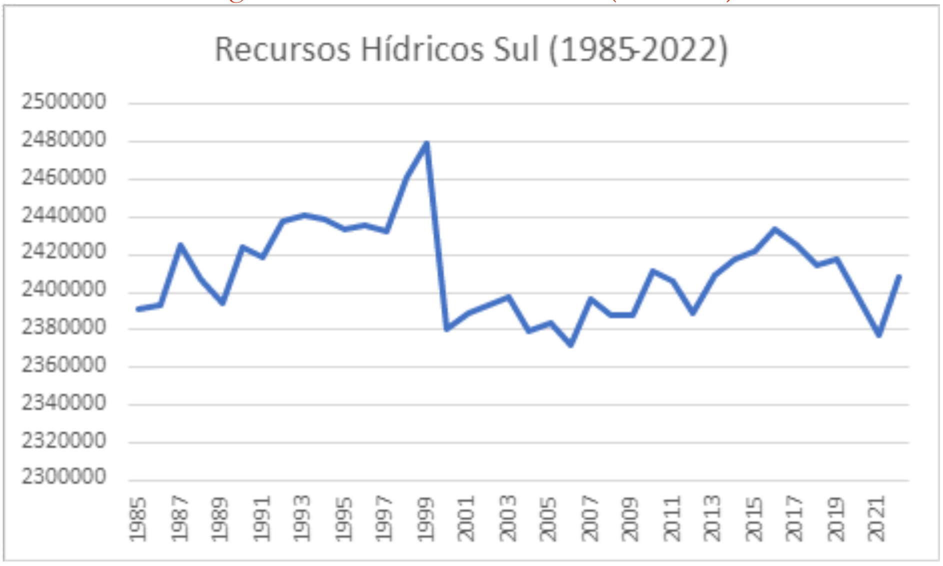
Fonte: produzidos pelos autores.

Figura 4 - Recursos hídricos Sudeste (área/ano)



Fonte: produzidos pelos autores.

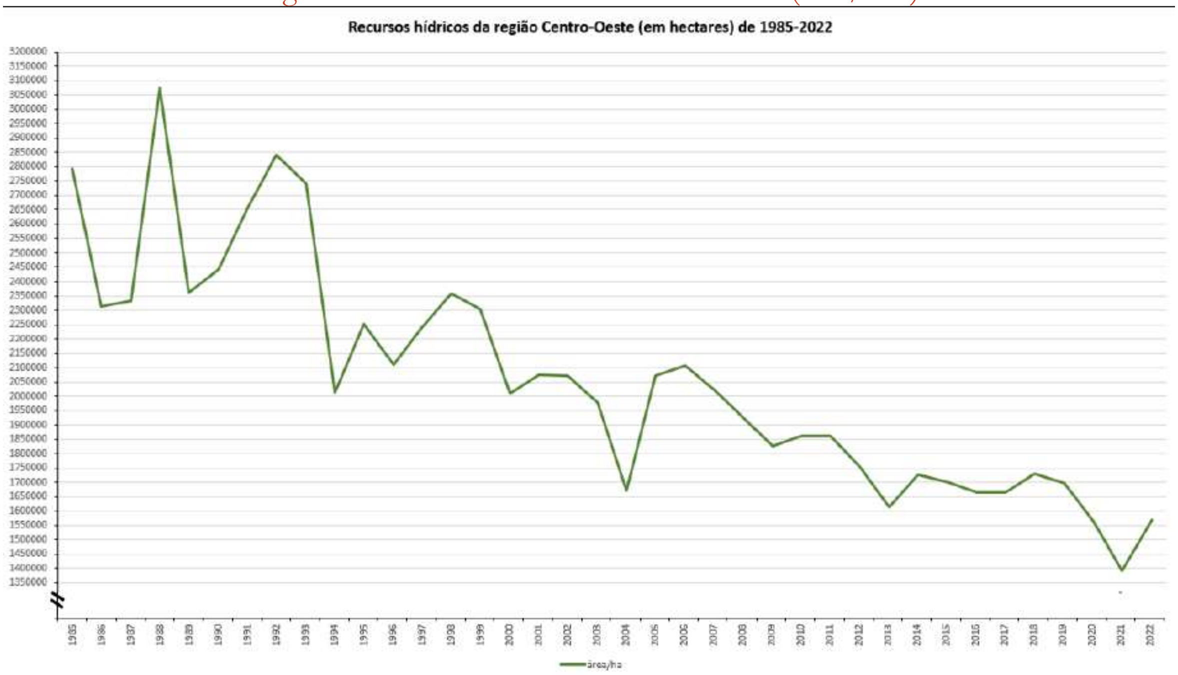
Figura 5 - Recursos hídricos Sul (área/ano)



Fonte: produzidos pelos autores.

Após discussão na aula sobre a produção dos gráficos acima, notamos alguns elementos incorretos na construção dos gráficos por meio da ferramenta automática do Excel. Com base em um vídeo produzido pela professora, produzimos novos gráficos, considerando esses elementos:

Figura 6 - Recursos hídricos Centro-Oeste (área/ano)



Fonte: produzidos pelos autores.

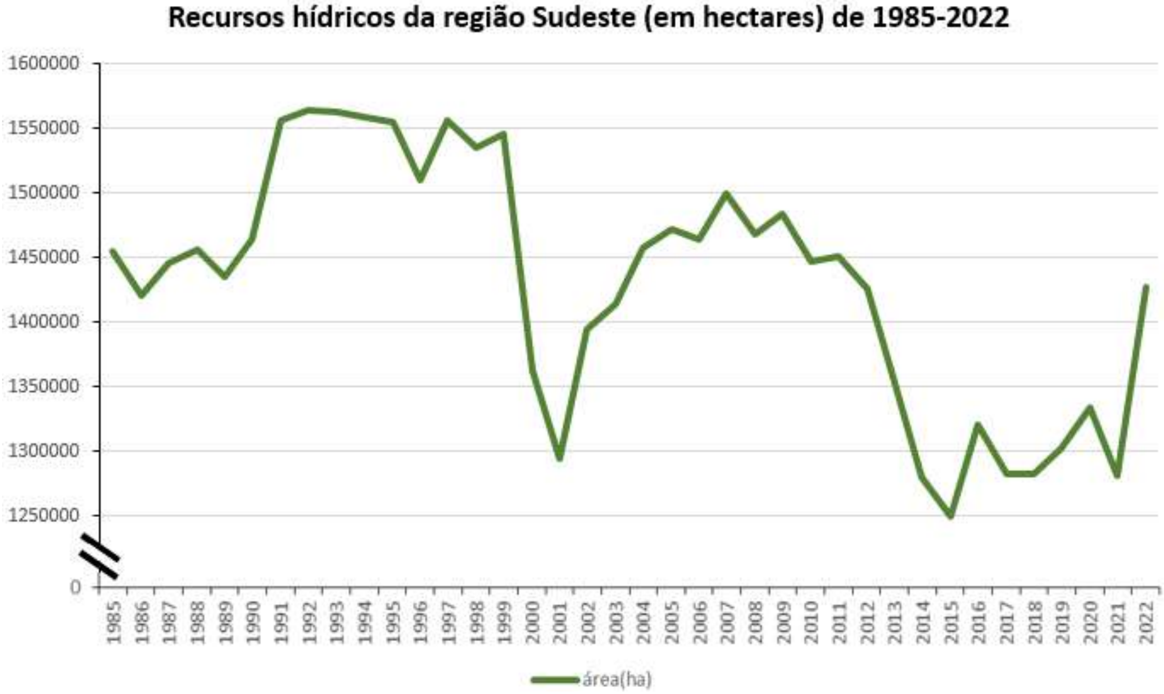
Figura 7 - Recursos hídricos Nordeste (área/ano)

Fonte: produzidos pelos autores.

Figura 8 - Recursos hídricos Norte (área/ano)

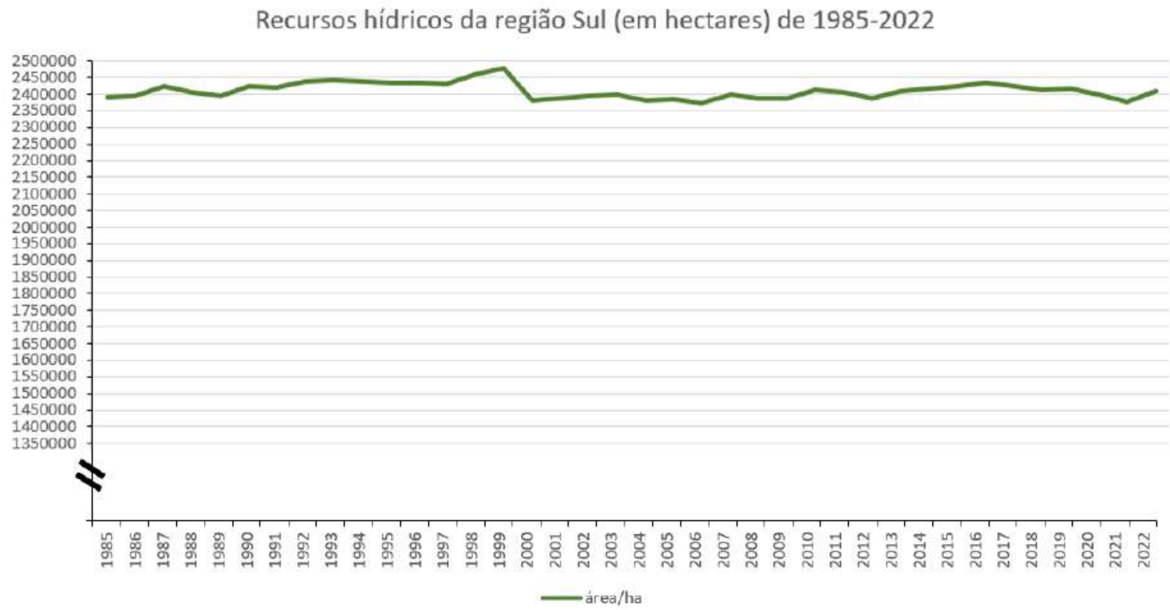
Fonte: produzidos pelos autores.

Figura 9 - Recursos hídricos Sudeste (área/ano)



Fonte: produzidos pelos autores.

Figura 10 - Recursos hídricos Sul (área/ano)



Fonte: produzidos pelos autores.

- Resposta *R*♥

Apresentar a discussão da resposta final para o estudo realizado: análises, tabelas e gráficos construídos etc.

O gráfico auxilia na percepção sobre a importância que cada região tem em sua utilização de recursos hídricos. Nos cenários atuais, evidencia-se que regiões mantivessem constante o seu volume de recursos, como as Regiões Sudeste, Norte e Nordeste.

A Região Centro-Oeste mostrou que seus recursos hídricos tiveram uma grande diminuição quando comparada aos primeiros anos.

Na Região Sul, percebemos que no gráfico não há oscilações quando comparado aos outros gráficos.

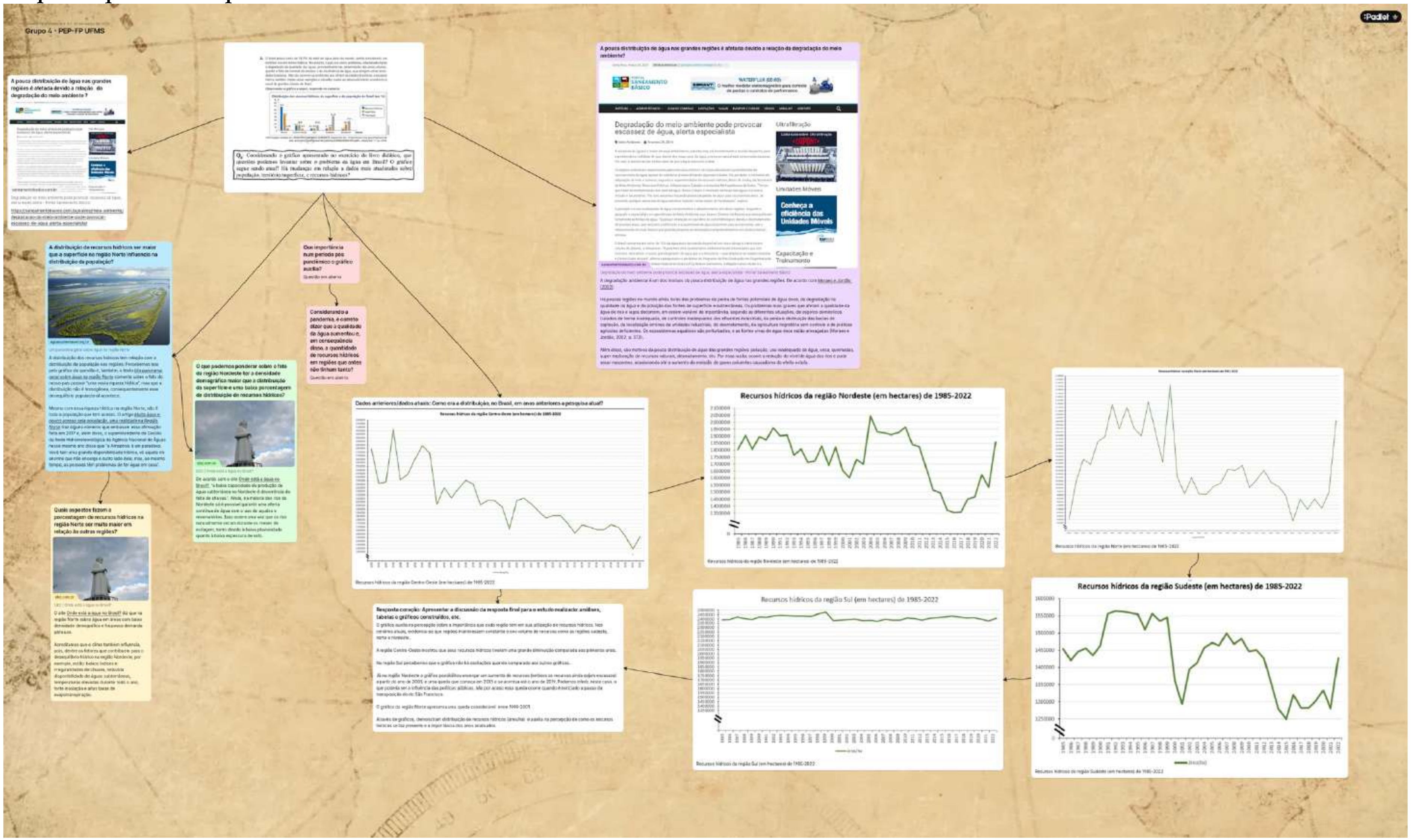
Já na região Nordeste o gráfico possibilitou enxergar um aumento de recursos (embora os recursos ainda sejam escassos) a partir do ano de 2003, e uma queda que começa em 2013 e se acentua até o ano de 2019. Podemos inferir, neste caso, o que poderia ser a influência das políticas públicas, não por acaso, essa queda ocorre quando é noticiado a pausa da transposição do rio São Francisco. O gráfico da Região Norte apresenta uma queda considerável entre 1999-2001. Os gráficos demonstram a distribuição de recursos hídricos (área/ha) e auxiliam na percepção de como os recursos hídricos se fazem presentes e a importância dos anos analisados.

- Questões em aberto

Apresentar as questões que o grupo não conseguiu responder ou as questões que o grupo acredita que poderiam ser estudadas mais profundamente.

Considerando a pandemia, é correto dizer que a qualidade da água aumentou e, em consequência disso, a quantidade de recursos hídricos em regiões que antes não tinham tanto?
Qual seria a importância, no período pós-pandêmico, do gráfico? Ou: Em que o gráfico pode auxiliar no período pós-pandêmico?

Mapa de questões-respostas



Parte II – Análise do PEP

<u>Dialética Questões-Respostas:</u> comentem sobre a utilidade desta ferramenta durante o PEP

- pontos positivos:

Organização de ideias, melhor visualização das questões e praticidade. Por meio desta ferramenta, conseguimos relacionar as questões e suas respectivas respostas.

- pontos negativos:

Dependendo da quantidade de questões, fica difícil visualizar o processo de estudo e pesquisa de um modo geral, amplo.

- Dificuldades encontradas?

No começo, apresentamos uma dificuldade em manusear as ferramentas, visto que nenhum integrante do grupo havia utilizado o *Padlet*. Além disso, houve uma certa dificuldade em relacionar a organização das questões no *Padlet* com o que se pedia na formação.

<u>Dialética mídia-meio:</u>

- Que mídia foram utilizadas durante o PEP?

Sites e artigos.

- Que objetos foram utilizados durante o PEP?

Excel e *Padlet*.

- Que conhecimentos estatísticos foram utilizados durante o PEP?

Proporção, razão, amplitude, porcentagem.

- Que dados foram utilizados durante o PEP?

Os dados sobre os recursos hídricos, em hectares, presentes em cada região do Brasil nos anos de 1985 a 2022.

- Que estudos poderiam ou precisam ser aprofundados?

Estudos relacionados à qualidade dos recursos hídricos das regiões do Brasil ao longo dos anos e, além disso, estudos da relação entre os recursos hídricos (sua qualidade e quantidade) e o período pandêmico.

- Dificuldades encontradas?

A partir dos dados utilizados, a professora sugeriu que fizéssemos os gráficos correspondentes às regiões. Encontramos dificuldades em construir esses gráficos. Construímos por meio da ferramenta automática do Excel, porém, não nos atentamos aos elementos que devem estar presentes nos gráficos, como, por exemplo, os valores, as categorias, a escala do gráfico nos eixos, a legenda.

Dialética Indivíduo-Coletivo: comentem sobre a organização do grupo durante o PEP.

- Discussões coletivas?
  Só discutimos coletivamente no momento em que nos reunimos para dialogar sobre os recursos hídricos no Brasil, quando foi apresentado a Q0.

- Divisão de tarefas?
  No início, as perguntas foram inseridas no *Padlet* de forma coletiva. Conforme a professora foi propondo novas tarefas, o grupo as cumpria conforme a disponibilidade de cada um.

- Qual a responsabilidade de cada um?

- Dificuldades encontradas?
  A falta de interação do grupo foi a dificuldade encontrada.

- O trabalho em grupo poderia ser melhorado? Em quais aspectos?
  Sim, trabalhar de forma mais coletiva e direcionada, visto que, cada um contribuía com as atividades de forma individual.

Módulo III – planejamento de atividade de investigação

Turma: 1º ano do Ensino Médio
Escola: Escola Estadual José Antônio Pereira
Conteúdo: Função afim
Currículo Novo Ensino Médio:

Números e Álgebra	**(MS.EM13MAT510)** Investigar conjuntos de dados relativos ao comportamento de duas variáveis numéricas, usando ou não tecnologias da informação, e, quando apropriado, levar em conta a variação e utilizar uma reta para descrever a relação observada.	Pares ordenados no plano cartesiano ou em tabelas em que representam ou não funções; Funções afins (função polinomial do 1º grau, linear, identidade e constante), taxa de variação; Gráficos de funções (crescimento/decrescimento); Razões trigonométricas objetivando a tangente em relação à inclinação do gráfico; Equação da reta: coeficiente angular.	A análise de dados/tabelas possibilita a representação por uma função polinomial de 1º grau. Citam-se os exemplos, conta de água, energia, gás, juros simples e/ou casos que determinado valor está em função de um valor fixo e de uma variável, estudo da taxa de variação da função que descreve tais situações, com o objetivo de estabelecer estimativas e previsões, observar padrões e fazer suposições, com ou sem auxílio de software de geometria dinâmica. Esta Habilidade está relacionada com as Habilidades **(MS.EM13MAT401)**, **(MS.EM13MAT507)** e **(MS.EM13MAT508)**.
Números e Álgebra	**(MS.EM13MAT501)** Investigar relações entre números expressos em tabelas para representá-los no plano cartesiano, identificando padrões e criando conjecturas para generalizar e expressar	Funções afins (função polinomial do 1º grau, linear, identidade e constante), taxa de variação; Gráficos de funções (crescimento/decrescimento);	Esta Habilidade demanda, entre outras, criar conjecturas. Assim, uma possível forma de desenvolvê-la é representar as cidades de Mato Grosso do Sul no plano cartesiano mediante pontos, utilizando o mapa do estado como plano de fundo, seja no papel ou em software/App. Neste contexto, tal Habilidade incentiva o desenvolvimento da curiosidade para aprender e a imaginação criativa, além de oportunizar um

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CURRÍCULO DE REFERÊNCIA DE MATO GROSSO DO SUL

ENSINO MÉDIO

MATEMÁTICA E SUAS TECNOLOGIAS - 1º ANO EM

COMPONENTE CURRICULAR: MATEMÁTICA

Eixo Temático	Habilidades	Objetos de Conhecimento	Sugestões Didáticas
	algebricamente essa generalização, reconhecendo quando essa representação é de função polinomial de 1º grau.	Razões trigonométricas objetivando a tangente em relação à inclinação do gráfico; Equação da reta: coeficiente angular.	olhar mais atento para o mapa e conjecturar se existem três cidades alinhadas. Para isso, sugere-se a construção de tabelas com as coordenadas dos pontos, para posteriormente investigar se existe uma reta que os contenha. Quando houver, recomenda-se obter a função polinomial do 1º grau que descreve o trajeto que liga as cidades. Esta Habilidade está relacionada com a Habilidade **(MS.EM13CHS606)**.

Números e Álgebra	**(MS.EM13MAT401)** Converter representações algébricas de funções polinomiais de 1º grau em representações geométricas no plano cartesiano, distinguindo os casos nos quais o comportamento é proporcional, recorrendo ou não a softwares ou aplicativos de álgebra e geometria dinâmica.	Funções afins e polinomiais do 1º grau; Gráficos de função afim; Estudo da variação de funções polinomiais de 1º grau: crescimento, decrescimento, taxa de variação da função.	Para o desenvolvimento desta Habilidade, recomenda-se a resolução e elaboração de situações-problemas do cotidiano, envolvendo a ideia de função polinomial do primeiro grau, por exemplo, o salário fixo de um vendedor acrescido de sua comissão em virtude das vendas realizadas ou a arrecadação de impostos de Mato Grosso do Sul. Tem-se como ponto de partida a utilização dos conhecimentos construídos e experiências, com o objetivo de entender e explicar o conceito de função matemática no contexto do cotidiano, de maneira determinada, proativa e confiante. Ainda, para além disso, sugere-se compreender e interpretar os métodos que descrevem esta realidade algébrica e geometricamente, por intermédio de textos e esquemas/diagramas. Diante disso, é possível desenvolver a curiosidade para aprender, no sentido de explorar ideias criativas para construir e reconstruir gráfico por planilha eletrônica/GeoGebra, explorar funções definidas por uma ou mais sentenças (tabela do Imposto de Renda, contas de luz, água, gás), em suas respectivas representações algébricas e geométricas, além de estudar domínios de validade, imagem, crescimento e decrescimento, com ou sem apoio de tecnologias digitais para organização a longo prazo. Em relação às representações algébricas e geométricas, essa Habilidade possibilita o desenvolvimento do raciocínio lógico, a capacidade de expressar opiniões e defender uma posição, caso seja necessário. Esta Habilidade está relacionada com a Habilidade **(MS.EM13CNT106)**.

Conteúdo	Função Afim
Habilidades	MS.EM13MAT401, MS.EM13MAT501, MS.EM13MAT510
Metodologia	Aula 1 1º momento: 5-10 minutos: organização da turma e apresentação dos professores envolvidos na atividade; 2º momento: 5-10 minutos: a sala será dividida em 5 grupos para realizarem algumas situações-problema (anexo 1) propostas pelos professores, baseada nas metodologias ativas; 3º momento: 10-25 minutos: serão distribuídos os problemas para os grupos fazerem a leitura e iniciarem a tarefa proposta; 4º momento: 5-10 minutos: socialização. Aula 2 1º momento: 5-10 minutos: organização da turma; 2º momento: 15-20 minutos: exemplificação da situação-problema no quadro (situação 6 em anexo). Nesse momento, será realizada a visita às obras, enunciando os conceitos principais relacionados ao tema e exemplificando o tipo de tarefa que se pretende como resultado do estudo de caso de cada grupo; 3º momento: 20-25 minutos: será entregue uma cartolina para cada grupo e os alunos elaborarão os gráficos, tabelas e informações da situação-problema nestas cartolinas. 4º momento: 5 minutos: socialização.

	Aula 3 1º momento: 5-10 minutos: organização da turma para apresentação das cartolinas; 2º momento: 25-35 minutos: cada grupo irá apresentar seus estudos e a construção das cartolinas que eles realizaram durante o desenvolvimento destas atividades, destacando o tipo de gráfico escolhido, as suas análises, a construção e análise das tabelas. 3º momento: 10-15 minutos: encerramento das atividades e comentários finais feitos pelos professores responsáveis.
Recursos didáticos	1- Quadro branco; 2- marcadores para quadro branco; 3- caderno; 4- lápis; 5- borracha; 6- caneta; 7- cartolina; 8- canetas coloridas para preenchimento das cartolinas; 9- lápis de cor; 10- calculadora ou calculadora do celular.
Avaliação	Avaliação ocorrerá com a composição de dois aspectos: avaliação formativa com peso 1, que se trata do processo do desenvolvimento das atividades de maneira satisfatória, e será registrada nas anotações do professor responsável, ao final de cada aula; e a avaliação do projeto final com peso 2, onde serão observados os aspectos estéticos e análise dos dados dos estudantes. A nota final poderia ser obtida pela média ponderada das atividades.

Situação 1:

As funções são utilizadas na representação cotidiana de situações que envolvam **valores constantes** e **variáveis**, sempre colocando um valor em função do outro. Por exemplo, ao abastecermos o carro no posto de gasolina, o preço a ser pago depende da quantidade de litros de combustível colocada no tanque ou mesmo do valor a ser pago por uma viagem de táxi, na qual o preço é calculado por uma parte fixa e uma quantidade que varia de acordo com o trajeto percorrido. Um motorista de táxi cobra R$ 4,30 de bandeirada (valor fixo) mais R$ 0,75 por quilômetro rodado (valor variável).

a) Escreva a equação que descreve esta situação.

b) Escreva a função que descreve esta situação.

c) Determine $f(0)$ e $f(x) = 0$.

d) Desenhe o gráfico desta função.

e) Determine o valor a ser pago por uma corrida relativa a um percurso de 18 quilômetros.

f) Determine a quantidade de quilômetros percorridos sendo que o custo da corrida foi de 48 reais.

Situação 2:

O preço de venda de um livro é de R$25,00 a unidade. Sabendo que o custo de cada livro corresponde a um valor fixo de R$ 4,00 mais R$ 6,00 por unidade:

a) Escreva a equação que descreve esta situação, sendo o lucro líquido (valor descontado das despesas) na venda de x livros, e o lucro obtido na venda.

b) Escreva a função que descreve esta situação.

c) Constitua uma função capaz de determinar o lucro líquido (valor descontado das despesas) na venda de x livros, e o lucro obtido na venda de 500 livros, 750 livros e 1000.

d) Construa o gráfico desta função, indique as vendas x = 500; x = 750; x=1.000.

e) É possível descobrir a quantidade de livros vendidos a partir do valor do lucro?

Situação 3:

O salário de um vendedor é composto de uma parte fixa no valor de R$1.320,00, mais uma parte variável de 8% sobre o valor de suas vendas no mês.

a) Escreva a equação que representa este caso.

b) Escreva a função que representa este caso.

c) Caso ele consiga vender R$380 000,00, calcule o valor de seu salário.

d) Caso ele consiga vender R $550.000,00, calcule o valor de seu salário.

e) Em um mês este funcionário ganhou 2.850,00. Quanto ele vendeu?

f) Desenhe o gráfico desta função.

Situação 4:
Um homem presta serviços autônomos e cobra os clientes da seguinte forma: R$ 60,00 é o valor fixo da visita e R$ 5,00 por hora de trabalho. Organize os dados no quadro abaixo.

Horas trabalhadas	Valor fixo	Valor adicional	Custo do trabalho
4 horas			
5 horas			
6 horas			
7 horas			

a) Qual é o custo do trabalho se o autônomo cobrar por 8 horas de trabalho?

b) Qual é a expressão matemática usada para calcular o custo do trabalho do autônomo de acordo com o preço de hora trabalhada?
c) Como seria a representação dessa situação em um gráfico, colocando as horas trabalhadas no eixo x e o custo do trabalho no eixo y?
d) Este gráfico representa uma função crescente ou decrescente? Por quê?
e) Qual a taxa de crescimento e decrescimento do gráfico? Justifique.

Situação 5:

Para produzir um determinado produto, uma indústria tem um custo fixo de R$ 32,00 mais R$ 1,50 por peça produzida. Preencha o quadro a seguir, de maneira a organizar os dados.

Quantidade de peças	Valor fixo	Valor adicional	Custo da produção
1 peça			
2 peças			
3 peças			
4 peças			

a) Qual é o custo da produção se a indústria produzir 10 peças?
b) Qual é a expressão matemática usada para calcular o valor do produto de acordo com a quantidade de peças produzidas?
c) Como seria a representação dessa situação em um gráfico, colocando a quantidade de peças produzidas para determinado produto no eixo x e o valor final do produto no eixo y?
d) Este gráfico representa uma função crescente ou decrescente? Por quê?
e) Qual a taxa de crescimento e decrescimento do gráfico? Justifique.

Situação 6 - exemplo:

Uma determinada espécie de pimenta, ao atingir 20 centímetros de altura, começa a crescer de forma linear. A cada dia que se passa, essa planta aumenta 2,5 centímetros. Assim:

a) Escreva a equação que descreve esta situação.

b) Escreva a função que descreve esta situação.

c) Desenhe o gráfico desta função.

d) Determine quantos centímetros essa planta terá com 10 dias.

e) Determine f(0) e f(x) = 0.

Bibliografia:

- **Questão inicial da atividade de investigação;**

Cada grupo terá uma situação para discutir, organizar e interpretar os dados, construir e analisar os gráficos.

- **Como se coloca a questão, o que se propõe fazer e que ferramentas estão disponíveis?**

A questão dá aos alunos dados que precisam ser organizados em um quadro. Esses dados serão interpretados e analisados por eles, a partir disso, trabalhamos o gráfico da função afim. As ferramentas disponíveis são: 1- Quadro branco; 2- marcadores para quadro branco; 3- caderno; 4- lápis; 5- borracha; 6- caneta; 7- cartolina; 8- canetas coloridas para preenchimento das cartolinas; 9- lápis de cor; 10- calculadora ou calculadora do celular.

- **Como o estudo está planejado para ser administrado em classe? Horário aproximado;**

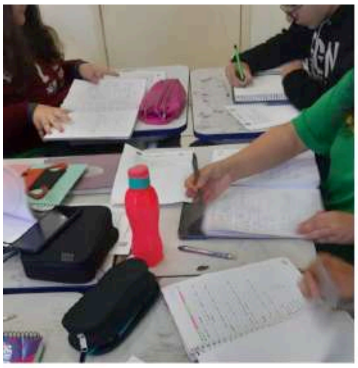
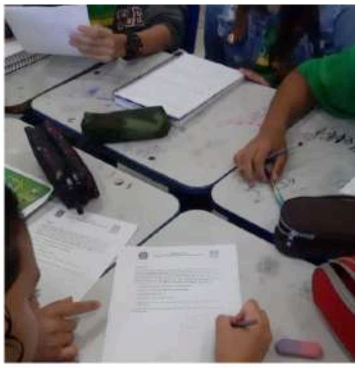
3 aulas de 50 min.

- **Como se espera que a atividade seja concluída?**

Os alunos concluem a atividade, sabendo construir gráficos de função afim e, a partir de coleta de dados, interpretá-los.

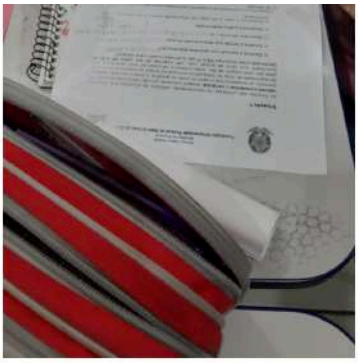
- **Qual/quais conteúdo(s) curricular(es) (em matemática e outros assuntos) pode(m) ser abordado(s) pela atividade proposta?**

Coleta, organização e análise de dados.

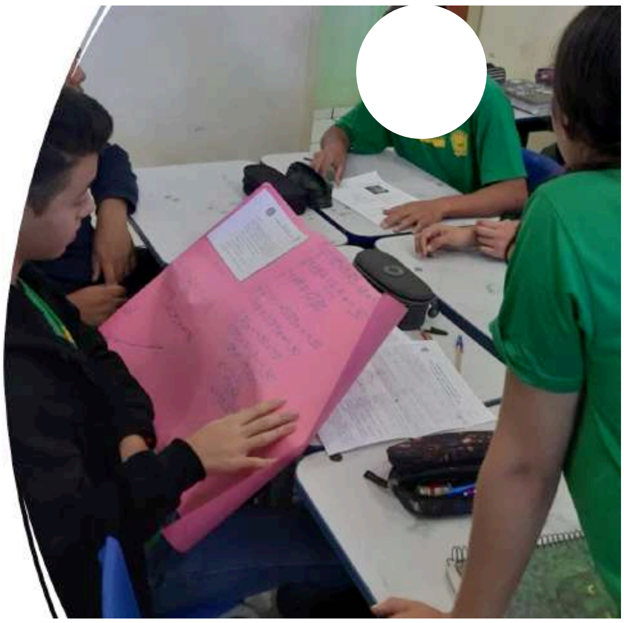
Situação 01

- Apresentaram uma interação muito boa.
- Um aluno no primeiro momento conseguiu interpretar corretamente o restante do grupo junto a ele conseguiu desenvolver a situação.
- Pesquisaram no caderno e em outro momento realizariam a pesquisa sobre o contúdo para tentar solucionar a atividade.

- Apresentaram todas as questões concluidas.
- Alguns integrantes faltaram devido ao frio, porem o grupo trabalhou em equipe e se mostraram bem interessados para terminar a atividade.

- Conseguiram interpretar a atividade completa e se ajudaram para poderem terminar a atividade.



Situação 02

- No primero momento o grupo leu a questão e com isso tentaram buscar no caderno atividades que relacionava a situação.
- Conseguiram realizar a alternativa a.
- Despois de conseguirem realizar a alternativa, não quiseram realizar mais a atividade.

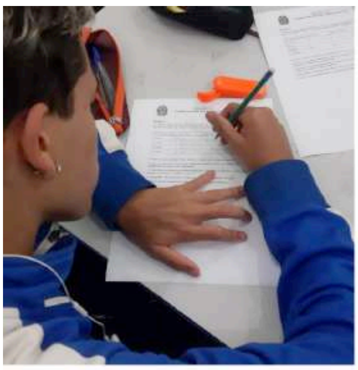
- Não se mostraram enganjados na atividade e tiveram dificuldades na execução das tarefas propostas.

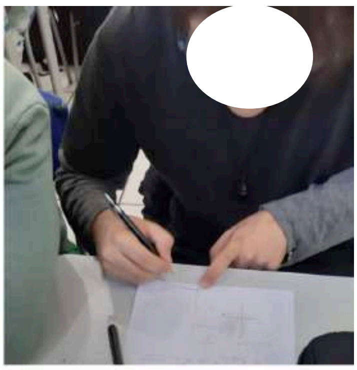
- Desenvolveram somente o que foi instruído e não se mostraram preocupados em se desenvolver.

Situação 03

- Fizeram mais questionamentos.
- Apresentaram dificuldades iniciais para desenvolvimento da atividade, porem mostraram vontade de realizarem a atividade .
- Fizeram o questionamento "aonde esse conteúdo será usado".
- Os integrantes do grupo não se dialogavam para realizarem o exercício, porem buscavam em seu caderno questões relacionada ao conteúdo.

- Apresentaram todas as quesões realizadas.
- Tentaram entregar tudo na aula e se organizaram para a apresentação

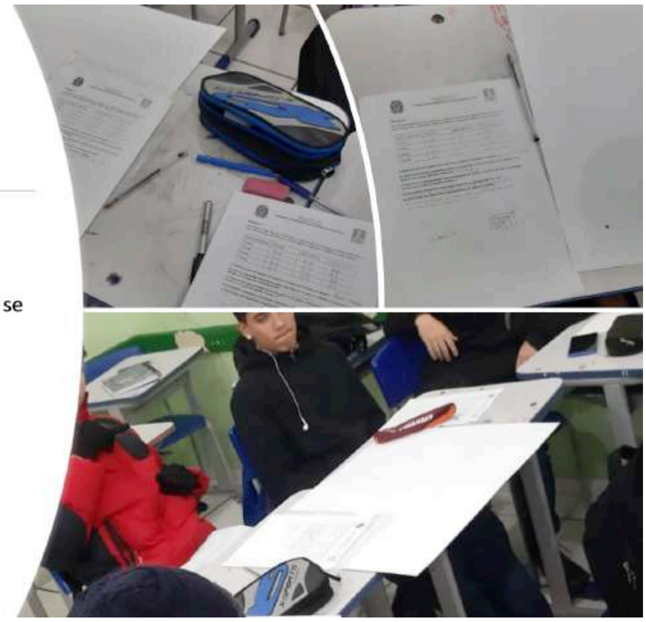




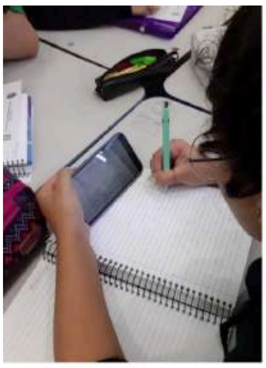
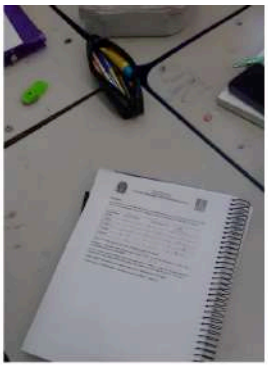
Situação 4

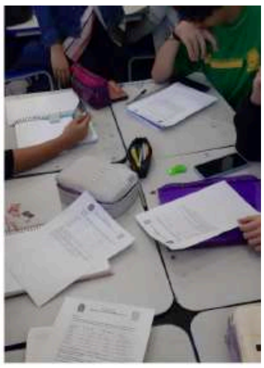
- O grupo desde o primeiro momento tentou realizar a atividade e faltando apenas duas alternativas para terminar a situação.
- Dialogavam muito.
- Fizeram os gráficos.

- Apresentaram a atividade completa.
- Se organizaram para todos do grupo poderem fazer um pouco e começaram a se aorganizar para a apresentação.



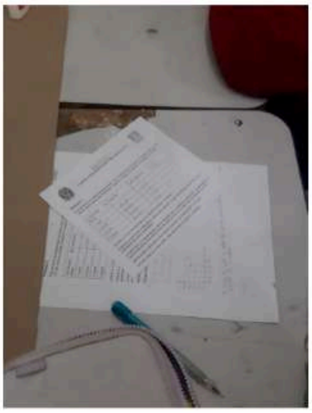
- Apresentaram o trabalho concluido.
- Tiveram grande proveito, porem somente um aluno tomou uma iniciativa para aapresentação, mas grande parte do grupo apresentou.

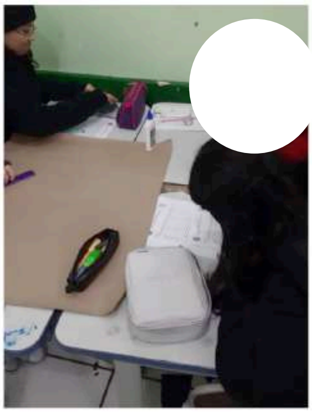





Situação 05

- Buscaram ajuda para poder realizar a atividade.
- Alguns integrantes do grupo não se mostravam interessados.





- Concluiram os exercicios e pediram ajuda para correção junto ao professor.
- Tentara se organizar para ninguem ficar sobrecarregado.

- Desenvolveram toda a situação.
- Conseguiram inteerpretar a situação.

Módulo IV – análises finais

Ecologia: condições e restrições

Uma última tarefa para casa...

1) Reunam-se em seus respectivos grupos;

2) Discutam e respondam a questão: a atividade que nós implementamos na escola foi, de fato, um PEP? Justifiquem.

3) Identifiquem condições (favoráveis) e restrições que impediram a implementação de percursos de estudo e pesquisa nas aulas que vocês ministraram. Pode ser qualquer tipo de aspecto: desde o número de alunos por turma, disponibilidade de recursos tecnológicos, motivação dos estudantes...

4) Tentem classificar e ver em que nível de codeterminação didática elas aparecem.

5) Preparem uma apresentação para a próxima sessão.

Slides da apresentação do dia 30/6/2023:

A atividade que nós implementamos na escola foi, de fato, um PEP? Justifique.

- Nós não entendemos a atividade que nós implementamos como um PEP. O percurso de estudo e pesquisa é caracterizado como como um processo de investigação dos estudantes e se faz a partir de uma questão Q0. A partir deste questionamento inicial se inicia a formulação de outras questões e, a partir daí, a pesquisa sobre o tema passa a ser trabalhado e o conteúdo que se organiza para responder as questões levantadas por eles.
- Deste modo, vemos que devido ao conjunto de condições e restrições que nós nos deparamos desde o início da formação do nosso grupo, tirando o foco do PEP e colocando o foco mais no conteúdo curricular que o professor da escola estava trabalhando.
- O PEP tem o momento de investigação e a proposta de atividade que fizemos não potenciou estes momentos de investigação.

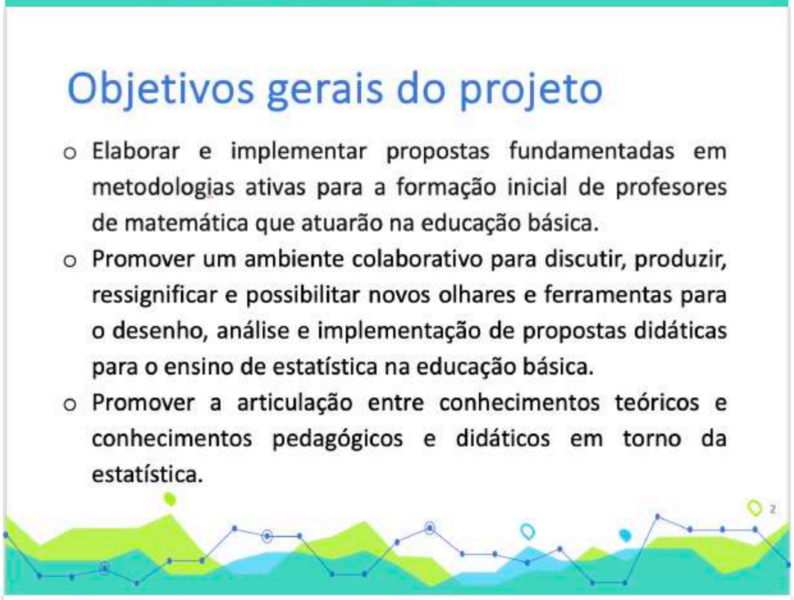
Condições e restrições

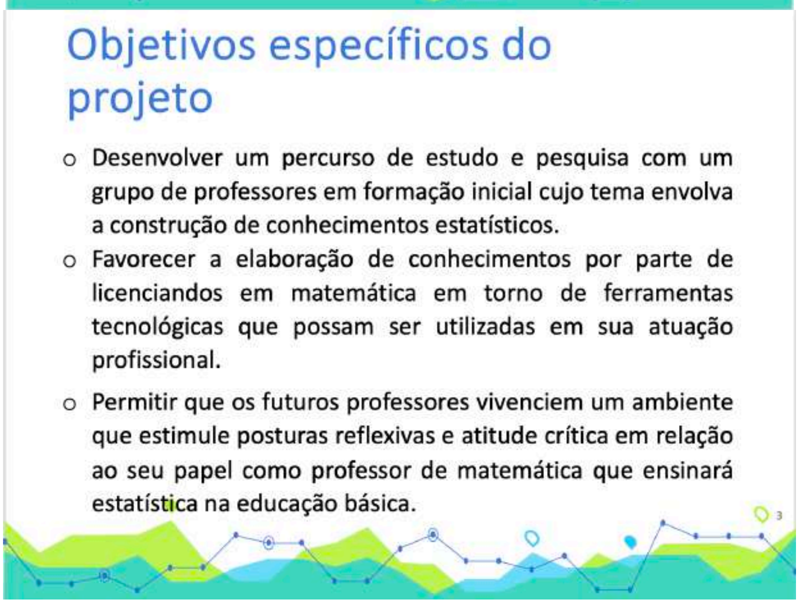
- Condição: Quantidade de alunos: 27 alunos foi uma quantidade boa para trabalhar.
- Restrição: Comprometimento parcial do grupo de estudantes
- Restrição: As aulas serem apenas uma vez na semana, nas quintas-feiras.
- Restrição: um feriado no meio do desenvolvimento das atividades.
- Restrição: a disponibilidade de apenas um integrante do grupo de residentes pedagógicos poder implementar a atividade em sala.
- Restrição: O tempo de aula – apenas 3 aulas de 50 minutos
- Restrição: Muitos alunos faltaram no segundo dia de implementação, influenciando no andamento da atividade.
- O currículo da escola que não nos permitiu implementar uma atividade de estatística.

Níveis de codeterminação didática

Civilização	Humana
Sociedade	Alunos
Escola	Currículo, duração e quantidade de aulas,
Pedagogia	A proposta didática
Disciplina	Matemática
Domínio	Álgebra
Setor	Função do primeira grau
Tema	Tabelas e gráficos
Questão	As situações trabalhadas

APPENDIX E.2: Slides of the face-to-face sessions





O curso é dividido em **16 encontros** com **foco em desenvolvimento de atividades de investigação relacionadas com estatística**.

Os encontros presencias ocorrerão todas as sextas-feiras das *20:40 às 22:40* (32h). Se completam com atividades assíncronas (28h) em horário livre. Certificado de 60 horas de curso se **80% de presença** e **entrega de todas as atividades em grupos.**

Público-alvo: professores de matemática em formação da educação básica **que atuam** nos **anos finais do Ensino Fundamental** e/ou no **ensino médio**.

Todos precisam assinar o TCLE - **TERMO DE CONSENTIMENTO LIVRE E ESCLARECIDO**

4

Espaços de comunicação

Google Sala de Aula

Criamos um **grupo no *WhatsApp*** restritamente para comunição a respeito do projeto. Regras básicas: Não enviar mensagens, imagens, vídeos, etc., que não façam parte do projeto de ensino de graduação.

5

Dinâmicas e recursos que serão mobilizados

Os participantes trabalharão em grupos, de preferência grupos formados por residentes que atuam na mesma escola e mesmo nível de escolaridade. Quem não estiver atuando (ou não for residente) escolherá o grupo que queira formar parte.

Mobilizaremos algumas TICs, principalmente navegando na internet, Google sala de aula, whatsapp e no Excel.

6

Fundamentos e contextualização: Uma mudança de paradigma
segundo a Teoria Antropológica do Didático (TAD)

Paradigma da visita às obras (ou *monumentalismo*)

CURRÍCULO COMO CONJUNTO DE OBRAS OU CONTEÚDOS

- Esse paradigma pode ser entendido como o ensino tradicional ao qual pertencemos;
- Trata-se de um paradigma dominante e que se encontra em decadência;
- Papel do aluno e papel do professor.

Paradigma do questionamento do mundo

CURRICULUM COMO CONJUNTO DE CUESTIONES

- O objetivo principal não é mais estudar uma "obra" pronta e acabado e sim uma questão (*Q*);
- Papel do aluno e papel do professor mudam.

Chevallard (2013). Enseñar matemáticas en la sociedad de mañana: alegato a favor de un contraparadigma emergente

7

Fundamentos e contextualização:

A formação de professores: visita ás obras ou questionamento do mundo?

Estamos imersos em um momento de debate sobre a profissão de professor e, em particular, sobre sua formação inicial e continuada.

Grande concordância de que a mudança de paradigma escolar deve andar de mãos dadas com a formação de professores como agente chave dessa mudança.

Vamos nos concentrar nas propostas dos Percursos de Estudo e Pesquisa (PEP) e da PEP para Formação de Professores (PEP-FP) (Ruiz-Olarría, 2015)

8

ESTRUTURA DA FORMAÇÃO

M0. Partirmos de uma **questão relacionada a profissão docente**: relacionada ao ensino da estatística nos anos finais do ensino fundamental
→ POSIÇÃO DE PROFESSOR

M1. Vivenciar um **processo de aprendizagem (PEP)** semelhante ao que poderia existir em aulas de matemática
→ POSIÇÃO DE ALUNO

M2. Análise coletiva do processo de ensino vivenciado com **ferramentas didáticas**
→ POSIÇÃO DE PROFESSOR-ANALISTA

M3. Adaptar e implementar (análise *in vivo*) o PEP experimentado a determinadas condições escolares
→ POSIÇÃO DE PROFESSOR-DESIGNER

M4. Análise *a posteriori* coletiva do PEP implementado e novas adaptações
→ POSIÇÃO DE PROFESSOR-ANALISTA-DESIGNER

9

Calendário de encontros

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5	21/04/2023	Apresentação das respostas finais da Q_0	M1 e M2
6	28/04/2023	Análise do processo de aprendizagem vivenciado	M2
7	05/05/2023	Apresentação e discussão coletiva das análises	M2
8	12/05/2023	Adaptações para implementação em sala de aula	M3
9	19/05/2023	Adaptações para implementação em sala de aula	M3
10	26/05/2023	Adaptações para implementação em sala de aula	M3
11	02/06/2023	Implementações em sala de aula	M3
12	09/06/2023	Implementações em sala de aula	M3
13	16/06/2023	Implementações em sala de aula	M3
14	23/06/2023	Discussões e análises finais	M4
15	30/06/2023	Discussões e análises finais	M4
16	07/07/2023	Discussões e análises finais (online)	M4

Atividade 1 – Nuvem de Palavras

Mentimeter

- Escreva cinco palavras que você associa à estatística:
- https://www.menti.com/al18he9dd6ke

11

Atividade 2 – Questões sobre o ensino de Estatística

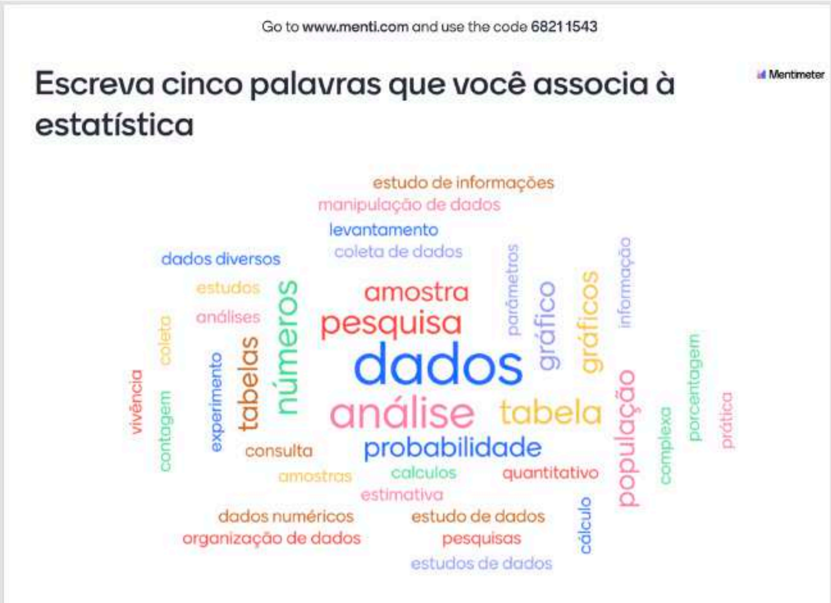
M0. Partir do **objetivo deste curso de formação**: questões relacionadas ao ensino de estatística na educação básica → POSICÃO DE PROFESSOR

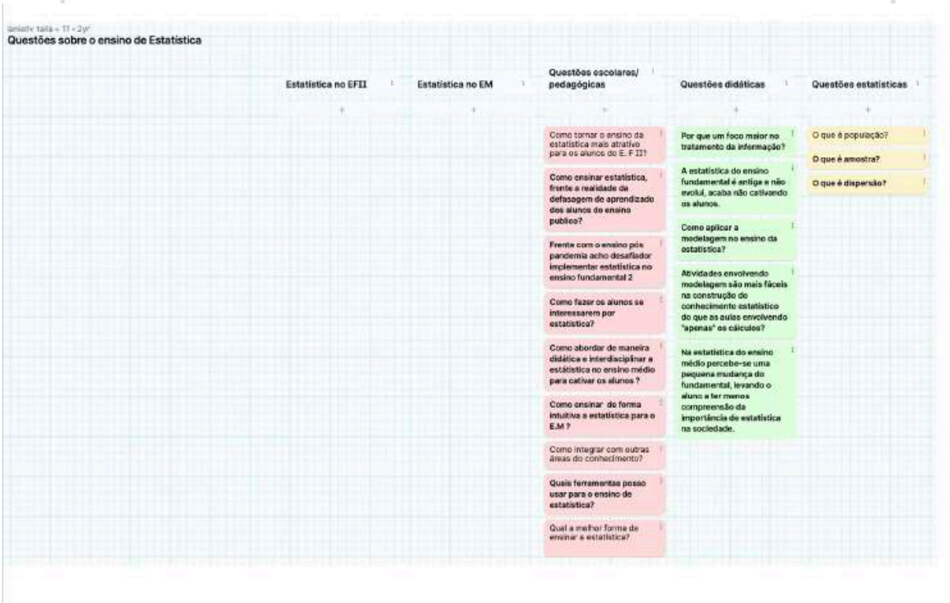
→ Coluna 1: Estatística EF2

→ Coluna 2: Estatística EM

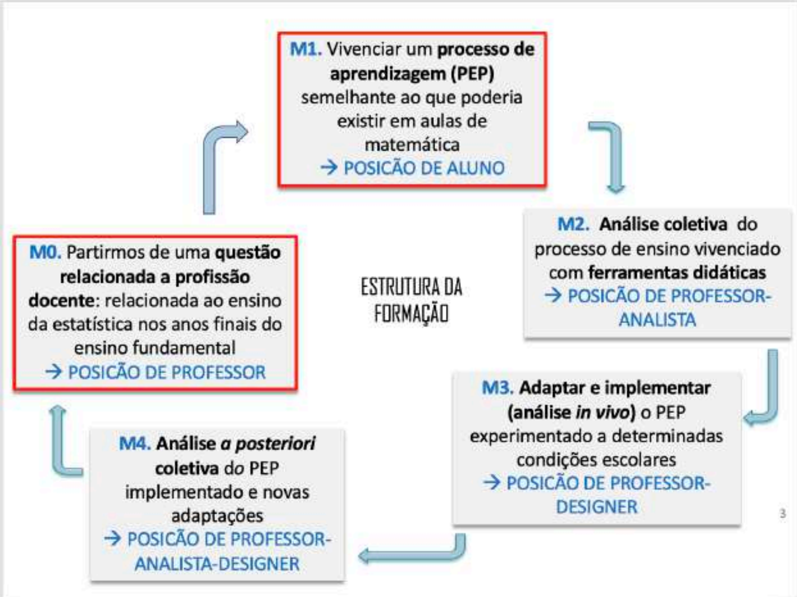
padlet https://padlet.com/janielly_taila/quest-es-sobre-o-ensino-de-estat-stica-j95sbj08d1bqo8jp

12


Go to www.menti.com and use the code 6821 1543
Mentimeter
Escreva cinco palavras que você associa à estatística
estudo de informações
manipulação de dados
levantamento
coleta de dados
dados diversos
estudos
análises
números
amostra
pesquisa
parâmetros
gráfico
gráficos
informação
coleta
tabelas
dados
vivência
contagem
experimento
análise
tabela
população
complexa
porcentagem
prática
consulta
probabilidade
amostras
calculos
quantitativo
estimativa
dados numéricos
estudo de dados
organização de dados
pesquisas
cálculo
estudos de dados


Questões sobre o ensino de Estatística
Estatística no EFII
Estatística no EM
Questões escolares/ pedagógicas
Questões didáticas
Questões estatísticas
Como tornar o ensino da estatística mais atrativo para os alunos do E. F II?
Como ensinar estatística, frente a realidade de defasagem de aprendizado dos alunos do ensino publico?
Frente com o ensino pós pandemia acho desafiador implementar estatística no ensino fundamental 2
Como fazer os alunos se interessarem por estatística?
Como abordar de maneira didática e interdisciplinar a estátistica no ensino médio para cativar os alunos ?
Como ensinar de forma intuitiva a estatística para o E.M ?
Como integrar com outras áreas do conhecimento?
Quais ferramentas posso usar para o ensino de estatística?
Qual a melhor forma de ensinar a estatística?
Por que um foco maior no tratamento da informação?
A estatística do ensino fundamental é antiga e não evolui, acaba não cativando os alunos.
Como aplicar a modelagem no ensino da estatística?
Atividades envolvendo modelagem são mais fáceis na construção do conhecimento estatístico do que as aulas envolvendo "apenas" os cálculos?
Na estatística do ensino médio percebe-se uma pequena mudança do fundamental, levando o aluno a ter menos compreensão da importância de estatística na sociedade.
O que é população?
O que é amostra?
O que é dispersão?


Articulações entre aspectos teóricos, pedagógicos e didáticos no ensino de estatística na perspectiva do questionamento do mundo
Projeto de Ensino de Graduação
Coordenadora: Sonia Maria Monteiro da Silva Burigato
Investigadoras/professoras participantes:
Marilena Bittar
Janielly Taila dos Santos Verbisck
Sessão 2- 24 de março de 2023
Campo Grande, MS
CAPES
UFMS
FUNDAÇÃO UNIVERSIDADE FEDERAL DE MATO GROSSO DO SUL
UNIVERSITAT DE BARCELONA



Calendário de encontros

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6	28/04/2023	Análise do processo de aprendizagem vivenciado	M2
7	05/05/2023	Apresentação e discussão coletiva das análises	M2
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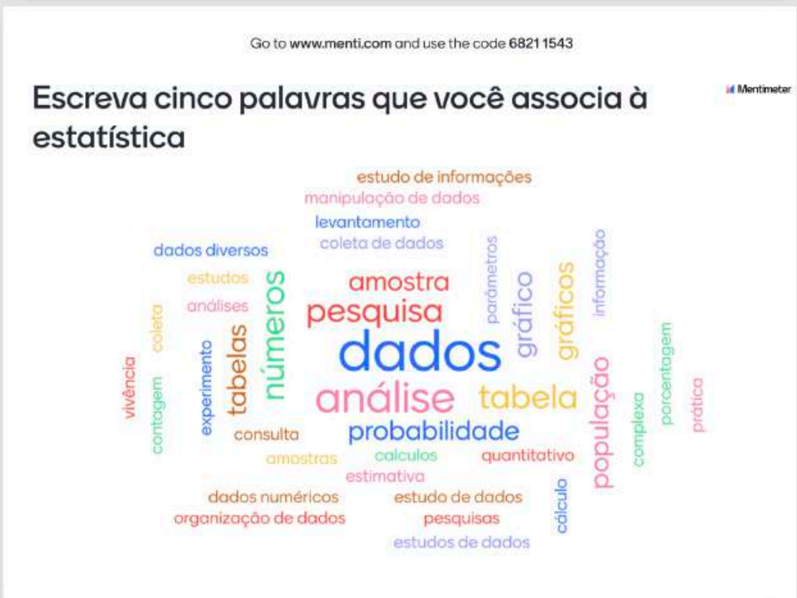
9	19/05/2023	Adaptações para implementação em sala de aula	M3
10	26/05/2023	Adaptações para implementação em sala de aula	M3
11	02/06/2023	Implementações em sala de aula	M3
12	09/06/2023 Não letivo	Implementações em sala de aula	M3
13	16/06/2023	Implementações em sala de aula	M3
14	23/06/2023	Discussões e análises finais	M4
15	30/06/2023	Discussões e análises finais	M4
16	07/07/2023	Discussões e análises finais (online)	M4



Atividade 2 (da sessão 1) – Questões sobre o ensino de Estatística

M0. Partir do **objetivo deste curso de formação**: questões relacionadas ao ensino de estatística na educação básica → POSICÃO DE PROFESSOR

→ Coluna 1: Estatística EF2

→ Coluna 2: Estatística EM

padlet https://padlet.com/janielly_taila/quest-es-sobre-o-ensino-de-estat-stica-j95sbj08d1bqo8jp

6

Pedagogia e didática

- **Pedagogo:** vem do grego, *paidagogos*, que era o escravo que acompanhava as crianças a escola.
- **Pedagogia**: organizar as condições gerais para que existam sistemas didáticos

---

- **Didático:** tudo relacionado ao estudo de algo ♥
- **Sistema didático**: S(X, Y, ♥) ♥ pode ser um saber o questão
- **Didática**: ciência que estuda o didático [economia, física, ...]

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Grupos

→ **Grupo 1:** https://padlet.com/janiellyverbisck/grupo-1-pep-fp-ufms-4u8y2ra0wonx5d9b
Drive: https://docs.google.com/document/d/1zsOUTVVhqw8kV1wCC29JF5Jqdaa70auEKzFIrRKyJC8/edit?usp=sharing

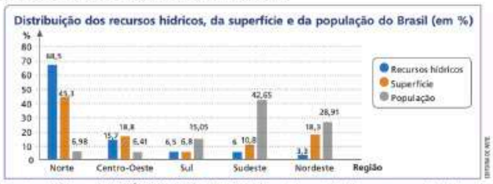
→ Grupo 2: https://padlet.com/janiellyverbisck/grupo-2-pep-fp-ufms-e5fvfmue5u0474cx

→ Grupo 3: https://padlet.com/janiellyverbisck/grupo-3-pep-fp-ufms-xea2uhxyxc4nvqsl

→ Grupo 4: https://padlet.com/janiellyverbisck/grupo-4-pep-fp-ufms-xsozj5rsg8c9ss6x

1. O Brasil possui cerca de 13,7% do total de água doce do mundo, sendo considerado um território rico em termos hídricos. No entanto, o país vive sérios problemas, relacionados tanto à degradação da qualidade das águas, principalmente nas proximidades das áreas urbanas, quanto à falta de controle do excesso e da insuficiência de água, que atingem várias localidades brasileiras. Não são somente as enchentes que afetam as cidades brasileiras: a escassez hídrica também impõe sérias restrições e elevados custos ao desenvolvimento econômico e social de grandes cidades do Brasil.

Observando o gráfico a seguir, responda no caderno:



Informações obtidas em: MINISTÉRIO DO MEIO AMBIENTE. Disponível em: <http://www.mma.gov.br/estruturas/sedr_proecotur/_publicacao/140_publicacao09062009025910.pdf>. Acesso em: 1º jul. 2018.

Q_0: Considerando o gráfico apresentado no exercício do livro didático, que questões podemos levantar sobre o problema da água em Brasil? O gráfico segue sendo atual? Há mudanças em relação a dados mais atualizados sobre população, território/superfície, e recursos hídricos?

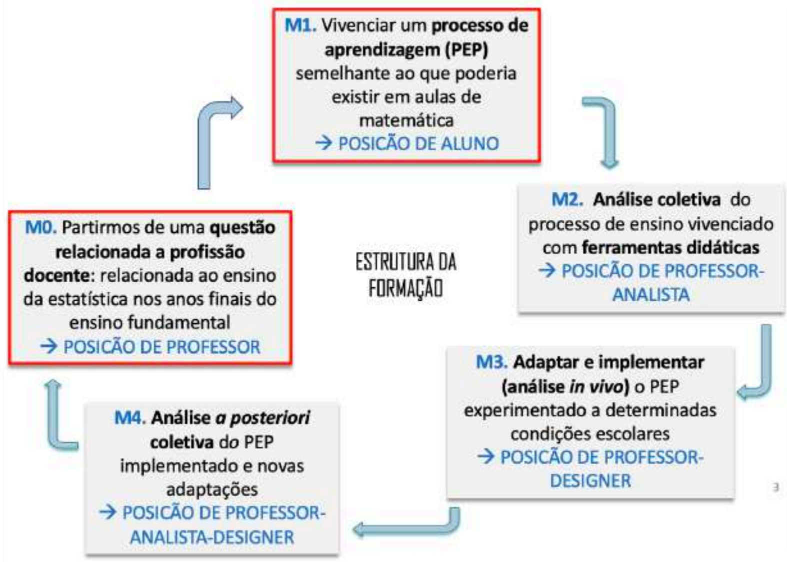
Articulações entre aspectos teóricos, pedagógicos e didáticos no ensino de estatística na perspectiva do questionamento do mundo

Projeto de Ensino de Graduação

Coordenadora: Sonia Maria Monteiro da Silva Burigato
Investigadoras/professoras participantes:
Marilena Bittar
Janielly Taila dos Santos Verbisck

Sessão 3– 31 de março de 2023
Campo Grande, MS

CAPES · UFMS Fundação Universidade Federal de Mato Grosso do Sul · Universitat de Barcelona



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4	14/04/2023	Investigação sobre Q_0 e preparação das respostas finais	M1
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7	05/05/2023	Apresentação e discussão coletiva das análises	M2
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9	19/05/2023	Adaptações para implementação em sala de aula	M3
10	26/05/2023	Adaptações para implementação em sala de aula	M3
11	02/06/2023	Implementações em sala de aula	M3
12	09/06/2023 Não letivo	Implementações em sala de aula	M3
13	16/06/2023	Implementações em sala de aula	M3
14	23/06/2023	Discussões e análises finais	M4
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1. O Brasil possui cerca de 13,7% do total de água doce do mundo, sendo considerado um território rico em termos hídricos. No entanto, o país vive sérios problemas, relacionados tanto à degradação da qualidade das águas, principalmente nas proximidades das áreas urbanas, quanto à falta de controle do excesso e da insuficiência de água, que atingem várias localidades brasileiras. Não são somente as enchentes que afetam as cidades brasileiras: a escassez hídrica também impõe sérias restrições e elevados custos ao desenvolvimento econômico e social de grandes cidades do Brasil.

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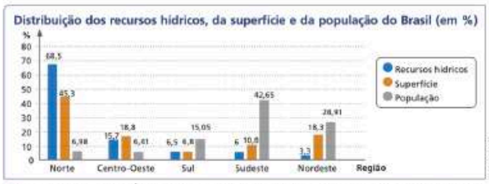

Informações obtidas em: MINISTÉRIO DO MEIO AMBIENTE. Disponível em: <http://www.mma.gov.br/estruturas/sedr_proecotur/_publicacao/140_publicacao09062009025910.pdf>. Acesso em: 1º jul. 2018.

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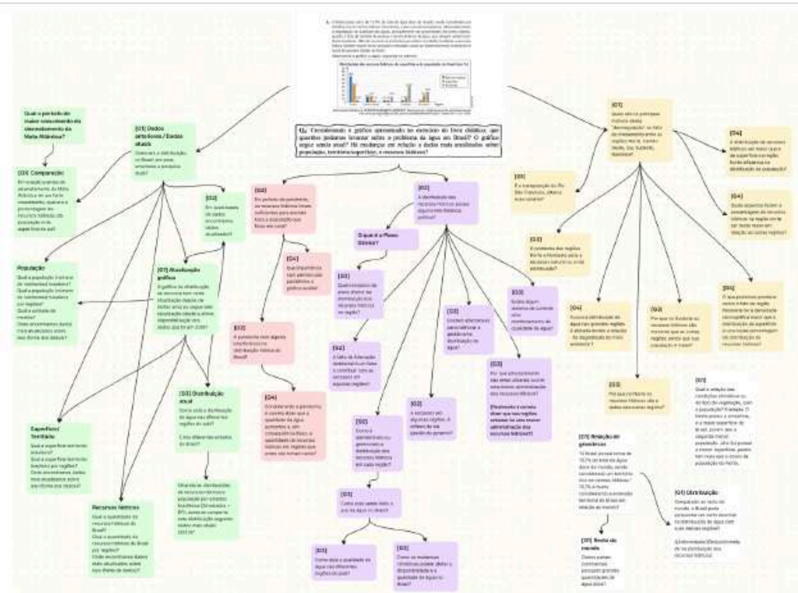

Bancos de dados oficiais

- População - Censo 2022: https://www.ibge.gov.br/estatisticas/sociais/populacao/22827-censo-demografico-2022.html?=&t=resultados
- Recursos hídricos: https://mapbiomas.org/estatisticas
- Território: https://www.ibge.gov.br/geociencias/organizacao-do-territorio/estrutura-territorial/15761-areas-dos-municipios.html?=&t=acesso-ao-produto

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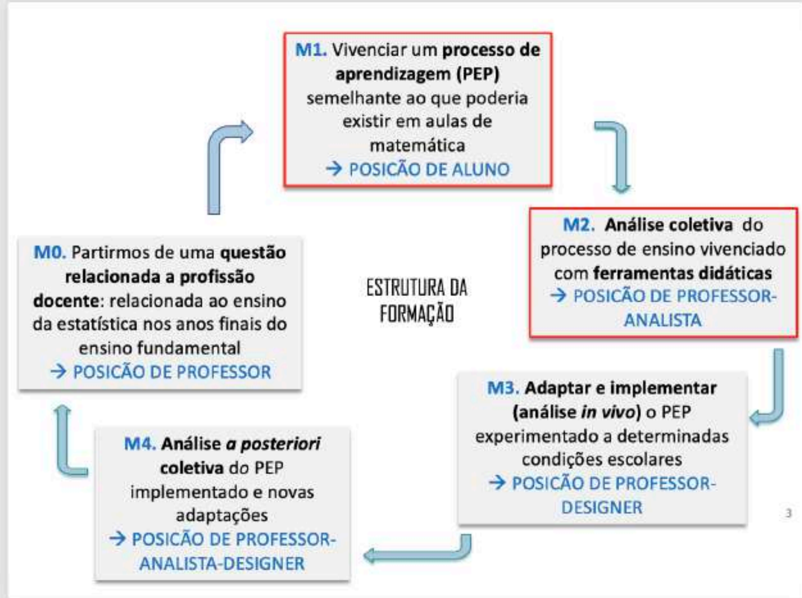
Articulações entre aspectos teóricos, pedagógicos e didáticos no ensino de estatística na perspectiva do questionamento do mundo

Projeto de Ensino de Graduação

Coordenadora: Sonia Maria Monteiro da Silva Burigato
Investigadoras/professoras participantes:
Marilena Bittar
Janielly Taila dos Santos Verbisck

Sessão 6– 28 de abril de 2023
Campo Grande, MS

CAPES — UFMS Fundação Universidade Federal de Mato Grosso do Sul — Universitat de Barcelona



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4	14/04/2023	Apresentação das respostas finais da Q_0	M1
5	21/04/2023 Não letivo	Assíncrono – finalização dos relatórios	M1 – M2
6	28/04/2023	Análise do processo de aprendizagem vivenciado	M2
7	05/05/2023	Análise do processo de aprendizagem vivenciado	M2
8	12/05/2023	Adaptações para implementação em sala de aula	M2 e M3
9	19/05/2023	Adaptações para implementação em sala de aula	M3
10	26/05/2023	Adaptações para implementação em sala de aula	M3
11	02/06/2023	Implementações em sala de aula	M3
12	09/06/2023 Não letivo	Implementações em sala de aula	M3
13	16/06/2023	Implementações em sala de aula	M3
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I – Aspectos conceituais

Comentários – aspectos conceituais

Quais são os principais motivos dessa "desregulação" ou falta de nivelamento entre as variáveis: Norte, Centro-Oeste, Sul, Sudeste, Nordeste?

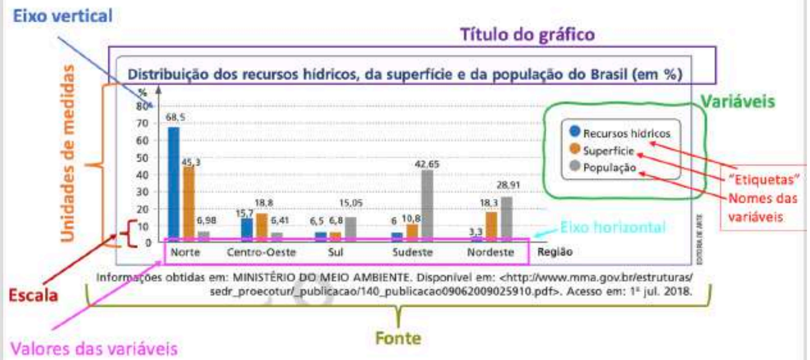
"As variáveis são as características que estão sendo analisadas em uma amostra ou população. Podem assumir valores numéricos ou não numéricos".

Informações obtidas em: MINISTÉRIO DO MEIO AMBIENTE. Disponível em: <http://www.mma.gov.br/estruturas/sedr_proecotur/_publicacao/140_publicacao09062009025910.pdf>. Acesso em: 1º jul. 2018.

Variáveis
- Quantitativas
  - Contínua
  - Discreta
- Qualitativas
  - Nominal
  - Ordinal

O tipo de variável está diretamente relacionado ao tipo de gráfico para representação dos dados!!

Comentários – aspectos conceituais



Coleta, organização e representação de dados

1. **Distribuição de frequência:** as tabelas de distribuição de frequência
2. **Variáveis quantitativas ou numéricas:** histogramas ou gráficos barras
3. **Variáveis qualitativas ou categóricas:** gráficos de barras (ou colunas), pictogramas, gráficos de setores

Tabelas e quadros

Você conhece a diferença???

Principais diferenças

Tabela
Recomenda-se o uso de tabelas para apresentar valores precisos; servindo para apresentar números e dados de forma organizada;
- Não há predominância de texto;
- Possui um conteúdo completo, portanto não há necessidade de se consultar o texto para entender o que está sendo ilustrado;
- É separada por colunas mas não possui necessariamente linhas;
- Possui cabeçalho para identificar o conteúdo de cada coluna;
- Nenhum espaço deve ficar em branco;
- A fonte deve ser mencionada no rodapé da tabela;
- Podem apresentar notas explicativas a fim de esclarecer a metodologia de como foi feita a coleta, fontes ou informações que ajudem a esclarecer o conteúdo da mesma.

- **Quadro**
- Há a predominância de texto;
- É separado por colunas e linhas;
- Indicação de dados numéricos não são obrigatórios, ou seja, podem aparecer ou não;
- Possui uma esquematização mais bem trabalhada;
- Seu conteúdo é mais descritivo;
- Diferencia-se de tabelas em sua apresentação pelas linhas verticais e horizontais para separação das casas.

Fonte: http://www.ciencia.ufma.br/voce-sabe-qual-a-diferenca-entre-quadro-tabela-e-figura-a-gente-explica/

A diferença primordial entre TABELAS e QUADROS está na FORMATAÇÃO

- As TABELAS seguem a normatização da ABNT 14.724 (2011), no subitem 5.9, o qual indica que devem ser padronizadas de acordo com as normas de apresentação tabular do IBGE (1993).

Esta norma indica que uma tabela deve possuir:
a) um título;
b) um cabeçalho;
c) um corpo contendo dados numéricos;
d) uma linha de fechamento;
e) uma fonte;
f) notas explicativas, se precisar.

Outro detalhe é que as tabelas...

NÃO POSSUEM FECHAMENTOS LATERAIS

Os QUADROS diferem das tabelas em seu conteúdo (não é numérico) e na formatação:

POSSUEM FECHAMENTOS LATERAIS

- Exemplos de Tabela de dupla entrada

Percentual de água nos órgãos do corpo humano

Órgão	Percentual
Cérebro	75%
Pulmões	86%
Fígado	86%
Músculos	75%
Coração	75%
Rins	83%
Sangue	81%

Fonte: C1, 8º ano, p. 27

Esperança de vida ao nascer – 2016

País	Esperança de vida ao nascer (em anos)
Japão	83,98
Canadá	82,20
Itália	83,40
França	82,70
Alemanha	81,00
Reino Unido	81,20
Estados Unidos	78,69
México	77,12
China	76,25
Brasil	75,51
Rússia	71,59
Índia	68,56
África do Sul	62,77

Fonte: COUNTRYECONOMY.COM. Disponível em: <https://pt.countryeconomy.com/demografia/esperanca-vida>. Acesso em: 7 set. 2018.

Fonte: C1, 8º ano, p. 130

Mortes no trânsito brasileiro por região, em 2015

Região	Quantidade de mortes
Sul	6064
Nordeste	12191
Norte	3419
Centro-Oeste	4069
Sudeste	12908

Fonte: C5, 8º ano, p. 193

- Exemplos de Tabela de distribuição de freqüências absoluta e relativa

Tabela 2

Grau de escolaridade dos funcionários da empresa X

Grau de escolaridade	Frequência absoluta	Frequência relativa (%)
Ensino Fundamental	6	5
Ensino Médio	30	25
Ensino Superior	48	40
Pós-graduação	36	30
Total	120	100

Fonte: Dados fictícios.

Fonte: C1, 8º ano, p. 214

Nacionalidade em um grupo de turistas

Nacionalidade	FA	FR (em %)
Brasileira	6	60%
Espanhola	3	30%
Argentina	1	10%
Total	**10**	**100%**

Tabela elaborada para fins didáticos.

Fonte: C2, 8º ano, p. 193

Frequências da escolha de sabor de sorvete preferido

Sabor	Frequência absoluta (FA)	Frequência relativa (FR em %)
Chocolate	5	25%
Morango	4	20%
Limão	10	50%
Baunilha	1	5%
Total	20	100%

Tabela elaborada para fins didáticos.

Fonte: C2, 9º ano, p. 270

- Exemplos de Tabela de distribuição em intervalo ou classes

Altura dos alunos da academia Saúde

Altura (em metro)	Número de alunos
$1{,}50 \vdash 1{,}58$	9
$1{,}58 \vdash 1{,}66$	11
$1{,}66 \vdash 1{,}74$	25
$1{,}74 \vdash 1{,}82$	30
$1{,}82 \vdash 1{,}90$	10
$1{,}90 \vdash 1{,}98$	5
Total	90

Fonte: Alunos da academia Saúde.

Fonte: C1, 8º ano, p. 54

Extensão territorial (km²)	Frequência (f)
$0 \vdash 250$	45
$250 \vdash 500$	18
$500 \vdash 750$	6
$750 \vdash 1000$	4
$1000 \vdash 1250$	2
Total	75

Fonte: C5, 8º ano, p. 207

DISTRIBUIÇÃO DAS UNIDADES DA FEDERAÇÃO DO PAÍS A SEGUNDO O NÚMERO MÉDIO DE ANOS CONCLUÍDOS (2019)

Número médio de anos concluídos	Frequência: quantidade de unidades da federação
$4 \vdash 5$	8
$5 \vdash 6$	7
$6 \vdash 7$	9
$7 \vdash 8$	2
$8 \vdash 9$	1

Dados obtidos pelo Instituto Nacional de Pesquisas Educacionais do país A em dezembro de 2019.

Fonte: C4, 8º ano, p. 173

• Exemplos de Tabela de distribuição de frequências (absoluta e relativa) por intervalos ou classes

DISTRIBUIÇÃO DOS FUNCIONÁRIOS DA EMPRESA ABC POR IDADE

Faixa etária (em ano)	Frequência absoluta	Frequência relativa
20 ⊢ 30	62	0,31
30 ⊢ 40	79	0,395
40 ⊢ 50	30	0,15
50 ⊢ 60	29	0,145
Total	200	1

Dados obtidos pela empresa ABC em outubro de 2019.

Fonte: C4, 8° ano, p. 173

DISTRIBUIÇÃO DAS PESSOAS DO GRUPO SEGUNDO A QUANTIDADE DE DIAS SEM DOR, APÓS USO DO MEDICAMENTO

Número de dias sem dor	Frequência absoluta	Frequência relativa
15 ⊢ 17	1	0,02
18 ⊢ 20	3	0,06
21 ⊢ 23	3	0,06
24 ⊢ 26	3	0,06
27 ⊢ 29	6	0,12
30 ⊢ **32**	**0**	**0**
33 ⊢ 35	2	0,04
36 ⊢ 38	3	0,06
39 ⊢ 41	2	0,04
42 ⊢ 44	4	0,08
45 ⊢ 47	7	0,14
48 ⊢ 50	4	0,08
51 ⊢ 53	5	0,1
54 ⊢ 56	3	0,06
57 ⊢ 59	4	0,08

Dados obtidos pelo Laboratório LB no 1º semestre de 2018.

Fonte: C4, 8° ano, p. 258

Medida de altura de um grupo de alunos

Medida de altura (em cm)	FA	FR (em %)
150 ⊢ 154	3	12%
154 ⊢ 158	8	32%
158 ⊢ 162	4	16%
162 ⊢ 166	9	36%
166 ⊢ 170	1	4%

Tabela elaborada para fins didáticos.

Fonte: C2, 8° ano, p. 203

• Exemplos de Quadro

Prova objetiva/Redação	Nota	
	Cintia	Sandro
Linguagens, Códigos e suas Tecnologias	628,9	680,4
Ciências Humanas e suas Tecnologias	672,1	602,8
Ciências da Natureza e suas Tecnologias	550	605,6
Matemática e suas Tecnologias	765	563,7
Redação	440	700

Fonte: C5, 8° ano, p. 199

. O quadro mostra as notas de quatro alunos do 8º ano na disciplina de Ciências. Observe:

Bruno	6,5	7,5	7,0	7,5	6,0
Camila	8,0	8,0	7,0	6,5	7,5
Marcela	5,0	5,5	4,5	5,5	6,0
Roberto	4,5	7,5	5,0	5,0	8,0

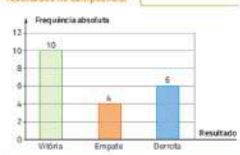
Fonte: C1, 8° ano, p. 23

Dia do mês	Temperatura (em °C)
1	15,5
3	14
5	13,5
7	18
9	19,5
11	20
13	13,5
15	13,5
17	18
19	20
21	18,5
23	13,5
25	21,5
27	20
29	16

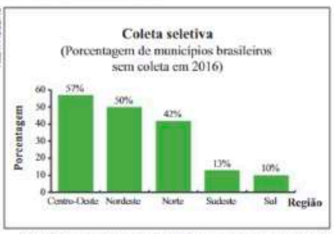
Fonte: C2, 9° ano, p. 300

• Exemplos de Gráfico de barras (ou colunas)

Frequência absoluta dos resultados no campeonato



Fonte: C2, 9° ano, p. 269

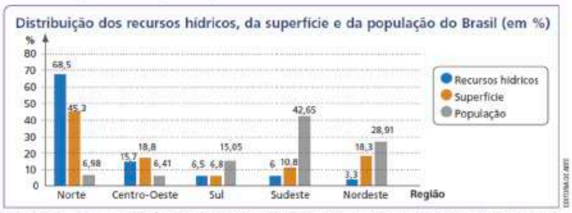


Dados obtidos em: GAMA, Mara. Cai geração de resíduos no país e mais cidades usam lixão, aponta estudo. *Folha de S.Paulo*, 1º set. 2017. Disponível em: <http://www1.folha.uol.com.br/cotidiano/2017/09/1914912-cai-geracao-de-residuos-no-pais-e-mais-cidades-usam-lixao-aponta-estudo.shtml>. Acesso em: 03 out. 2017.

Dados obtidos em: HAYDÉE, Lygia. Os 10 parques nacionais mais visitados do Brasil em 2013. *Exame*, 7 abr. 2017. Disponível em: <https://exame.abril.com.br/estilo-de-vida/os-10-parques-nacionais-mais-visitados-do-brasil-em-2013/>. Acesso em: 03 out. 2017.

Fonte: C3, 8° ano, p. 66

• Exemplos de Gráfico de múltiplas barras (ou colunas)



Informações obtidas em: MINISTÉRIO DO MEIO AMBIENTE. Disponível em: <http://www.mma.gov.br/estruturas/sedr_proecotur/_publicacao/140_publicacao09062009025910.pdf>. Acesso em: 1º jul. 2018.

Fonte: C1, 8º ano, p. 26

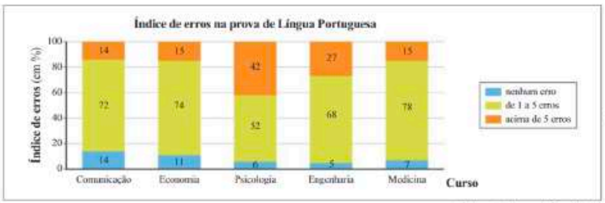
Fonte: C4, 9º ano, p. 109

Dados obtidos em: <https://cidades.ibge.gov.br/brasil/pesquisa/22/28120?ano=2016>. Acesso em: 13 jun. 2018.

• Exemplos de Gráfico de barras (ou colunas) sobrepostas

Fonte: Diretoria da empresa.

Fonte: C1, 9º ano, p. 186



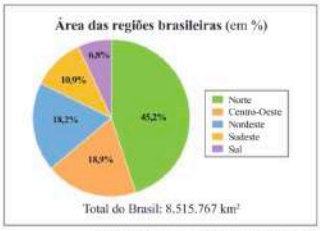
Dados obtidos pela faculdade.

Fonte: C3, 8º ano, p. 217

• Exemplos de Gráfico de setores

Gráficos elaborados para fins didáticos.

Fonte: C2, 9º ano, p. 277



Dados obtidos em: *Almanaque Abril 2015*, São Paulo: Abril, 2015. p. 654.

Fonte: C3, 9º ano, p. 267

Gráfico 1

Informações obtidas em: LOBO, J. **A bicicleta**. Disponível em: <www.ta.org.br/temp/2013/smtr_ta.pdf>. Acesso em: 3 fev. 2018.

Gráfico 2

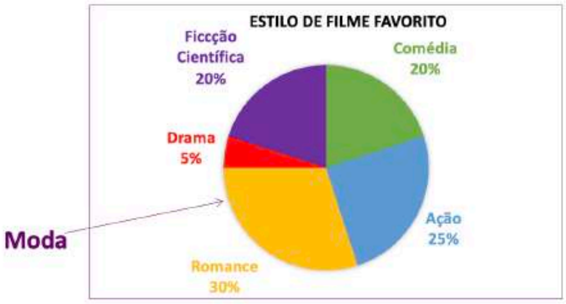
Informações obtidas em: TRANSPORTE ATIVO. **Introdução ao mundo cicloviário**. Disponível em: <www.ta.org.br/educativos/imc/IMG/IMC_02.pdf>. Acesso em: 3 fev. 2018.

Fonte: C1, 8º ano, p. 117

Representação gráfica de variáveis qualitativas ou categóricas

GRÁFICO DE SETORES

O **gráfico de setores** é uma forma de representar dados estatísticos onde a população é representada por um **círculo** e cada subconjunto (ou parte) é representado por um **setor de um círculo**, com o tamanho de cada setor indicando a frequência.



Representando amostras através de gráficos

Tabela: Tipo de filme favorito

Comédia	Ação	Romance	Drama	Ficção Científica	Total
4	5	6	1	4	20

	Comédia	Ação	Romance	Drama	Ficção Científica	Total
Frequência absoluta	4	5	6	1	4	20
Frequência relativa	$\frac{4}{20} = 0,2 = 20\%$	$\frac{5}{20} = 0,25 = 25\%$	$\frac{6}{20} = 0,3 = 30\%$	$\frac{1}{20} = 0,05 = 5\%$	$\frac{4}{20} = 0,2 = 20\%$	100%

	Comédia	Ação	Romance	Drama	Ficção Científica	Total
Frequência absoluta	4	5	6	1	4	20
Frequência relativa	20%	25%	30%	5%	20%	100%
Setor circular	$\frac{4}{20} \cdot 360° = 72°$	$\frac{5}{20} \cdot 360° = 90°$	$\frac{6}{20} \cdot 360° = 108°$	$\frac{1}{20} \cdot 360° = 18°$	$\frac{4}{20} \cdot 360° = 72°$	360°

- Exemplos de Gráfico de linhas (ou

Dados obtidos em: <http://www.anatel.gov.br/dados/2015-02-04-18-43-59>. Acesso em: 5 set. 2018.

Fonte: C4, 8º ano, p. 67

Gráfico elaborado para fins didáticos

Fonte: C2, 9º ano, p. 273

Fonte: UNITED NATIONS. **World Population Prospects**: The 2017 Revision. Disponível em: <https://esa.un.org/unpd/wpp/DVD/Files/1_Indicators%20(Standard)/EXCEL_FILES/1_Population/WPP2017_POP_F01_1_TOTAL_POPULATION_BOTH_SEXES.xlsx>. Acesso em: 2 out. 2018.

Fonte: C5, 8º ano, p. 191

• Exemplos de Gráfico de múltiplas linhas (ou segmentos)

Dados obtidos pela Escola Ponte Feliz em janeiro de 2019.

Fonte: C4, 9° ano, p. 109

Dados obtidos em: <http://www.denatran.gov.br/estatistica/237-frota-veiculos>. Acesso em: 13 jun. 2018.

Fonte: C4, 9° ano, p. 108

• Exemplos de Histograma

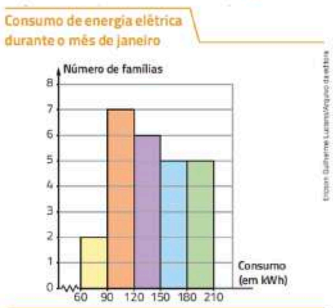


Gráfico elaborado para fins didáticos.

Fonte: C2, 9° ano, p. 277

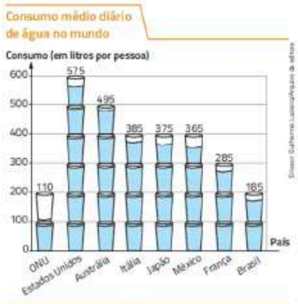
Fonte: Dados fictícios.

Fonte: C1, 8° ano, p. 217

Fonte: Dados fictícios.

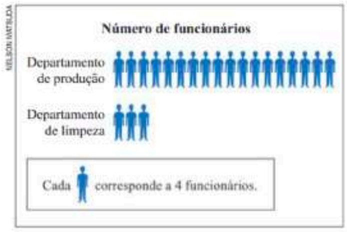
Fonte: C1, 8° ano, p. 215

• Exemplos de Pictograma



Fonte de consulta: BRASIL ESCOLA. Geografia. Disponível em: <https://brasilescola.uol.com.br/geografia/consumo-agua-no-mundo.htm>. Acesso em: 3 set. 2018.

Fonte: C2, 9° ano, p. 277



Dados obtidos pela empresa Aquitem.

Fonte: C3, 8° ano, p. 71

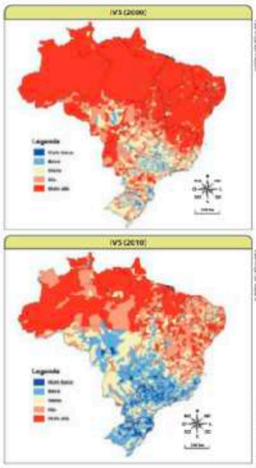
Dados obtidos pela prefeitura de Quatro Ventos em janeiro de 2020.

Fonte: C4, 9° ano, p. 52

• Exemplos de Cartograma

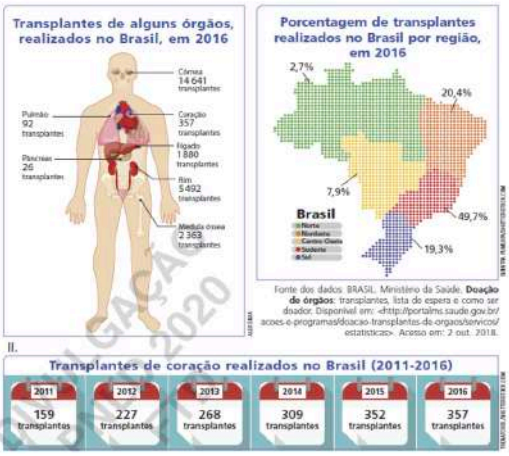
Elaborado a partir de: Climatempo. Disponível em: <https://www.climatempo.com.br>. Acesso em: 29 set. 2017.

Fonte: C3, 8° ano, p. 71



Fonte: C3, 9° ano, p. 109

• Exemplos de Infográfico



Fonte: C5, 8° ano, p. 192

O que fazer com as variáveis quantitativas ou numéricas?

Com variáveis numéricas podemos lidar com **valores discretos** (ano de nascimento das crianças em uma sala de aula, quantas horas pratica esportes por semana, etc.) ou **valores contínuos** (medição de altura ou mão das crianças em uma sala de aula)

- A variável quantitativa ou numérica indica as **quantidades**,
- Podemos **operar** com seus valores
- A forma de distribuição pode ser resumida numericamente através de: a **média**, a **mediana**, a **moda**, a **amplitude** e sua variabilidade
- Eles podem ser representados através dos **histogramas ou gráficos de barras**

Medidas de dispersão: amplitude

"A **amplitude** de um conjunto, em Estatística, é a diferença entre o maior elemento desse conjunto e o menor. Em outras palavras, para encontrar a amplitude de uma lista de números, basta subtrair o menor elemento do maior."

Exemplo:

→ 46,025 (São Paulo) – 0,635 (Roraima) = 45,39
→ (em milhões

Em um livro didático ...

Variância

Para definir a variância de um conjunto de dados, precisamos antes conhecer o valor do **desvio** de cada dado. O desvio é calculado pela diferença entre cada dado e a média do conjunto.

Por exemplo, em um grupo de 3 pessoas com 5 anos, 15 anos e 40 anos, temos a média de 20 anos. Então, o desvio do valor 5 anos é -15 ($5 - 20 = -15$), o desvio do valor 15 anos é -5 ($15 - 20 = -5$) e o desvio do valor 40 anos é 20 ($40 - 20 = 20$).

Assim, definimos a variância.

A **variância (V)** de um conjunto de dados é a média aritmética dos quadrados dos desvios.

Por exemplo, vamos calcular a variância nos grupos **A**, **B** e **C** citados na página anterior.

- Grupo **A**: 20, 20, 20, 20, 20, 20.
  $MA = 20$ Desvios: $20 - 20 = 0$; todos iguais a 0. $V = 0$
  Quando todos os valores são iguais, dizemos que não houve dispersão e, por isso, a variância é igual a 0.
- Grupo **B**: 22, 23, 18, 19, 20, 18.
  $MA = 20$
  Desvios: $22 - 20 = 2$; $23 - 20 = 3$; $18 - 20 = -2$; $19 - 20 = -1$; $20 - 20 = 0$; $18 - 20 = -2$
  Como a variância é a média aritmética dos quadrados dos desvios, temos:
  $$V = \frac{2^2 + 3^2 + (-2)^2 + (-1)^2 + 0^2 + (-2)^2}{6} = \frac{4 + 9 + 4 + 1 + 0 + 4}{6} = \frac{22}{6} = 3{,}6$$
- Grupo **C**: 6, 62, 39, 4, 8, 1.
  $MA = 20$
  Desvios: $6 - 20 = -14$; $62 - 20 = 42$; $39 - 20 = 19$; $4 - 20 = -16$; $8 - 20 = -12$; $1 - 20 = -19$.
  $$V = \frac{(-14)^2 + 42^2 + 19^2 + (-16)^2 + (-12)^2 + (-19)^2}{6} = \frac{3\,082}{6} \simeq 513{,}6$$

A variância é suficiente para diferenciar a dispersão dos grupos: o grupo A
grupo **C** tem uma dispersão maior do que a do grupo **B** ($513{,}6 > 3{,}6$).

Porém, não é possível expressar a variância na mesma unidade dos valore
da variável, pois os desvios dos dados são elevados ao quadrado. Então, defini
-se outra medida de dispersão, chamada **desvio-padrão**.

Em um livro didático ...

Desvio-padrão

O **desvio-padrão (*DP*)** é a raiz quadrada da variância.

O desvio-padrão facilita a interpretação dos dados, pois é expresso na mesma unidade dos valores observados (do conjunto de dados).

No exemplo que estamos analisando, vamos usar uma calculadora e a aproximação racional nas raízes quadradas.

- Grupo **A**: $DP = \sqrt{0} = 0$ (A dispersão é de 0 ano. Não há dispersão.)
- Grupo **B**: $DP = \sqrt{3{,}6} \simeq 1{,}9$ (A dispersão é de, aproximadamente, 1,9 ano. É uma pequena dispersão.)
- Grupo **C**: $DP = \sqrt{513{,}6} \simeq 22{,}6$ (A dispersão é de, aproximadamente, 22,6 anos. É uma dispersão grande.)

A variância e o desvio-padrão são sempre números positivos ou nulos.

O que fazer com as variáveis qualitativas ou categóricas?

- As variáveis qualitativas ou categóricas indicam atributos ou qualidades
- **Não podemos operar** com elas. Não são permitidas operações numéricas.
- Dependendo do uso em escala, alguma ordem pode ser distinguida (ordinal), mas em outra não (nominal).
- Podemos determinar:
  - ✓ **frequências** [frequências relativa e absoluta], acompanhadas **da tabela de distribuição de frequências**
  - ✓ Os **gráficos de barras (ou colunas)**, **pictogramas**, e **gráficos de setores**
  - ✓ a **moda**: o valor em um conjunto de dados que ocorre com mais frequência

II – Construções e análises dos gráficos por grupos

41

Comentários – Grupo 1

Atualização Gráfica: O gráfico da distribuição de recursos tem certa atualização depois de tantos anos ou segue sem atualização desde a última disponibilização dos dados que foi em 2018 ? Que questionamentos podemos fazer em relação a esta comparação (sobre o gráfico do exercício do livro e o gráfico com dados atualizados)?

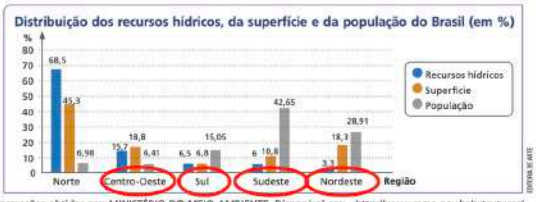
Questões derivadas:

Qual a quantidade de recursos hídricos do Brasil?

Qual a quantidade de recursos hídricos do Brasil por regiões?
Onde encontramos dados mais atualizados sobre isso (fonte de dados)?

Qual a superfície territorial brasileira?
Qual a superfície territorial brasileira por regiões?
Onde encontramos dados mais atualizados sobre isso (fonte dos dados)?

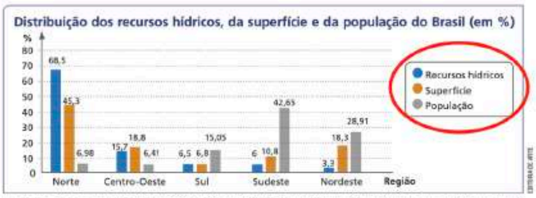
Em quais bases de dados encontramos dados atualizados?



Informações obtidas em: MINISTÉRIO DO MEIO AMBIENTE. Disponível em: <http://www.mma.gov.br/estruturas/sedr_proecotur/_publicacao/140_publicacao09062009025910.pdf>. Acesso em: 1º jul. 2018.

?

Região	Distribuição Populacion	Distribuição Territorial	Distribuição de Recursos Hidrografic
Região Norte	9%	45,25%	60,15%
Região Nordeste	27%	18,23%	10,20%
Região Sudeste	42%	10,86%	7,80%
Região Sul	15%	6,77%	13,20%
Região Centro-Oeste	8%	18,87%	8,65%



Informações obtidas em: MINISTÉRIO DO MEIO AMBIENTE. Disponível em: <http://www.mma.gov.br/estruturas/sedr_proecotur/_publicacao/140_publicacao09062009025910.pdf>. Acesso em: 1º jul. 2018.

?

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Região Sul	15%	6,77%	13,20%
Região Centro-Oeste	8%	18,87%	8,65%

Comentários – Grupo 1

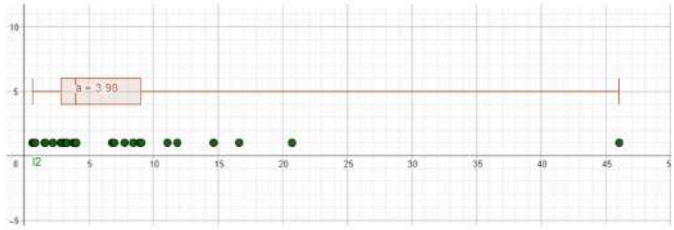
Sugestões:

Reconstruir o gráfico corrigindo os erros, e focar na questão para realizar a análise e construir uma resposta.

Atualização Gráfica: O gráfico da distribuição de recursos tem certa atualização depois de tantos anos ou segue sem atualização desde a última disponibilização dos dados que foi em 2018 ? Que questionamentos podemos fazer em relação a esta comparação (sobre o gráfico do exercício do livro e o gráfico com dados atualizados)?

Comentários – Grupos 2 e 3

Olhando as distribuições de recursos hídricos e população por estados brasileiros (26 estados + DF), como se comporta esta distribuição segundo dados mais atuais (2022)?



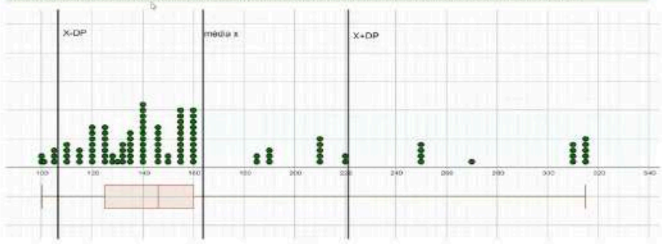
Sugestões:

Focar na questão para realizar a análise e construir uma resposta.

Um exemplo...

Uma pesquisa realizada com 106 apartamentos à venda em um bairro da cidade, com relação ao tamanho, apontou os seguintes valores em metros quadrados:

Quais são as informações representadas? Qual a amplitude dos dados? Qual a região de concentração? O que significa o intervalo da média e desvio-padrão? O que significa o boxplot? Qual informação a localização da média fornece?



A partir da observação dessas representações, redija um texto apresentando os dados para um interessado em comprar apartamento.

Exemplo retirada a partir de uma formação continuada realizada pela PUC-SP – 2022

- **Quais informações representadas?**
  Temos um gráfico de pontos, onde cada ponto verde representa um apartamento e seu respectivo tamanho (em metros quadrados) apresentados no eixo horizontal. Junto, temos o boxplot (diagrama de caixa) que representa a distribuição dos dados divididos em quatros partes (que são os quartis), a linha vertical vermelha dentro do retângulo é a mediana Q2 (segundo quartil) e os lados verticais paralelos do retângulo representam o Q1 (primeiro quartil) e Q3 (terceiro quartil).
  Além disso, o "x" é a média e o DP o desvio padrão.

- **O que significa o boxplot?** O boxplot ordena os dados em ordem crescente e os divide em 4 partes (quartis). A mediana é o segundo quartil (Q2) que, nessa situação, é aproximadamente 145, ou seja, ao menos 50% dos dados estão abaixo disso e a outra metade acima. O primeiro quartil é aproximadamente 125, o terceiro quartil é 160. Então conseguimos dizer a distribuição dos dados: o valor mínimo parece ser 100 e o último valor (que chamamos de "bigode" é aproximadamente 315).

- Então a **região de concentração** dos dados está no intervalo entre 100 e 160 (75% dos dados).

- **Amplitude:** $\simeq (315 - 100) = 215$. A amplitude é a diferença entre o maior valor e o menor.

- **O que significa o intervalo da média e do desvio padrão?** Em média temos que neste bairro os apartamentos a venda possuem aproximadamente 164m². Porém, nota-se que temos um gráfico boxplox assimétrico, pois possuímos dados bem dispersos, logo, os valores dos dados extremos influenciam no resultado da média. O desvio padrão [Desvio padrão $= (220 - 164) = 56$] mostra a dispersão dos dados em relação à média, isto é, como está a distribuição destes. Quanto maior esse valor, maior é a dispersão dos dados. Neste caso, os dados não estão "bem" distribuídos. Como vimos, 75% dos dados estão entre 100 e 160, e a média é de aproximadamente 164. Era de se esperar que a média estivesse neste intervalo, entretanto ela é maior, isso mostra que existem alguns valores maiores (como os que estão situados entre 300 e 320) que afetam o cálculo da média. Por exemplo, dizer a um comprador que *em média os apartamentos possuem 164 m²* não seria o mais adequado, até porque não há apartamentos nessa média (164). A média não é o valor típico. A média é fortemente influenciada pelos extremos.

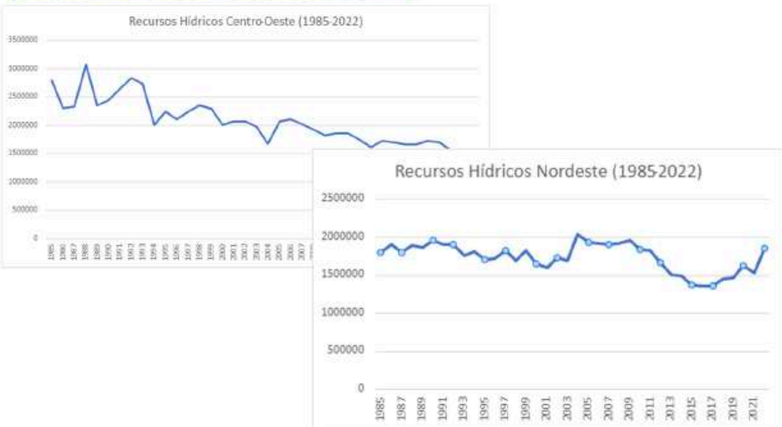
A partir da descrição das informações fornecidas pelos gráficos, recomendamos:

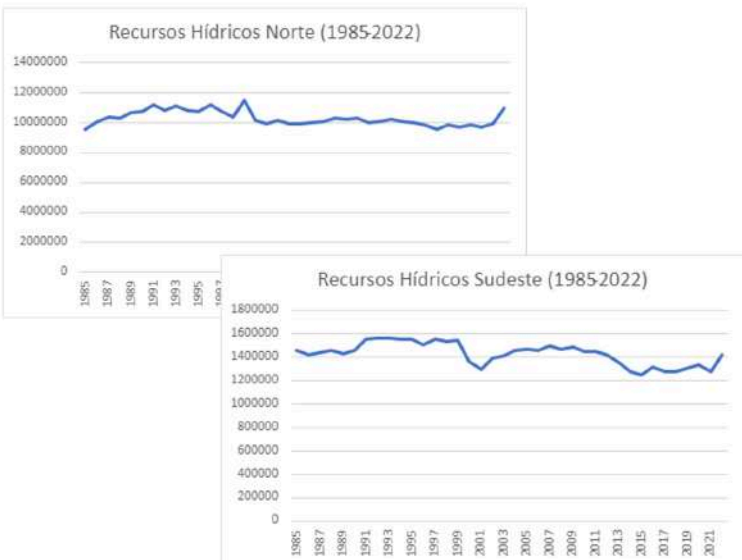
Será mais fácil encontrar apartamentos de 100 a 160 m^2. Como há mais opções nessa faixa, pode ser possível achar preços mais "em conta" e negociar.

Podíamos inferir que como tem poucos apartamentos acima de 160 m^2, a procura deve ser maior e o preço alto.

Comentários – Grupo 4

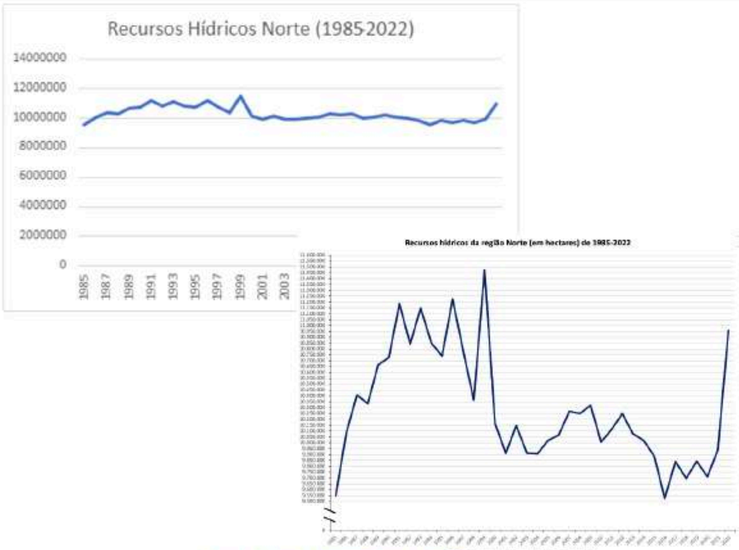
Como era a distribuição, no Brasil, em anos anteriores a pesquisa atual? Que comparações podemos fazer em relação a anos anteriores?





Sugestões:

- Reconstruir os gráficos com "quebra no eixo vertical" e todos com a mesma escala (por exemplo, 50.000);
- Focar na questão para realizar a análise e construir uma resposta.



Exemplo de construção: https://youtu.be/4fuQJw2qf2g

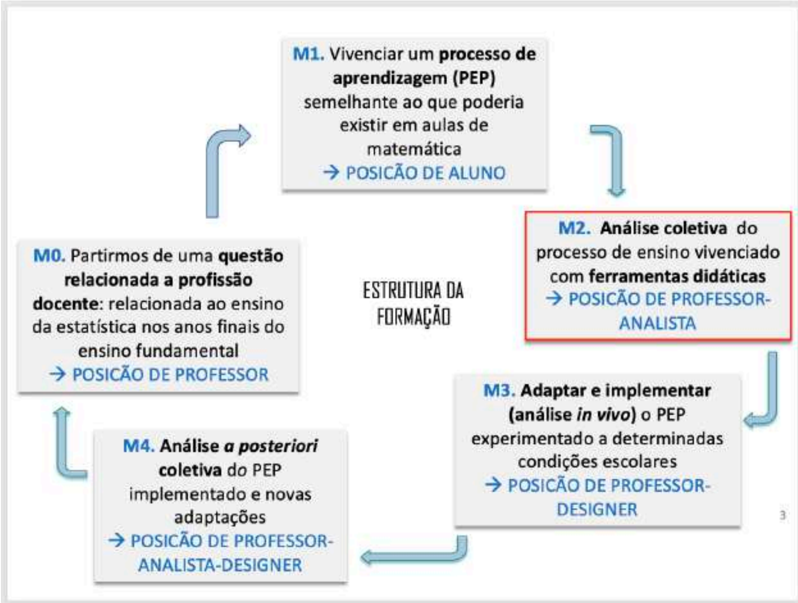
Articulações entre aspectos teóricos, pedagógicos e didáticos no ensino de estatística na perspectiva do questionamento do mundo

Projeto de Ensino de Graduação

Coordenadora: Sonia Maria Monteiro da Silva Burigato
Investigadoras/professoras participantes:
Marilena Bittar
Janielly Taila dos Santos Verbisck

Sessão 7- 05 de maio de 2023
Campo Grande, MS

CAPES · UFMS FUNDAÇÃO UNIVERSIDADE FEDERAL DE MATO GROSSO DO SUL · UNIVERSITAT DE BARCELONA



Calendário de encontros

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8	12/05/2023	Adaptações para implementação em sala de aula	M2 e M3
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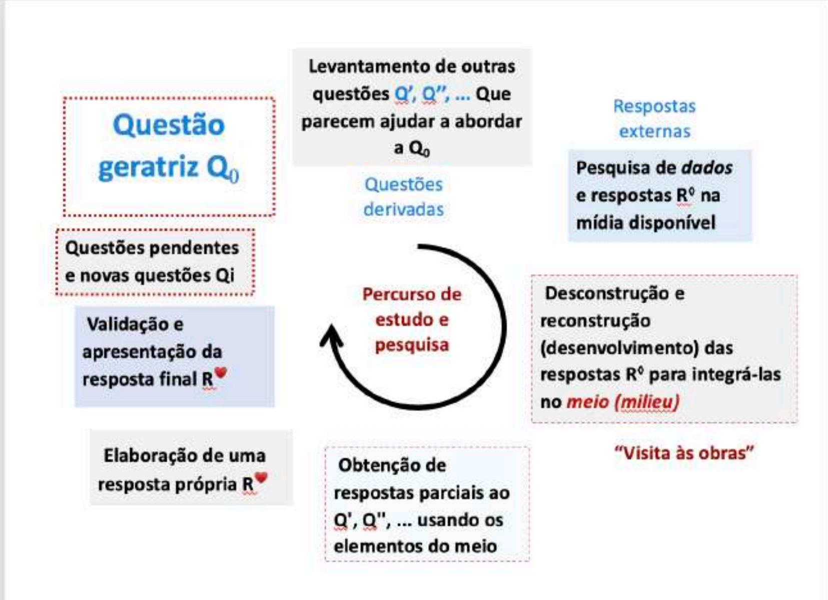
10	26/05/2023	Adaptações para implementação em sala de aula	M3
11	02/06/2023	Implementações em sala de aula	M3
12	09/06/2023 Não letivo	Implementações em sala de aula	M3
13	16/06/2023	Implementações em sala de aula	M3
14	23/06/2023	Discussões e análises finais	M4
15	30/06/2023	Discussões e análises finais	M4
16	07/07/2023	Discussões e análises finais (online)	M4

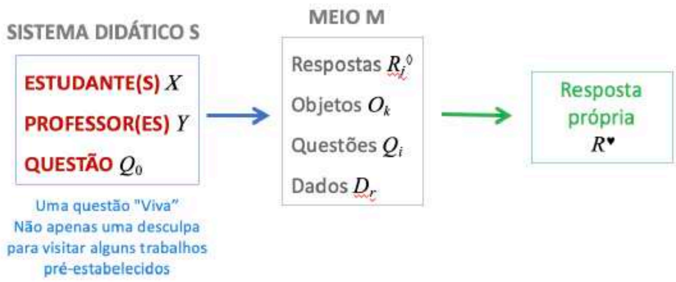
Na sessão passada...

Fizemos algumas retomadas:

- Aspectos conceituais
- Comentários sobre as construções análises de gráficos de cada grupo
- Reorganização do PEP no Mapa de questões-respostas nos Padlets.



Esquema herbartiano



$$[S(X;Y;Q_0) \rightarrow \{R_j^\lozenge, O_k, Q_i, D_r\}] \hookrightarrow R^\heartsuit$$

Esquema herbartiano

- Uma **questão** Q_0 (a questão 'geratriz') e um grupo de **estudantes** X e **supervisor(es)** Y

  → sistema didático $S(X;\ Y;\ Q_0)$

- O objetivo é elaborar sua própria **resposta** (coletiva) $R^\heartsuit$ para Q_0

$$[S(X;\ Y;\ Q_0)] \hookrightarrow R^\heartsuit$$

- Para elaborar $R^\heartsuit$, o Sistema didático cria um *meio (milieu)* M:

$$[S(X;\ Y;\ Q_0) \curvearrowright \mathrm{M}] \hookrightarrow R^\heartsuit$$

- O **meio (milieu)** $\boldsymbol{M}$ **evolui** durante o processo de investigação.

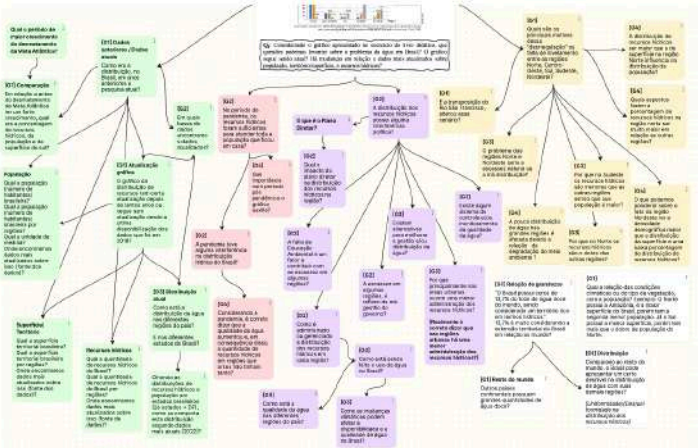
$$[S(X;Y;Q_0) \rightarrow \{R_j^\lozenge, O_k, Q_i, D_r\}] \hookrightarrow R^\heartsuit$$

EXEMPLOS DE S(X, Y, Q)

- Professor com alunos resolvendo um problema
- Estudante de doutorado e o problema de tesis
- Professores com educadores sobre o ensino de estatística e probabilidade no ensino médio
- Filhos com seus pais para aprender como prender seus sapatos
- Dois amigos, um ensinando português ao outro

$[\boldsymbol{S(X;Y;Q_0)} \rightarrow \{R_j^{\Diamond}, O_k, Q_i, D_r\}] \rightarrow R^{\heartsuit}$

EXEMPLOS DE Q_0 e Q', Q'', etc...



$[S(X;Y;\boldsymbol{Q_0}) \rightarrow \{R_j^{\Diamond}, O_k, \boldsymbol{Q_i}, D_r\}] \rightarrow R^{\heartsuit}$

EXEMPLOS DE OBJETOS

- Todos os conhecimentos que já temos
- Excel, calculadoras, celular, computador
- Bases de dados
- Artigos, livros, expertos ao nosso redor

$[S(X;Y;Q_0) \rightarrow \{R_j^{\Diamond}, \boldsymbol{O_k}, Q_i, D_r\}] \rightarrow R^{\heartsuit}$

EXEMPLOS DE DADOS

Prévia da população calculada com base nos resultados do Censo Demográfico 2022 até 25 de dezembro de 2022	
BRASIL E UNIDADES DA FEDERAÇÃO	**POPULAÇÃO**
Brasil	**207.750.291**
Região Norte	**17.834.762**
Rondônia	1.616.379
Acre	829.780
Amazonas	3.952.262
Roraima	634.805
Pará	8.442.962
Amapá	774.268
Tocantins	1.584.306
Região Nordeste	**55.389.382**
Maranhão	6.800.605
Piauí	3.270.174
Ceará	8.936.431
Rio Grande do Norte	3.303.953
Paraíba	4.030.961
Pernambuco	9.051.113
Alagoas	3.125.254
Sergipe	2.211.868
Bahia	14.659.023
Região Sudeste	**87.348.223**
Minas Gerais	20.732.660
Espírito Santo	3.975.100
Rio de Janeiro	16.615.526
São Paulo	46.024.937
Região Sul	**30.685.598**
Paraná	11.835.379
Santa Catarina	7.762.154
Rio Grande do Sul	11.088.065
Região Centro-Oeste	**16.492.326**
Mato Grosso do Sul	2.833.742
Mato Grosso	3.784.239
Goiás	6.950.976
Distrito Federal	2.923.369

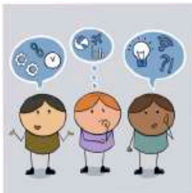
Fonte: IBGE. Diretoria de Pesquisas - DPE - Coordenação Técnica do Censo Demográfico - CTD

Ano	Norte	Centro-Oeste	Nordeste	Sul	Sudeste	Total
1985	9.550.744,05	2.793.413,43	1.806.214,43	2.390.959,46	1.455.553,59	17.996.884,96
1986	10.091.713,08	2.312.419,49	1.906.156,55	2.393.714,83	1.421.075,57	18.125.079,53
1987	10.406.819,42	2.332.338,18	1.805.553,47	2.424.560,49	1.445.090,35	18.414.961,91
1988	10.335.122,86	3.075.002,48	1.898.275,39	2.407.378,62	1.456.186,61	19.171.965,96
1989	10.664.379,79	2.362.806,57	1.872.914,30	2.394.640,35	1.435.031,10	18.729.571,92
1990	10.729.683,58	2.440.949,40	1.959.873,15	2.423.997,69	1.463.823,00	19.018.326,82
1991	11.187.162,66	2.654.824,58	1.901.811,26	2.419.145,29	1.556.108,30	19.719.050,09
1992	10.842.121,76	2.841.410,47	1.907.527,95	2.438.036,53	1.564.099,96	19.593.196,67
1993	11.149.072,46	2.743.347,33	1.763.444,31	2.440.586,55	1.562.898,53	19.659.349,18
1994	10.851.602,22	2.012.902,27	1.808.188,48	2.438.565,47	1.558.718,21	18.669.974,66
1995	10.740.153,48	2.250.639,92	1.711.498,87	2.433.503,38	1.554.419,09	18.690.214,73
1996	11.224.947,50	2.111.116,35	1.723.254,17	2.435.822,63	1.509.885,93	19.005.026,59
1997	10.777.898,29	2.241.511,29	1.830.739,57	2.431.989,95	1.556.678,67	18.838.817,77
1998	10.366.763,20	2.358.200,18	1.690.269,85	2.460.938,65	1.535.079,27	18.411.251,13
1999	11.472.858,79	2.301.801,04	1.821.823,27	2.479.468,95	1.545.320,76	19.621.272,80
2000	10.166.407,09	2.009.313,67	1.656.109,31	2.380.849,84	1.363.386,87	17.576.066,78
2001	9.910.448,87	2.073.814,02	1.602.188,42	2.388.965,20	1.294.996,67	17.270.413,19
2002	10.147.105,63	2.070.126,70	1.733.216,74	2.393.402,34	1.394.524,42	17.738.375,83
2003	9.912.390,95	1.978.728,64	1.698.789,19	2.397.212,37	1.414.779,39	17.401.900,55
2004	9.907.514,46	2.065.932,91	2.043.707,13	2.379.619,54	1.457.953,30	17.854.727,34
2005	10.021.026,28	2.071.377,19	1.930.599,00	2.384.028,29	1.471.677,16	17.878.707,93
2006	10.066.067,34	2.107.615,91	1.924.576,49	2.371.906,69	1.464.550,85	17.934.717,28
2007	10.268.679,18	2.022.948,27	1.911.326,97	2.396.705,06	1.499.959,30	18.099.618,78
2008	10.249.130,03	1.925.415,92	1.926.148,54	2.387.516,83	1.468.254,19	17.956.465,51
2009	10.321.281,97	1.827.533,83	1.966.203,34	2.388.346,15	1.484.341,74	17.987.707,02
2010	10.006.179,20	1.861.197,40	1.839.291,73	2.411.143,62	1.446.590,02	17.564.401,98
2011	10.117.480,06	1.861.548,00	1.824.797,48	2.405.949,66	1.451.463,02	17.661.238,21
2012	10.248.685,40	1.750.949,84	1.671.759,78	2.388.633,47	1.425.350,29	17.485.378,78
2013	10.077.710,13	1.613.656,66	1.513.386,48	2.408.641,12	1.353.087,84	16.966.482,22
2014	10.021.440,31	1.725.683,88	1.496.975,49	2.417.527,97	1.280.465,60	16.942.093,25
2015	9.887.994,40	1.699.594,09	1.373.907,98	2.421.871,15	1.249.848,57	16.633.216,18
2016	9.525.771,22	1.665.714,37	1.354.460,28	2.433.277,68	1.320.773,39	16.299.996,93
2017	9.840.565,23	1.663.425,66	1.357.987,54	2.424.680,46	1.282.747,02	16.569.405,91
2018	9.697.253,31	1.729.522,92	1.450.685,10	2.414.525,99	1.282.146,74	16.574.134,00
2019	9.842.852,84	1.697.227,10	1.468.296,44	2.417.876,81	1.303.022,82	16.729.276,01
2020	9.710.126,95	1.565.211,69	1.623.029,38	2.398.945,42	1.334.258,42	16.631.571,85

$$[S(X;Y;Q_0) \rightarrow \{R_j^{\lozenge}, O_k, Q_i, \mathbf{D_r}\}] \rightarrow R^{\heartsuit}$$

EXEMPLOS Respostas externas $R^{\lozenge}$

- Obras que acreditamos ser úteis e vamos estudar
  - Recursos hídricos no Brasil
  - Construção de tabelas: Quais os principais elementos de uma tabela?
  - Gráficos estatísticos - analisar e decidir que tipo de gráficos construir para a representação dos dados; Quais os principais elementos de um gráfico?
  - É melhor construir gráficos de barras ou gráficos de linha para representar estes dados? Por quê? Com series temporais é mais habitual construir gráficos de linhas. Há informações sobre este tema?
  - Gráficos com series tomam valores distintos. Como comparar?
  - Comparações de perda em porcentagens (1990 - 2022), cálculo da variação anual (absoluta e em porcentagens)
  - ...



$$[S(X;Y;Q_0) \rightarrow \{\mathbf{R_j^{\lozenge}}, O_k, Q_i, D_r\}] \rightarrow R^{\heartsuit}$$

Esquema herbartiano

- Elementos do processo de estudo (investigação)

$$[S(X;Y;Q_0) \rightarrow \{R_j^{\lozenge}, O_k, Q_i, D_r\}] \rightarrow R^{\heartsuit}$$

Dialéticas

- Dinâmicas do processo de estudo (investigação)
  - Questões e respostas
  - Mídia e meio
  - Indivíduo e coletivo

Dialética questões e respostas

- Para abordar uma questão, geram-se questões derivadas
- Buscam-se respostas já disponíveis (em nosso meio ou em mídias acessíveis)
- Estas respostas são estudadas, validadas e desenvolvidas
- Durante o estudo surgem novas questões
- É um processo infinito!
- Obs.: no contexto escolar, são valorizadas muito mais as respostas já estabelecidas e pouco as perguntas. Precisamos tentar atuar em direção contrária, valorizando as questões que motivaram a resposta.

$$[S(X;Y;Q_0) \rightarrow \{R_j^{\lozenge}, O_k, Q_i, D_r\}] \rightarrowtail R^{\heartsuit}$$

Dialética mídia e meio

- Durante a pesquisa, utilizamos todos os conhecimentos e instrumentos que temos disponíveis em nosso meio. São coisas seguras, são coisas seguras, sobre as quais não temos (ou temos pouca) dúvida.
- Para avançar, necessitamos novas informações, respostas, conhecimentos, instrumentos, etc., que buscamos em distintas fontes de informação: as mídias.
- Estas informações, conhecimentos, instrumentos, etc. não são nossos. Temos que estuda-los, desconstruir, reconstruir, validar para integrá-los em nosso meio. Nos "apropriamos" deles para poder explorá-los na elaboração de respostas.

$$[S(X;Y;Q_0) \rightarrow \{R_j^{\lozenge}, O_k, Q_i, D_r\}] \rightarrowtail R^{\heartsuit}$$

Dialética mídia e meio

- Deste modo, o que encontramos nas mídias são incorporados (parcialmente) em nosso meio e este vai evoluindo.
- Também devemos ser capazes de obter novas informações de nosso meio, utilizá-las como mídia.
- Para analisar esta dialética, olhamos de onde saem as informações, dados e respostas externas e como seu acesso é gerenciado (mídia). Também perguntamos como são validadas e transformadas; e com que materiais elaboram-se as respostas próprias finais ou intermediárias (meio).

$$[S(X;Y;Q_0) \rightarrow \{R_j^{\lozenge}, O_k, Q_i, D_r\}] \rightarrowtail R^{\heartsuit}$$

Dialética indivíduo e coletivo

- A pesquisa é um processo coletivo realizado por X sob a direção de Y.
- Neste processo, pequenos grupos X_i são gerados e o trabalho individual x_i também é realizado. Os xi e os yi devem se organizar para trabalhar juntos
- Há muitos formatos de trabalho, desde o puramente individual (x_i coletando dados em seu computador) até o coletivo total (X e Y discutindo os resultados juntos).
- Também podemos falar de um meio coletivo (da sala/classe) e de meios individuais (de cada aluno): há aprendizado conjunto e aprendizado coletivo.
- Para analisar esta dialética, podemos perguntar sobre os papeis (quem faz o quê) nas descrições da dialética anterior.

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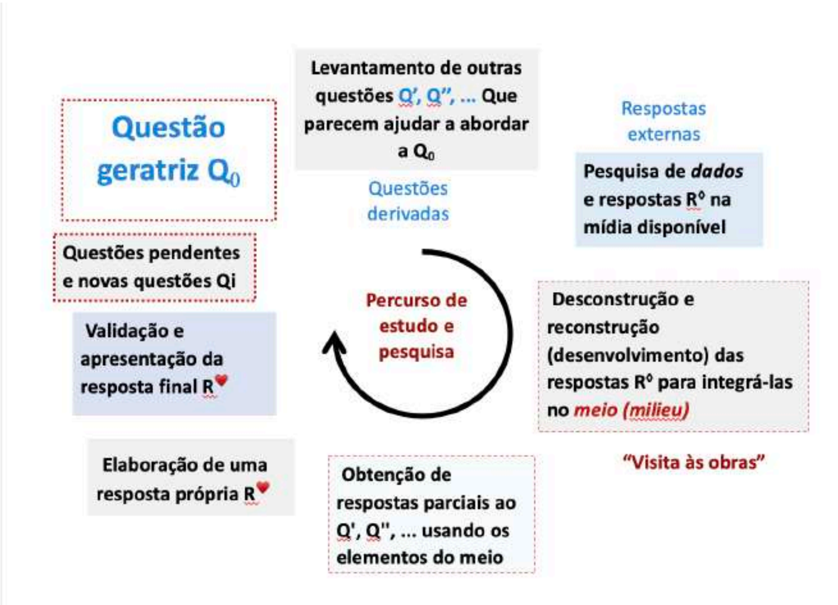


Conhecimentos estatísticos

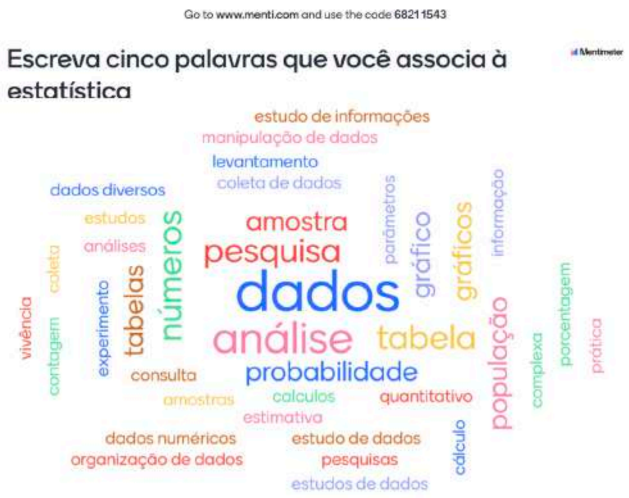
Segundo as análises dos grupos

- "Amostra: populacional, territorial e de recursos hídricos;
Mediana: utilizamos na análise do gráfico de caixa;
Porcentagem: na interpretação do gráfico da questão inicial proposta;
Boxplot e o dotplot: na construção dos gráficos."

- "Mediana utilizada no diagrama de caixa, amostra, população, média e variáveis."

- "Proporção, razão, amplitude, porcentagem."

Relembrando a sessão 1...



O tratamento de dados

"[...] todos os cidadãos precisam desenvolver habilidades **para coletar, organizar, representar, interpretar e analisar dados em uma variedade de contextos**, de maneira a fazer julgamentos bem fundamentados e tomar as decisões adequadas. Isso inclui raciocinar e utilizar conceitos, representações e índices estatísticos para descrever, explicar e predizer fenômenos."

(BNCC, p. 274)

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EDUCAÇÃO É A BASE

O tratamento de dados

"Com relação à estatística, os primeiros passos envolvem **o trabalho com a coleta e a organização de dados de uma pesquisa de interesse dos alunos**. O planejamento de como fazer a pesquisa ajuda a compreender o papel da estatística no cotidiano dos alunos. Assim, a leitura, a interpretação e a construção de tabelas e gráficos têm papel fundamental, bem como a forma de produção de texto escrito para a comunicação de dados, pois é preciso compreender que o texto deve sintetizar ou justificar as conclusões. No Ensino Fundamental – Anos Finais, a expectativa é que os alunos saibam planejar e construir relatórios de pesquisas estatísticas descritivas, incluindo medidas de tendência central e construção de tabelas e diversos tipos de gráfico. Esse planejamento inclui **a definição de questões relevantes e da população a ser pesquisada**, a decisão sobre a necessidade ou não de usar amostra e, quando for o caso, a seleção de seus elementos por meio de uma adequada técnica de amostragem. (BNCC, p. 275)

BASE NACIONAL COMUM CURRICULAR

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Objetos de conhecimento e habilidades

Anos finais do ensino fundamental

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Objetos de conhecimento	Habilidades
Coleta de dados, organização e registro. Construção de diferentes tipos de gráficos para representá-los e interpretação das informações.	(EF**06**MA33) Planejar e **coletar dados de pesquisa** referente a práticas sociais escolhidas pelos alunos e **fazer uso de planilhas eletrônicas para registro, representação e interpretação das informações**, em tabelas, vários tipos de gráficos e texto.
Planejamento de pesquisa, coleta e organização dos dados, construção de tabelas e gráficos e interpretação das informações	(EF**07**MA36) Planejar e realizar pesquisa envolvendo **tema da realidade social**, identificando a necessidade de ser censitária ou de usar amostra, e **interpretar os dados** para comunicá-los por meio de relatório escrito, tabelas e gráficos, com o apoio de planilhas eletrônicas.

Competência específica e habilidades
Ensino médio

Competência específica	Habilidades
Articular conhecimentos matemáticos ao propor e/ou **participar de ações para investigar desafios do mundo contemporâneo** e tomar decisões éticas e socialmente responsáveis, com base na **análise de problemas de urgência social**, como os voltados a situações de saúde, sustentabilidade, [...] entre outros, recorrendo a conceitos, procedimentos e linguagens próprios da Matemática.	(EM13MAT202) Planejar e executar pesquisa amostral sobre questões relevantes, **usando dados coletados diretamente ou em diferentes fontes**, e comunicar os resultados por meio de relatório contendo gráficos e interpretação das medidas de tendência central e das medidas de dispersão (amplitude e desvio padrão), utilizando ou não recursos tecnológicos.
(EM13MAT408) Construir e interpretar tabelas e gráficos de frequências, **com base em dados obtidos em pesquisas por amostras estatísticas**, incluindo ou não o uso de softwares que inter-relacionem estatística, geometria e álgebra.	
(EM13MAT409) Interpretar e comparar **conjuntos de dados estatísticos** por meio de diferentes diagramas e gráficos, como o histograma, o de caixa (box-plot), o de ramos e folhas, reconhecendo os mais eficientes para sua análise.	

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EDUCAÇÃO É A BASE

Conhecimentos estatísticos

- Pesquisa, Coleta de dados – em banco de dados oficial;
- Organização dos dados – tabelas no Excel, ...;
- Construção de gráficos – principais elementos de um gráfico, escalas, gráficos de linhas (ou segmentos), gráficos de barras, gráficos de pontos (dotplot), gráficos de caixa (boxplot);
- Análise dos gráficos construídos;
- Frequências absolutas e frequências relativas;
- Medidas de tendência central – média e mediana;

Atividade Módulo 3: Adaptações para implementação em sala de aula

- Trabalho (em seus grupos) para propor um projeto de atividade adaptado aos alunos do ensino fundamental ou ensino médio.
- Os grupos pensarão em:
  - Questão inicial da atividade de investigação.
  - Como se coloca a questão, o que se propõe fazer e que ferramentas estão disponíveis?
  - Como o estudo está planejado para ser administrado em classe: horário aproximado
  - Como se espera que a atividade seja concluída
  - Qual/quais conteúdo(s) curricular(es) (em matemática e outros assuntos) pode(m) ser abordado(s) pela atividade proposta.

Sessão 14

Sessões	Datas previstas	Atividades propostas	Módulos
1	17/03/2023	Introdução do projeto de ensino + $Q_{0\text{-}FP}$	M0
2	24/03/2023	Apresentação da Q_0 e início do PEP	M1
3	31/03/2023	Investigação sobre Q_0: trabalho com bancos de dados. Novas questões derivadas	M1
4	14/04/2023	Apresentação das respostas finais da Q_0	M1
5	21/04/2023 Não letivo	Assíncrono – finalização dos relatórios	M1 – M2
6	28/04/2023	Análise do processo de aprendizagem vivenciado	M2
7	05/05/2023	Análise do processo de aprendizagem vivenciado	M2
8	12/05/2023	Análise + Adaptações para sala de aula	M2 – M3
9	19/05/2023	Adaptações para implementação em sala de aula	M3
10	26/05/2023	Adaptações para implementação em sala de aula	M3
11	02/06/2023	Implementações em sala de aula	M3
12	09/06/2023 Não letivo	Implementações em sala de aula	M3
13	16/06/2023	Implementações em sala de aula	M3
14	23/06/2023	Implementações em sala de aula + Discussões e análises finais	M3 – M4
15	30/06/2023	Discussões e análises finais + Festa de despedida	M4
16	07/07/2023	Elaboração de relatório final sobre análise *a posteriori*	M4

Grupo 3 – andamento e finalização da implementação

- Aulas 5 e 6 (01/06/23): Neste dia não estavam presentes todos os integrantes de cada grupo;
- Os 7 grupos se reuniram novamente para finalizar a atividade (com a elaboração do cartaz de apresentação dos resultados)

Grupo 3

Vamos relembrar as questões levantadas por vocês em nosso primeiro encontro?

Questões escolares/ pedagógicas	Questões didáticas	Questões estatísticas
Como tornar o ensino da estatística mais atrativo para os alunos do E. F II?	Por que um foco maior no tratamento da informação?	O que é população?
Como ensinar estatística, frente a realidade da defasagem de aprendizado dos alunos do ensino publico?	A estatística do ensino fundamental é antiga e não evolui, acaba não cativando os alunos.	O que é amostra?
Frente com o ensino pós pandemia acho desafiador implementar estatística no ensino fundamental 2	Como aplicar a modelagem no ensino da estatística?	O que é dispersão?
Como fazer os alunos se interessarem por estatística?	Atividades envolvendo modelagem são mais fáceis na construção do conhecimento estatístico do que as aulas envolvendo "apenas" os cálculos?	
Como abordar de maneira didática e interdisciplinar a estátistica no ensino médio para cativar os alunos ?	Na estatística do ensino médio percebe-se uma pequena mudança do fundamental, levando o aluno a ter menos compreensão da importância de estatística na sociedade.	
Como ensinar de forma intuitiva a estatística para o E.M ?		
Como integrar com outras áreas do conhecimento?		
Quais ferramentas posso usar para o ensino de estatística?		
Qual a melhor forma de ensinar a estatística?		

Ecologia: condições e restrições

- Nossa Q_0 do M0 se centra em questionar: **Como ensinar estatística nos anos finais do ensino fundamental e ensino médio?**

- Esta questão evolui quando questionamos o conhecimento a ser ensinado: **Como evitar que o ensino de estatística (no EF e EM) se limite a um conjunto isolado de conceitos e técnicas em vez de ser uma ferramenta para resolver (ou investigar) questões relevantes da nossa sociedade?**

Ecologia: condições e restrições

Em nossas pesquisas inseridas na TAD e com enfoque na transição de paradigmas (da visita às obras para o questionamento do mundo), buscamos realizar uma **análise ecológica**:

- E mudar a razão de ser do conhecimento implica claramente uma mudança na forma como ele é concebido. Mas essa mudança não é trivial, ela é marcada por:
  - Condições que podem favorecer a transição
  - Restrições que impedem ou dificultam

Ecologia: condições e restrições

Estas condições/restrições não são todas controláveis e modificáveis pelo professor. Pensar isso seria ilusório e acrescentaria uma falsa pressão aos professores.

Uma ferramenta teórica crucial para analisar de onde vêm as condições e restrições é a **escala de níveis de codeterminação didática** (Chevallard, 2002).

Ecologia: condições e restrições

Níveis genéricos / superiores

Civilização Sociedades Escolas Pedagogias

Disciplinas Domínios Setores Temas Questões

Níveis específicos / inferiores

Ecologia: condições e restrições

Por exemplo, podemos caracterizar algumas restrições ao nível pedagógico no EF e EM em relação ao ensino de estatística (e dos demais domínios):

- As questões que justificam a razão de ser de um conteúdo curricular tendem a ficar implícitas;
- O intervalo de validade das técnicas apresentadas não costuma ser objeto de estudo;
- Se forem estudadas questões, o mais comum é que sejam necessárias apenas as técnicas fornecidas pelo docente.

Ecologia: condições e restrições

Uma última tarefa para casa...

1) Reunam-se em seus respectivos grupos;

2) Discutam e respondam a questão: a atividade que nós implementamos na escola foi, de fato, um PEP? Justifiquem.

3) Identifiquem condições (favoráveis) e restrições que impediram a implementação de percursos de estudo e pesquisa nas aulas que vocês ministraram. Pode ser qualquer tipo de aspecto: desde o número de alunos por turma, disponibilidade de recursos tecnológicos, motivação dos estudantes...

4) Tentem classificar e ver em que nível de codeterminação didática elas aparecem.

5) Preparem uma apresentação para a próxima sessão.